TURBO-GENERATOR MODEL WITH MAGNETIC SATURATION

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Summary

In this paper a mathematical model of turbo-generators is published. It is suitable for their large signal dynamical analysis. The equations are written in a d–q coordinate system fixed to the rotor. One stator and two rotor windings are used in both directions. The effects of iron saturation are approximately taken into account as regards the main field inductances only. The model includes the mechanical transient process as well.

By adding to the eight-time-constant model of the generator the model of the excitation regulator, various transient processes can be simulated, among them some that would be undesirable to investigate experimentally due to safety considerations or to the danger of damages, e.g. a synchronization in an erroneous phase position.

Introduction

Several authors have dealt with computer simulation of turbo-generators, both in this country and abroad. The complexity of the model needed is determined by the investigation to be carried out; e.g. it is usual to model the synchronous generator by a single reactance X_d and the internal e.m.f. (pole-voltage) of the machine for the calculation of power distribution or voltage and current distribution in a network. To calculate fault currents, one- or two-reactance models are generally employed. The generator models used for the development of the excitation system and for the adjustment of the regulator involve three to six electrical time constants, and it is usually desirable to take the mechanical transients and, approximately, the saturation as well into account. In the following, such a model is presented.

Turbo-generator model with no iron saturation

The model developed is suitable for salient pole synchronous machines as well. The stator has been presumed to be electrically and magnetically symmetrical and to be equipped with a symmetrical three-phase winding. The number of windings taken into account on the rotor depends on the purpose of the investigation and on the accuracy required. The iron core is usually modelled by separate windings. The number of windings is restricted by the increase of computer time and by the determinability of their values. In the present case, in direction d, a further winding is assumed beside the excitation winding, modelling the damping effect of the iron core as well as the effect of the damping rods. In direction q, two short-circuited windings have been reckoned with, both model the damping effect of the iron core.

In the present case, the data of the windings can be determined from the usual data obtained by designers' caculations and by measurements on the generator. Namely, the quantities τ_d'', τ_q', τ_q'', X_l, R_s, R_f, X_d', X_d'', X_d, X_q', X_q'', X_q yield the data of the model windings. According to our experience, in some cases the time constants τ_{do}' and τ_d' obtained from the model are considerably less than the measured values (see Appendix A.2.). This is due to the insufficient modelling of the iron core. This problem can be overcome by assuming a further winding in direction d. The determination of the model is presented in the Appendix with the aid of the data of the synchronous machine type ORH 476 of Ganz Electric Works (Appendix A.2.).

Relative quantities are employed in the equations. The base values are the peak values of phase quantities at rated operating conditions, the rated electrical angular frequency, the rated apparent power and the "apparent" torque (calculated from the apparent power). The time base is the reciprocal value of the rated network frequency.

The equations of the synchronous machine

The vector of state variables is:

$$X^T = (\Psi_d, \Psi_q, \Psi_f, \Psi_{Q1}, \Psi_{md}, \Psi_{mq}, W, \delta)\,.$$

It should be noted that the main field fluxes Ψ_{md} and Ψ_{mq} have been selected as state variables in order to simplify the computation of saturation. So, the damping winding fluxes Ψ_{D1} and Ψ_{Q2} can be expressed by state variables and thus they are not independent variables. The fluxes are measured in a rotating (d, q) coordinate system fixed to the rotor. W is measured in a stator coordinate system.

The voltage equations of the machine are given in Appendix A.1. According to these equations, the electrical model of the synchronous machine can be drawn as shown in Fig. 1.

The equations, including the mechanical equations A.1.2. have to be arranged so that the derivatives of the state variables are put on the left-hand side, while the non state variables should be eliminated. The final result is given in the Appendix.

According to the equations, the signal-flow diagram of the machine can be drawn as shown in Fig. 2.

