SELF-EXCITED VIBRATION IN TWO-DIMENSIONAL CUTTING PROCESS

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Summary

Test has been conducted in rough machining to study the self-excited vibration of cutting system, modelled as a "two-degree of freedom system with time lag." The system consists of a workpiece with grooves assumed to vibrate relatively to the rigid tool in oblique-cutting.

Introduction

The dynamic behaviour of cutting tool is one of the most important characteristics of assessment with respect to their chatter [1].

Self-excited vibration that occurs violently in metal cutting operations puts limit to the practically possible size of cut referred to as the regenerative chatter. It is one of the various kinds of vibrations that arises in metal cutting [2].

In the early stages of the development of machine tool chatter analysis, the coefficients relating to the variations in chip thickness, . . . etc. of the cutting forces have been regarded as constants for a wide range of cutting conditions. More recently, it has been found that variations in the limit of stability for cutting conditions, particularly at high cutting speeds, can be explained only if the cutting coefficients vary with cutting speed, depth of cut, feed, etc. [3].

Nomenclature

dP_x : dynamic cutting force in x-axis
dP_y : dynamic cutting force in y-axis
Ω : angular speed [rad/s]
r : feed rate [mm/s]
N : cutting speed [rpm]
t_0 : dept of cut [mm]

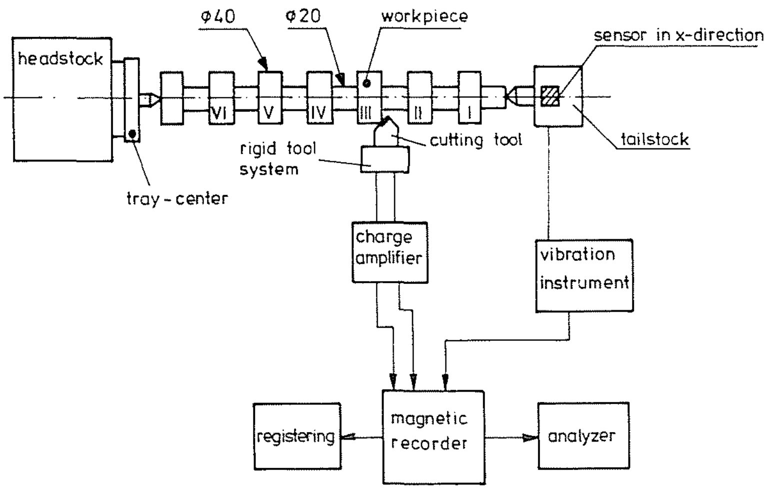

Fig. 1. The block-diagram for measuring cutting forces and vibrations in two-dimensional system

Test was performed at various depths of cut (0.05 – 0.7 mm), at constant feed of 0.1 mm/rev, with variable cutting speeds (47.5 – 750 rpm) and the cutting velocity was 0.05 – 1.57 m/s.

Mechanical model

Figure 2 shows the mechanical model of the system with two-degree of freedom.

The following assumptions are considered:

— the vibratory system of the mechanical model is linear,

— the direction of the variable component of the cutting force P is constant,

— the variable component of the cutting force depends only on vibrations in x-axis and in y-axis,

— the value of the variable component of cutting force varies proportionally and instantaneously with the two vibrational displacements x and y,

— the regenerative chatter and the mode-coupling principles are taken into consideration.

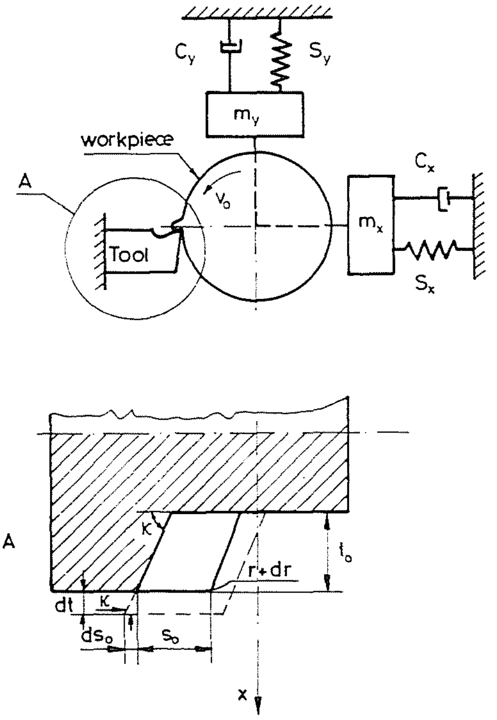

Fig. 2. Mechanical model of two-degree of freedom system

Theory

Under steady-state cutting conditions the following parameters are used.

$$V = R\Omega = 2\Pi RN\,, \quad N = \frac{\Omega}{2\Pi}, \quad \Omega = \frac{2\Pi}{T} \quad \text{and} \quad r = \frac{S_0\Omega}{2\Pi}$$

