# SOME PROBLEMS CONCERNING THE MULTISTAGE STARTING OF CAGE INDUCTION MOTORS 

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## Summary

The transient switching current of three-phase cage motors is the smaller, the smaller is the phase shift between the applied voltage and the voltage induced by the action of the rotor currents in the moment of switch-over. Authors extend the analysis of the star-delta switching-well known from literature-to motors started in four steps and point out the importance of choosing the combination of connections the most advaniageous in respect of phase relationships.

## Introduction

The following phenomena occur during the multistage starting of cage induction motors: after switching off the stator, the flux maintained by rotor currents induces a voltage in the stator winding. The transient surge current due to switching on the new stage depends on the difference between the voltage applied to the phase winding and the voltage induced at that moment in the phase winding. This voltage difference is the smaller, the smaller is the phase shift between the two voltages. This phase shift can be, however, influenced by the appropriate choice of phase relationships of consecutive connections.

The analysis of the two-stage (star-delta) starting can be found in [1]. The phase relationships of the three- or fourstage starting have not be elaborated, as far as we know.

## Objective

The aim of this paper is to point out the importance of the choice of consecutive connection combinations from the viewpoint of the switch-over transient currents at the multistage starting of cage motors. The study of the problem became interesting by examining a starting difficulty met in practice.

## The time variation of the voltage remaining in the stator winding after switching off

The phenomena occurring in induction motors at switching off are treated in detail in [1] and [2]. The spacial vector of the rotor linkage $\psi_{r}$ maintains in the moment of fast switching off its value before switching off, after that its amplitude decays according to the open-circuit rotor time constant $T_{\text {ro }}$ and its angular velocity bound to the rotor decreases to value $\omega=\omega_{1}(1-s)$ :

$$
\begin{equation*}
\psi_{r}=\psi_{r o} \mathrm{e}^{-\frac{t}{T_{r o}} \mathrm{e}^{j \omega_{1}(1-s)}} \tag{1}
\end{equation*}
$$

The value of the slip $s$ is considered as being constant, because its change during the usually small switch-over time (e.g. 0.03 s ) can be neglected as related to the starting time of the drive.

The mutual part of the linkage $\psi_{r}$ induces a voltage in the stator winding, whose value based on the relationship $u_{s}=\frac{L_{m}}{L_{r}} \frac{\mathrm{~d} \psi_{r}}{\mathrm{~d} t}$ is given by

$$
\begin{equation*}
\mathbf{u}_{s}=\frac{L_{m}}{L_{r}} \frac{\mathrm{~d} \psi_{r}}{\mathrm{~d} t}=\frac{L_{m}}{L_{r}}\left[-\frac{1}{T_{r o}}+j \omega_{1}(1-s)\right] \mathrm{e}^{-\frac{t}{T_{r o}} \psi_{r o}{ }^{j \omega_{1}(1-s)}} \tag{2}
\end{equation*}
$$

with $\psi_{r o}$ - spatial vector of the rotor linkage in the moment of switching off
$\psi_{r}$ - spatial vector of the rotor linkage at the time $t$ after switching off
$\omega_{1}$ - angular frequency of the mains
$s$ - momentary value of the slip
$T_{\text {ro }}$ - open-circuit rotor time constant
$\mathbf{u}_{s}$ - spatial vector of the stator voltage
$L_{m} \quad$ - mutual inductance of the stator and rotor
$L_{r} \quad$ - total inductance of the rotor
The informatory values of numerical relationships at medium size machines ( $10-100 \mathrm{~kW}$ ) based on [1] are:

$$
\frac{L_{m}}{L_{r}} \approx 0.95 ; \quad s \approx 3 \% ; \quad T_{r o}=\frac{X_{r}}{\omega_{1} R_{r}} \approx 1.3 \mathrm{~s}
$$