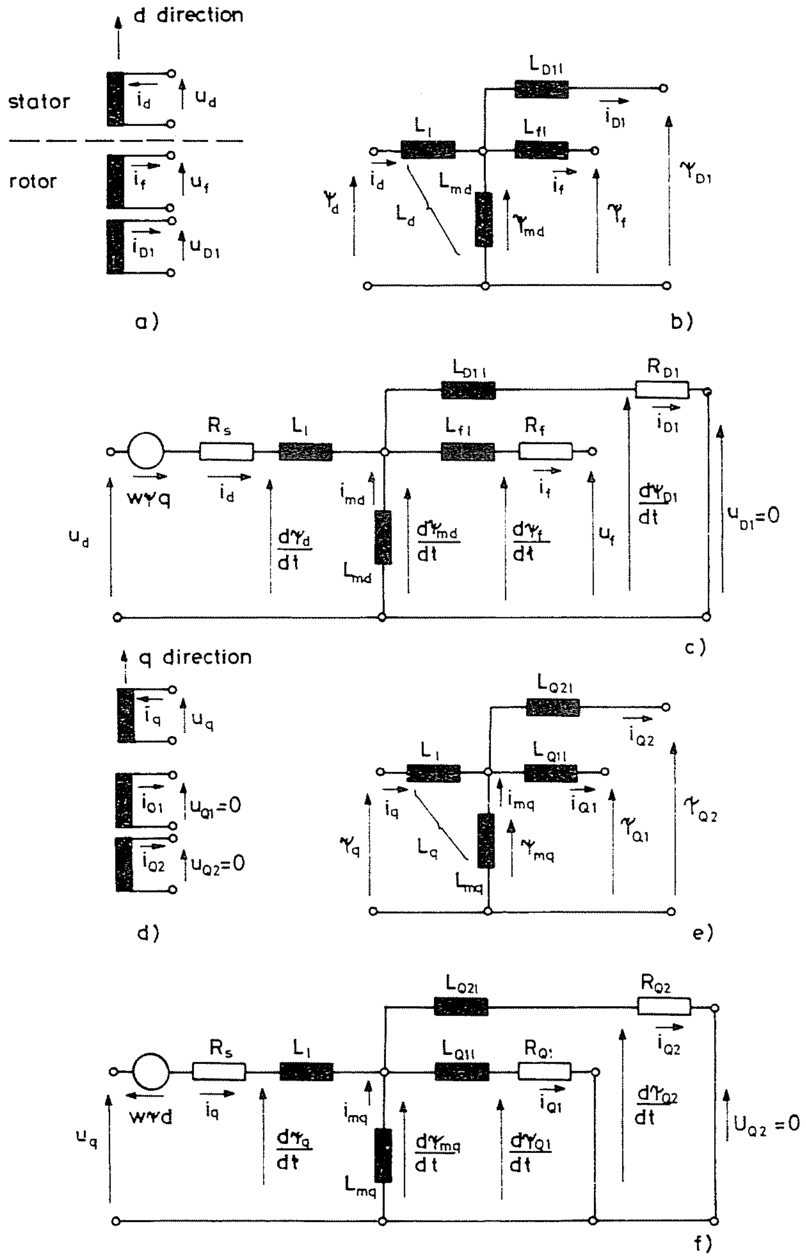

Fig. 1. Equivalent circuit of a synchronous machine in a synchronously rotating coordinate system.