The mean cutting force is defined by:

$$P = C_P q^{X_P}$$

where

C_P: factor depends on the material of the workpiece, cutting conditions and the geometry of the tool

x_P: exponential constant

q: chip area

$$q = t_0 S_0$$

$$\frac{\partial P}{\partial t_0} = C_P x_P t_0^{x_P - 1} S_0^{x_P} \tag{1}$$

S_0 : nominal feed [mm/rev]
V : cutting velocity [m/s]
K_{sx} :chip thickness coefficient in x-axis [N/m]
K_{sy} : chip thickness coefficient in y-axis [N/m]
K_{1x} : dynamic chip thickness coefficient in x-axis [N/m]
K_{1y} : dynamic chip thickness coefficient in y-axis [N/m]
K_{Ω} : angular speed coefficient [N s]
K_V : cutting velocity coefficient [Ns/m]
T : time lag at each revolution [s]
v : displacement coefficient
C_0 : feed rate factor
R : radius of workpiece [mm]
dS_0 : chip thickness variation
dr : feed rate variation
$d\Omega$: angular speed variation
dt_0 : depth of cut variation
dV : cutting velocity variation
m_x : mass in x-axis [Ns^2/m]
m_y : mass in y-axis [Ns^2/m]
C_x : damping coefficient in x-axis [Ns/m]
C_y : damping coefficient in y-axis [Ns/m]
S_x : stiffness in x-axis [N/m]
S_y : stiffness in y-axis [N/m]

Experimental procedures

Test was performed on a horizontal lathe powered by a 5.5 kw d.c. motor, having center to center length 1500 mm. A workpiece of A42 steel, 350 mm long and 40 mm outside diameter was clamped between the headstock and tailstock centers.

From *Fig. 1.* it is obvious that the workpiece used had grooves which means that the stiffness of the system was artificially decreased. The section III (Fig. 1) was considered for investigation.

Tool geometry:

overhang	$L = 20$ mm
relief angle	$\alpha = 6°$
rake angle	$\gamma = 12°$
tip angle	$\varepsilon = 78°$
nose radius	$r = 0.4$ mm

size of the tool shank 25×22.5 mm.

let

$$K_S = \frac{\partial P}{\partial t_0} \tag{2}$$

from Eq. (1) and (2)

$$\frac{\partial P}{\partial S_0} = \frac{t_0}{S_0} K_S . \tag{3}$$

Under steady-state cutting conditions the cutting force is defined as

$$P = f(S_0, t_0, V).$$

Therefore, the cutting force variation becomes

$$\mathrm{d}\,P = \left(\frac{\partial P}{\partial S_0}\right) \mathrm{d}\,S_0 + \left(\frac{\partial P}{\partial t_0}\right) \mathrm{d}\,t_0 + \left(\frac{\partial P}{\partial V}\right) \mathrm{d}\,V, \tag{4}$$

where

$$V = R\Omega\,, \qquad \mathrm{d}\,V = R\,\mathrm{d}\,\Omega,$$

$$\frac{\partial P}{\partial V} = K_V\,, \qquad K_\Omega = K_V R. \tag{5}$$

Substituting Eqs (2), (3) and (5) into Eq. (4)

$$\mathrm{d}\,P = \frac{t_0}{S_0} K_S\,\mathrm{d}\,S_0 + K_S\,\mathrm{d}\,t_0 + K_\Omega\,\mathrm{d}\,\Omega\,. \tag{6}$$

Under the dynamic cutting conditions the cutting force is a function of four independent factors $P(S_0, r, V, t_0)$. The cutting force variation for small changes in these factors can be written as

$$\mathrm{d}\,P = K_1\,\mathrm{d}\,S_0 + K_2\,\mathrm{d}\,r + K_3\,\mathrm{d}\,\Omega + K_4\,\mathrm{d}\,t_0 \tag{7}$$

where K_1, K_2, K_3 and K_4 being the dynamic coefficients which can be determined from sophisticated dynamic cutting tests. To determine the dynamic coefficients, the following conditions are considered:

I-Condition

Ω = constant, t_0 = constant and S_0 = variable

$$\mathrm{d}\,\Omega = \mathrm{d}t_0 = 0 \quad \text{and} \quad \mathrm{d}r = \frac{\Omega}{2\Pi}\,\mathrm{d}S_0\,.$$