The value of $\psi_{r o}$ in Eq. (2) with the preceding numerical values, is obtained from the complexor diagram for nominal load by the relationship:

$$
\begin{equation*}
j \omega_{1} \psi_{r o} \approx 0.9 \mathbf{u}_{s o} \mathrm{e}^{-j \varphi_{0}}, \quad \text { with } \quad \varphi_{0} \approx 10^{\circ} \tag{3}
\end{equation*}
$$

where
$\mathbf{u}_{s o}$ - spatial vector of the stator voltage at the moment of switching off. The switching off is considered momentary

If the complexor of the stator voltage in phase "a" was $\mathbf{U}_{a o}$ in the moment of switching off, its value based on Eq. (2) and (3) varies in the function of time as follows:

$$
\begin{equation*}
\mathbf{U}_{a i} \approx 0.95\left[\frac{j}{\omega_{1} T_{r o}}+(1-s)\right] \mathrm{e}^{-\frac{1}{T_{r o}} 0.9 \bar{U}_{a o} \mathrm{e}^{-j\left(\varphi_{0}+s \omega_{1} 1\right)}} \tag{4}
\end{equation*}
$$

The first member in square brackets can be neglected as compared to the second one.

Let us assume a time $t=0.03 \mathrm{~s}$ for the switching over. Here the ratio $t / T_{\text {ro }}$ can be neglected too, and the voltage will be, at the moment of switching on the next stage:

$$
\begin{equation*}
\mathbf{U}_{a i} \approx 0.83 \mathbf{U}_{a 0} \mathrm{e}^{-j \varphi} \tag{5}
\end{equation*}
$$

with:

$$
\varphi=\varphi_{0}+\frac{180}{\pi} \omega_{1} \quad s t \approx 10^{\circ}+16^{\circ}=26^{\circ}
$$

The transient current due to switching over will be the smallest under the considered conditions, if the phase voltage of the following stage lags by $26^{\circ}$ to the phase voltage of the preceding stage. The phase relationships are to be chosen in a way that the aforementioned condition could be the best approximated.

## The correct realization of the star-delta starting

The precedings are well demonstrated by the comparison of the two possible variants of the star-delta starting [1], see Fig. 1. (The phases of voltages are symbolized in the figure by the positions of the windings.)


Fig. I. Two variants of the star-delta switching

In connection $I$ the new phase voltage $\mathbb{U}_{a I}$ is leading by $30^{\circ}$, as relative to the original phase voltage $\mathrm{U}_{a} \mathrm{Y}$, so the voltage difference $\Delta \overline{\mathbf{U}}_{I}$ is large at the moment of the following switching on. Much more advantageous is combination $I I$, where $\mathbf{U}_{a I I}$ lagging by $30^{\circ}$, is nearly in phase with voltage $\mathbf{U}_{a i}$ and so the difference $\Delta \mathrm{U}_{I I}=\mathrm{U}_{a I I}-\mathrm{U}_{a i}$ is small.

## The combinations of connections at four-stage starting

The study of phase relationships of switching at four-stage starting became necessary for the solution of a practical starting problem [6].

Under difficult starting conditions one or two intermediary connections can be applied between the star and delta connections. The mixed connection consists of the delta connection of parts of the partitioned phase windings. The number of terminals of the machine should be increased from 6 to 9 . Two different mixed connections could be realized by windings partitioned into three sections and with 12 terminals, but instead of this the winding has two inequal sections and the two different connections are obtained by interchanging the beginnings and ends of the windings.

From the viewpoint of the phase relationships the mutual position of the two sections of a winding is decisive. Essentially there are three possibilities of partitioning. At single-layer windings the number of slots per pole and phase is separated into two groups and the phase shift of their voltages is $30^{\circ}$. Depending on wich sections form the delta, two different connections are possible (Fig. 2. a and b). At double-layer windings the two layers represent the two sections and in the simplest case the spatial position of the two layers can be the same, i.e. the two partial voltages are in phase (Fig. 2. c) [3], [4], [5]. In the figure the axes of the windings indicate the phase position of the voltage complexors.