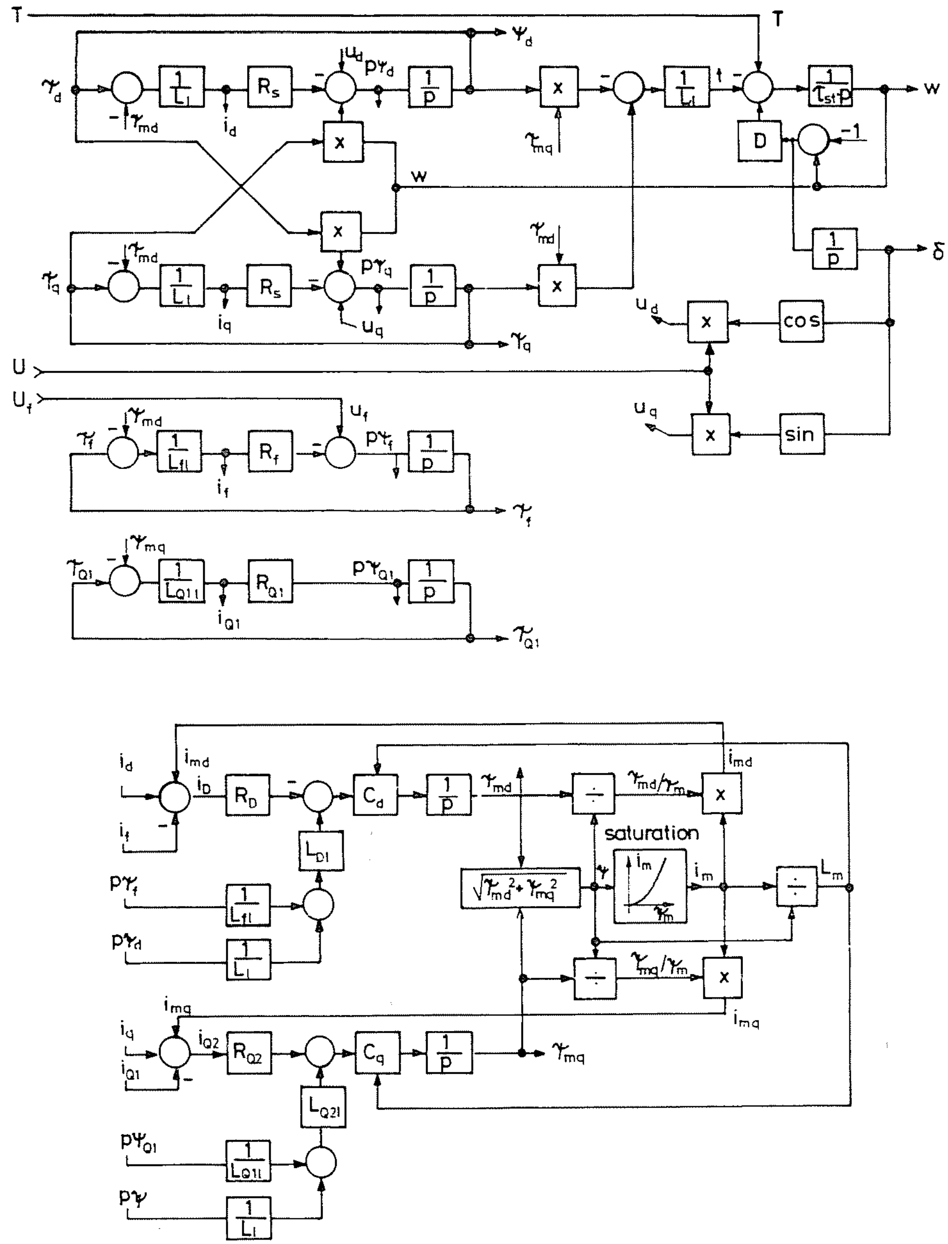

Fig. 2. Mathematical computational model of synchronous machine

Simulation of the electrical network

In the investigated case the generator is connected to an infinite bus through a transimssion line modelled by elements R_e and L_e is examined. If the resistance and inductance of the transmission line are added to those of the stator of the synchronous machine, the equations are formally identical with the equations A.1.4 in the Appendix; of coarse, the meaning of the variables is different, e.g. u_d and u_q are the components of the voltage of the infinite bus, Ψ_d and Ψ_q are "fluxes" of the infinite network, while L_l and R_s are understood as L_l+L_e and R_s+R_e, respectively. Similarly, δ is the load angle with respect to the infinite bus.

Approximation of iron saturation

The machine to be modelled, as most turbo-generators, has a practically constant air-gap, thus, neglecting the effects of the slots of the excitation winding, magnetic conductivity can be taken as constant in every direction.

For modelling the saturation, the following approximations are used:

— The non-sinusoidal flux density distribution along the perimeter of the air gap is substituted by its fundamental harmonic.

— The saturation of the main field inductances is taken into account, the leakage inductances are assumed to be constant.

With these presumptions, the no-load characteristic giving the relationship between the terminal voltage u and the excitation current i_f in relative units at synchronous angular speed $w=1$ is identical to the relationship between the main field flux Ψ_m and the magnetizing current i_m. As the turbogenerator has a cylindrical rotor, the magnetization curve can be taken practically identical in every direction, thus the resultant magnetizing current vector $\bar{i}_m$ and the main field winding flux $\bar{\Psi}_m$ are unidirectional and the relationship between their absolute values is given by the excitation curve of the machine:

$$\bar{\Psi}_m = L_m \bar{i}_m \tag{2}$$

where L_m is the saturated main field inductance.

It should be stressed, that only an approximation method is used here to take into consideration the effect of saturation by recomputing the value of L_m in each Runge-Kutta step in the model equations given in A.2.

Results

The aim of modelling was the simultaneous investigation of the turbogenerator and the excitation system. The examinations carried out with the entire system are not covered by this paper, a few cases are only presented concerning the synchronous generator without excitation system and its excitation at constant voltage. The results presented are:

a) Abrupt decrease of the excitation voltage from its rated value by 10%, Fig. 4.a;

b) Decreasing the voltage of the infinite network by 2%, Fig. 4.b;

c) Decreasing the mechanical driving torque from its rated value to zero, Fig. 4.c;

d) Erroneous synchronizing operation at opposition. Before synchronization, the terminal voltage of the synchronous machine was equal to that of the infinite network, but the angle between them was 180 degrees, Fig. 4.d;

e) Examination of excitation. The excitation voltage of the synchronous machine was increased from zero to over its rated value. In this case, a very great impedance was inserted between the synchronous machine and the infinite network, thus eliminating the necessity to modify the machine equations Fig. 4.e.

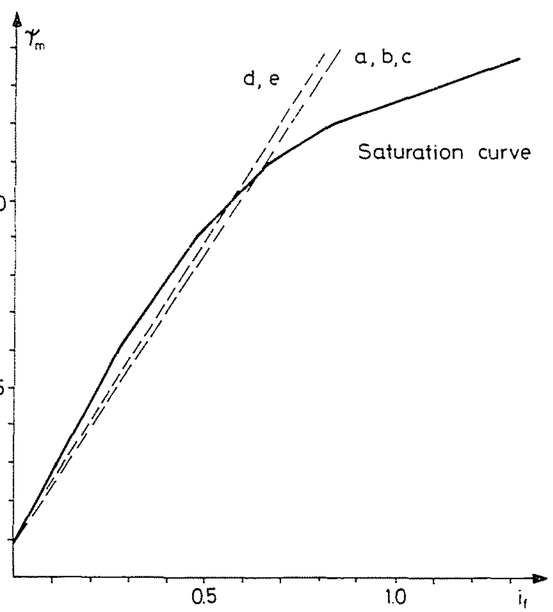

Fig. 3. Saturation curve of the synchronous machine

The saturation characteristic of the synchronous machine is shown in Fig. 3. The transient processes investigated involved a relatively low change in the main field flux in cases a), b), c) and d), while the change was naturally considerable in case e). For the sake of comparison, the no-saturation cases have also been indicated in the diagrams. In case of no saturation the magnetizing curve of the machine was replaced by a straight line crossing the original magnetizing curve in the initial working point, see Fig. 3. In case e) this line belongs to the rated flux.

Of course the transient processes shown in the diagrams have been checked as regards initial and final values as well as time constants; but this is not detailed here.

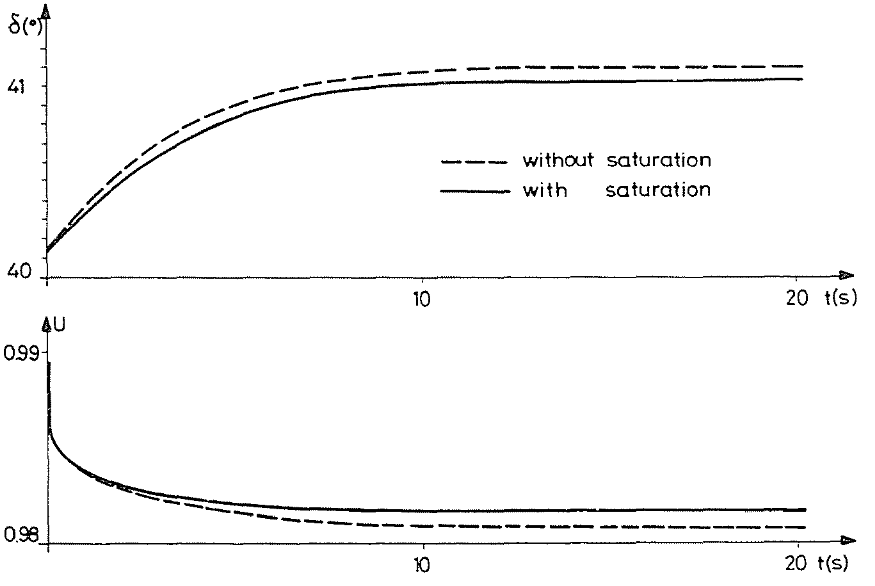

Fig. 4.a. Transient of a synchronous machine: decreasing u_f by 10% from rated working point. ....without saturation, ___ with saturation

In Figs 4.a b and c, a difference of only 5–10% in the final values can be seen with regard to the change. The differences being small is in part explained by the fact that the machine was only slightly saturated in the working points examined, see Fig. 3.