Considering that our aim is only to show the importance of the choice of phase relationships, it is sufficient to deal only with case $b$.

a)

b)

c)

Fig. 2. The variants of the mixed connection

Here a simple relationship can be written for the starting torque in function of the partitioning of the winding. If the rate of the number of turns for the winding section denoted by 1 is " $a$ " (i.e. $a=1$ corresponds to the starconnection and $a=0$ to the delta-connection), then the known ratio [5] can be used:

$$
\begin{equation*}
\frac{M_{i a}}{M_{i Y}}=\frac{3}{1+a+a^{2}} \tag{6}
\end{equation*}
$$

with
$M_{i a}$ - the starting torque if the relative number of turns of the sections connected to the vertices of the delta is " $a$ "
$M_{i Y}$ - the starting torque in star connection ( $a=1$ )
The proportional (i.e. corresponding to a geometrical series) grading of starting torques in the four stages requires the values $a_{1}=1$ (star), $a_{2}=0.705$, $a_{3}=0.333$ and $a_{4}=0$ (delta) obtained from Eq. (6).

If only one tapping is allowed, the following condition must be necessarily satisfied:

$$
\begin{equation*}
a_{2}+a_{3}=1 \tag{7}
\end{equation*}
$$

So a compromise is to be adopted and the proportional grading can only be approximated. The partitioning of the winding in the rate of $1 / 3-2 / 3$ gives a reasonable approximation: $a_{2}=2 / 3 ; a_{3}=1 / 3$ and this has been used for the complexor diagrams which follow.

Figure 3 shows a connection met in practice which caused starting difficulties [6] and Fig. 4 gives a possible, improved solution.

Parts a. in the figures show the connections. The beginnings of windings are denoted by 1 , the ends by 2 and the intermediary tappings by 3 . Stage II contains a delta formed by the smaller sections and stage III. gives a delta formed by the larger sections simply by interchanging terminals 1 and 2.

Parts $b$. of the figures show the voltages of the phase windings with respect to the line voltage of the mains chosen as reference.

Parts c. of the figures show the sequence of the voltages applied to the phase windings in each stage.

The numerical values are given in a table, too. The symbols used in the table are:
$U_{a} / U_{R S} \quad$ - the relative value of the voltage applied to the phase winding $x$
$\Delta \alpha \quad$ - the change of the phase angle of the phase voltage at switching over.
$\beta=\Delta \alpha+\varphi \quad$ - the switch-over angle showing the deviation from the ideal switch-over.


Fig. 3. The original four-stage starting


a)

II.

c)
I.

b)

Fig. 4. The proposed improved four-stage starting

Table
Phase relationships of switchings

| Stage | $U_{a} / U_{R s}$ | Fig. 3. |  |  | Fig. 4. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\alpha$ | $\Delta \alpha$ | $\beta$ | $\alpha$ | $\Delta \alpha$ | $\beta$ |
| I | 0.577 | $-30^{\circ}$ |  | $6.6^{\circ}$ | $32.6^{\circ}$ | $-30^{\circ}$ | $-36.6^{\circ}$ |
| II | 0.688 | $-23.4^{\circ}$ | $-6.6^{\circ}$ | $19.4^{\circ}$ |  |  |  |
| III | 0.832 | $46.1^{\circ}$ | 69.5 | $95.5^{\circ}$ | -36.1 | $20.1^{\circ}$ | $-73.9^{\circ}$ |
| IV | 1.0 | $0^{\circ}$ | $-46.3^{\circ}$ | $11.3^{\circ}$ |  |  |  |

In the solution of Fig. 3 the angle $\beta=95.5^{\circ}$ of changing from stage II to stage III, caused a transient current which could not be tolerated. It should be mentioned, that for this solution it has been tried to delay this switch-over by a time relay with the aim to wait until the rotor comes to the right position, but during the several tenths of a second required for this, the slip of the drive with relatively short mechanical time constant has been increased and this caused a current resulting in break-down.

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