The most striking difference is encountered in the investigation of excitation. In such examinations, the effect of saturation is not negligible. The two curves in the initial stage are clearly seen (Fig. 4.e) to run together and only split in the neighbourhood of the rated terminal voltage. The effect of the iron core and of the damping rods can be clearly recognized in the excitation

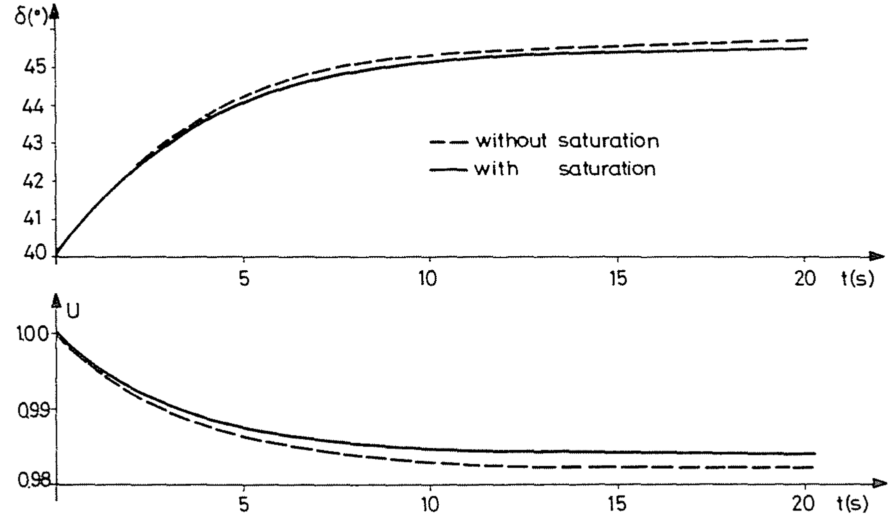

Fig. 4.b. Transient of a synchronous machine: decreasing u by 2% from rated working point

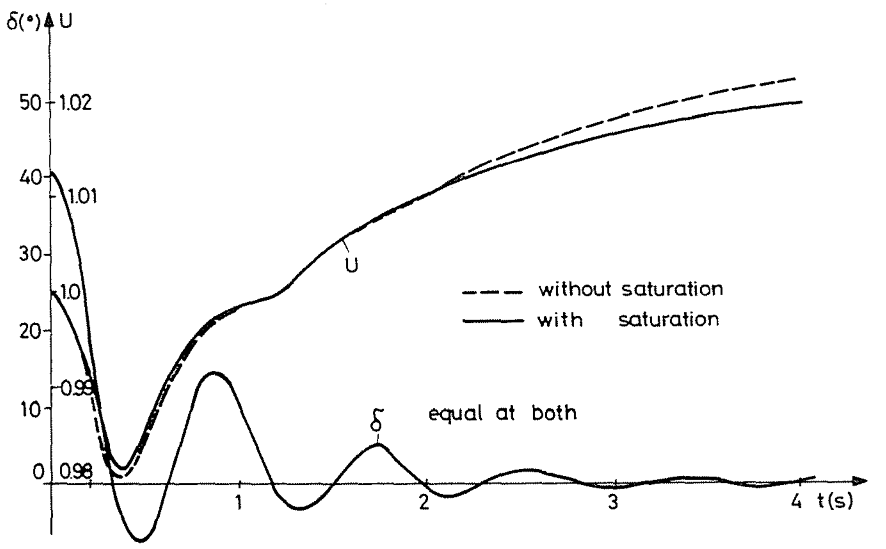

Fig. 4.c. Transient of a synchronous machine, decreasing driving torque to zero from rated working point equal at both

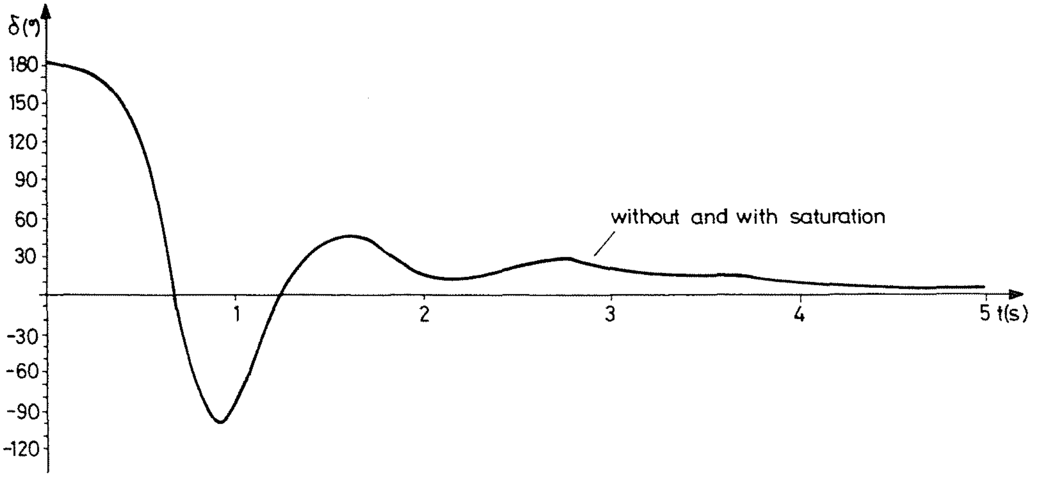

Fig. 4.d. Transient of a synchronous machine, erroneous synchronization in opposition

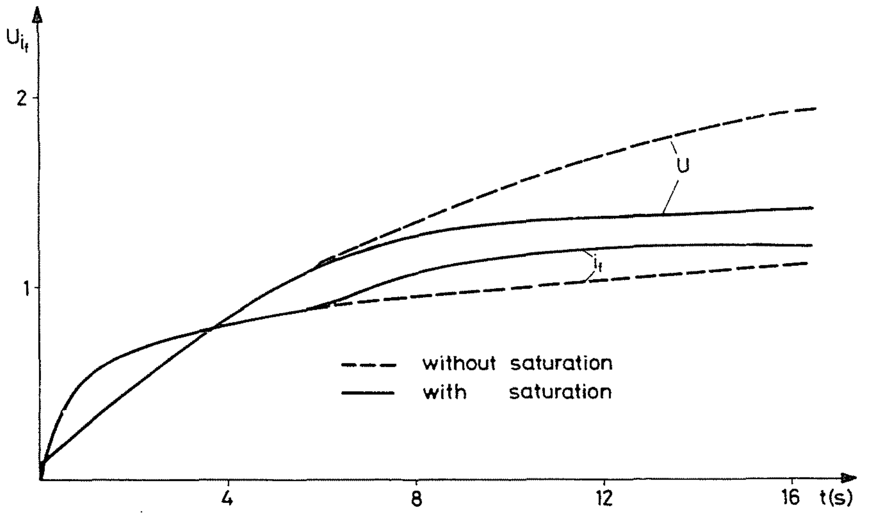

Fig. 4.e. Transient of synchronous machine: exciting from zero excitation current

current. The former add a counter-excitation to that of the excitation winding in the first few seconds, thus the current of the latter can increase faster as compared to the time constant T_{do}. Therefore, it can be stated that taking saturation into account, even in the simplified manner presented, results are of a considerably higher accuracy, although in case of transients with low-range signal variation the no-saturation model gives a good approximation, too.

Appendix

Notations:

Subscripts

$D1$, $D2$:	damping windings in direction d on rotor;
e:	elements of the transmission line;
f:	excitation winding in direction d on rotor;
d:	d direction component or quantity;
n:	rated value;
m:	magnetizing branch, maximal value;
l:	leakage type inductance;
s:	stator quantity;
$Q1$, $Q2$:	damping windings in direction q on rotor;
q:	q direction component or quantity;
o:	quantity measured at open stator terminals;
a:	armature quantity.

Character notations:

C:	generally to denote a constant or a coefficient;
D:	damping factor of the turbine-generator system;
L:	inductance;
T:	mechanical torque, constant in time;
t:	electrical torque, time-varying;
p:	operator d/dt;
R:	resistance;
τ:	time constant;
u, i:	stator voltage and current (without subscript); winding voltages and currents (with subscript);
w:	electrical angular velocity of rotor;
X:	reactance;
δ:	load angle, angle between pole voltage and terminal voltage.

Superscripts:

$'$:	transient quantity;
$''$:	sub-transient quantity;

In general, lower case letters denote time functions, while upper case letters denote constant values.

A.1 Model of the turbo-generator

The equations of the generator are: 6 winding model

$$u_d = -i_d R_s + \frac{d\psi_d}{dt} - w\psi_q$$

$$u_q = -i_q R_s + \frac{d\psi_q}{dt} + w\psi_d$$

$$u_f = i_f R_f + \frac{d\psi_f}{dt}$$

$$u_{D1} = 0 = i_{D1} R_{D1} + \frac{d\psi_{D1}}{dt}$$

$$u_{Q1} = 0 = i_{Q1} R_{Q1} + \frac{d\psi_{Q1}}{dt}$$

$$u_{Q2} = 0 = i_{Q2} R_{Q2} + \frac{d\psi_{Q2}}{dt} \qquad \text{A.1.1}$$

The mechanical equations are:

$$T - t - (w-1)D = \tau_{st}\frac{dw}{dt}$$

$$\frac{d\delta}{dt} = w - 1 \qquad \text{A.1.2}$$

The above equations are to be re-arranged to contain the state variables and their derivatives. The currents are expressible with the aid of the fluxes by using Fig. 1. The elimination of ψ_{Q2} and ψ_{D1} is slightly more complex. According to the model:

$$i_{D1} = i_{md} - i_f + i_d = \frac{\psi_{md}}{L_{md}} - \frac{\psi_f - \psi_{md}}{L_{fl}} - \frac{\psi_d - \psi_{md}}{L_l}.$$

On the other hand, from Fig. 1:

$$\psi_{D1} = \psi_{md} + i_{D1} L_{D1l}$$

Differentiation yields:

$$\frac{d\psi_{D1}}{dt} = \frac{d\psi_{md}}{dt} + L_{D1l}\frac{di_{D1}}{dt} =$$

$$= \frac{d\psi_{md}}{dt}\left[1 + L_{D1l}\left(\frac{1}{L_{fl}} + \frac{1}{L_l} + \frac{1}{L_{md}}\right)\right] - L_{D1l}\left(\frac{d\psi_f}{L_{fl}\,dt} + \frac{d\psi_d}{L_l\,dt}\right)$$

Substituting this into the voltage equation of the $D1$ winding:

$$u_{D1}=0=\left(\frac{\psi_{md}}{L_{md}}-\frac{\psi_f-\psi_{md}}{L_{fl}}-\frac{\psi_d-\psi_{md}}{L_l}\right)R_{D1}+$$
$$+\frac{d\psi_{md}}{dt}\left[1+L_{D1l}\left(\frac{1}{L_{md}}+\frac{1}{L_{fl}}+\frac{1}{L_l}\right)\right]-L_{D1l}\left(\frac{d\psi_f}{L_{fl}\,dt}+\frac{d\psi_d}{L_l\,dt}\right) \qquad \text{A1.3}$$

In direction q, the equation of the damping winding $Q2$ can be derived similarly. Expressing the time-derivatives from the state equations of the synchronous machine, a form suitable for a computer aided solution is obtained:

$$\frac{d\psi_d}{dt}=u_d-\frac{\psi_d-\psi_{md}}{L_l}R_s+w\psi_q; \qquad u_d=U\cos\delta$$

$$\frac{d\psi_q}{dt}=u_q-\frac{\psi_q-\psi_{mq}}{L_l}R_s-w\psi_d; \qquad u_q=U\sin\delta$$

$$\frac{d\psi_f}{dt}=u_G-\frac{\psi_f-\psi_{md}}{L_{fl}}R_f$$

$$\frac{d\psi_{Q1}}{dt}=-\frac{\psi_{Q1}-\psi_{mq}}{L_{Q1l}}R_{Q1}$$

$$\frac{d\psi_{md}}{dt}=-R_{D1}C_d\left(\frac{\psi_{md}}{L_{md}}-\frac{\psi_f-\psi_{md}}{L_{fl}}-\frac{\psi_d-\psi_{md}}{L_l}\right)+$$
$$+L_{D1l}C_d\left(\frac{d\psi_f}{L_{fl}\,dt}+\frac{d\psi_d}{L_l\,dt}\right)$$

$$\frac{d\psi_{mq}}{dt}=-R_{Q2}C_q\left(\frac{\psi_{mq}}{L_{mq}}-\frac{\psi_{Q1}-\psi_{mq}}{L_{Q1l}}-\frac{\psi_q-\psi_{mq}}{L_l}\right)+$$
$$+L_{Q2l}C_q\left(\frac{d\psi_{Q1}}{L_{Q1l}\,dt}+\frac{d\psi_q}{L_l\,dt}\right)$$

$$\frac{dw}{dt}=[T-t-D(w-1)]\frac{1}{\tau_{st}}$$

$$\frac{d\delta}{dt}=w-1 \qquad \text{A.1.4}$$

where

$$t=\psi_d i_q-\psi_q i_d=\frac{\psi_d\psi_{mq}-\psi_q\psi_{md}}{L_l}$$

is the electrical torque, and

$$\frac{1}{C_d}=1+L_{D1}\left(\frac{1}{L_{md}}+\frac{1}{L_{fl}}+\frac{1}{L_l}\right)$$

$$\frac{1}{C_q}=1+L_{Q2}\left(\frac{1}{L_{mq}}+\frac{1}{L_{Q1l}}+\frac{1}{L_l}\right)$$

A.2 Determination of the data of the turbo-generator type ORH 476 of Ganz Electric Works

Measurement data: in relative units, if not otherwise indicated

Quantities of rated operation:

$$S_n=212.5\text{ MVA}$$
$$U_n=15.5\text{ kV}$$
$$I_n=7915\text{ A}$$
$$\cos\varphi_n=0.8$$

maximal inductive reactive power: $Q=0.35\ S_n$
Quantities of excitation windings: $U_{fn}=410$ V, $I_{fn}=2300$ A

A few points of the excitation curve:

U	0.6	0.9	1	1.1	1.2	1.3	p.u.
I_f	350	570	670	800	1000	1350	A

Short-circuit characteristic: $I_f=1342$ A at $I_n=7915$ A
Further data: saturated/non-saturated

$X_l=0.154/0.193$
$X''_d=0.19/0.21$
$X'_d=0.26/0.3$
$X''_q=0.2/0.27$
$X'_q=0.8/1.17$
$\tau''_d=0.016$ s saturated
$\tau'_d=1.3$ s saturated
$\tau'_{do}=9.8$ s saturated
$\tau_d=0.4$ s non-saturated

$R_s=0.001536$
$R_f=0.001205$

$\tau'_q=0.935$ s non-saturated
$\tau''_q=0.01154$ s non-saturated

According to reference [1], the definitions of the various quantities with the notations of Fig. 1 at the 8 time-constant 6 winding model:

(* denotes the operation of calculation of parallel resultant, i.e. e.g. $a*b=a\cdot b/(a+b)$)

$$L_d''=L_l+L_{md}*L_{fl}*L_{D1l}$$

$$L_d'=L_l+L_{md}*L_{fl}$$

$$L_q''=L_l+L_{mq}*L_{Q1l}*L_{Q2l}$$

$$L_q'=L_l+L_{mq}*L_{Q1l}$$

$$\tau_{d0}'=\frac{L_{fl}+L_{md}}{R_f};\qquad \tau_d'=\frac{L_{fl}+L_{md}*L_l}{R_f};\qquad \tau_d''=\frac{L_{D1l}+L_{md}*L_l*L_{fl}}{R_{D1}}$$

$$\tau_q'=\frac{L_{Q1l}+L_{mq}*L_l}{R_{Q1}};\qquad \tau_q''=\frac{L_{Q2l}+L_{mq}*L_l*L_{Q1l}}{R_{Q2}}$$

Since the flux in the machine during the transients examined is around its rated value, the leakage-type reactances were taken into account by their saturated values e.g. at L_l. The resistances in direction q were determined from the non-saturated quantities. L_m was derived from the saturation curve $L_m=L_{md}=L_{mq}$.

$L_m=1.831/2.1$	$R_s=0.001536$
$L_l=0.154/0.193$	$R_f=0.001205$
$L_{fl}=0.1125$	$R_{D1}=0.0233$
$L_{D1l}=0.0545$	$R_{Q1}=0.00772$
$L_{Q1l}=0.99$	$R_{Q2}=0.0675$
$L_{Q2l}=0.0495$	

With these data, $\tau_{do}'=5.80$ s and $\tau_d'=0.673$ s was obtained from the definitive equations. Both are about 50% of the measured values. The error could be eliminated by assuming a further damping winding in direction d, thus taking the damping effect of the iron core into account more accurately. The time constant of this winding is in the order of that of the excitation winding. The parameters of the new model have been calculated to make the value of τ_{do}' equal to its measured value after substituting the winding $D2$ and the excitation winding by one winding of an equivalent time constant. The resultant time constant of the two windings was in our case approximately equal to the sum of the time constants of the two windings, and in order to avoid a change in the leakage of the equivalent winding, the value

$$L_{fl}=L_{D2l}=0.225$$

was taken.

Thus $R_{D2}=0.0015$ was obtained.

The time constants, in the new model were $\tau'_{do}=9.8$ s and $\tau'_{d}=1.21$ s which constitute an acceptable approximation.

Reference

KOVÁCS, K. P.–RÁCZ, I.: "Transient processes in A C machines". Akadémiai Kiadó, Budapest, 1969. (in German)

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