Substituting these in Eqs (6) and (7)

$$K_2 = \left(\frac{t_0}{S_0} K_S - K_1\right)\frac{2\Pi}{\Omega}\,. \tag{8}$$

II-Condition

S_0 = constant, t_0 = constant and Ω = variable

$$\mathrm{d}r=\frac{S_0}{2\Pi}\mathrm{d}\Omega \quad \text{and} \quad \mathrm{d}S_0=\mathrm{d}t_0=0,$$

from Eqs (6) and (7)

$$K_3=K_\Omega-\left(\frac{t_0}{S_0}K_S-K_1\right)\frac{S_0}{2\Pi R}T. \tag{9}$$

III-Condition

S_0 = constant, Ω = constant and t_0 = variable

$$\mathrm{d}S_0=\mathrm{d}\Omega=\mathrm{d}r=0, \text{ and}$$

similarly from Eqs (6) and (7)

$$K_4=K_S \tag{10}$$

Substituting Eqs (8), (9) and (10) into Eq. (7)

$$\mathrm{d}P=K_1\,\mathrm{d}S_0+\left(\frac{t_0}{S_0}K_S-K_1\right)\frac{2\Pi}{\Omega}\mathrm{d}r+$$
$$+\left[\frac{K_\Omega}{R}-\left(\frac{t_0}{S_0}K_S-K_1\right)\frac{S_0}{2\Pi R}T\right]\mathrm{d}\Omega R+K_S\,\mathrm{d}t_0. \tag{11}$$

Where, $\mathrm{d}S_0$, $\mathrm{d}t_0$, $\mathrm{d}r$ and $\mathrm{d}\Omega$ are functions of time and are independent from each other, calculated as

$$\mathrm{d}S_0=\mathrm{d}S_{0x}+\mathrm{d}S_{0y}=(x-x_T)-v(y-y_T)$$
$$\mathrm{d}t_0=\mathrm{d}t_{0x}+\mathrm{d}t_{0y}=x-vy$$
$$\mathrm{d}r=\mathrm{d}r_x+\mathrm{d}r_y=\dot{x}-v\dot{y}$$

Vibration in x-axis

The cutting velocity V is constant in x-axis, i.e., $\mathrm{d}V=0$. Therefore, from Eq. (11) the dynamic cutting force in x-axis can be written as:

$$\mathrm{d}P_x=K_{1x}[x-x_T-v(y-y_T)]+K_{S_x}(x-vy)+$$
$$+\left(\frac{t_0}{S_0}K_{S_x}-K_{1x}\right)\frac{2\Pi}{\Omega}(\dot{x}-v\dot{y}). \tag{12}$$

Where, $\left(\frac{t_0}{S_0}K_{S_x}-K_{1x}\right)$ is the feed rate coefficient in x-axis.

The feed rate factor C_0 considered by Knight [3] is

$$C_0 = \frac{K_S - K_1}{K_1}. \qquad (13)$$

In x-axis Eq. (13) becomes

$$C_0 = \frac{\frac{t_0}{S_0} K_{S_x} - K_{1x}}{K_{1x}}.$$

Therefore, the dynamic chip thickness coefficients may be defined as

$$K_{1x} = \frac{t_0}{S_0} K_{S_x} \frac{1}{1+C_0}, \qquad (14)$$

and

$$K_{1y} = \frac{t_0}{S_0} K_{S_y} \frac{1}{1+C_0}. \qquad (15)$$

Vibration in y-axis

The cutting velocity V is variable and the vibration y brings a change in the angular speed

$$d\Omega = \frac{y}{R}$$

Similarly, the dynamic cutting force in y-axis was determined from Eq. (11) as follows

$$dP_y = K_{1y}[(x - x_T) - v(y - y_T)] + K_{S_y}(x - vy) + \left(\frac{t_0}{S_0} K_{S_y} - K_{1y}\right) T\dot{x} +$$
$$+ \left[K_{V_y} - \left(\frac{t_0}{S_0} K_{S_y} - K_{1y}\right) T\left(v + \frac{S_0}{2\Pi R}\right)\right]\dot{y} \qquad (16)$$

Where $\left(\frac{t_0}{S_0} K_{S_y} - K_{1y}\right)\left(v + \frac{S_0}{2\Pi R}\right)$ is the feed rate coefficient in y-axis.

The displacement coefficient v is calculated as

$$v = \frac{t_0}{\sqrt{R^2 - (R - t_0)^2}}. \qquad (17)$$

The stability condition

The equations of motion of the mechanical model with two-degree of freedom system are written as:

$$m_x\ddot{x}+C_x\dot{x}+S_x x=-\mathrm{d}P_x \tag{18}$$

$$m_y\ddot{y}+C_y\dot{y}+S_y y=-\mathrm{d}P_y \tag{19}$$

Substituting Eq. (12) into Eq. (18) and Eq. (16) into Eq. (19), Eqs (18) and (19) can be represented in matrix form

$$\mathbf{A}\ddot{\mathbf{X}}+\mathbf{B}\dot{\mathbf{X}}+\mathbf{D}\mathbf{X}+\mathbf{E}\acute{\mathbf{X}}=0 \tag{20}$$

where,

A, **B**, **D** and **E** are constants

and

$$\ddot{\mathbf{X}}=\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix},\quad \dot{X}=\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix},\quad X=\begin{bmatrix}x\\ y\end{bmatrix}\quad \text{and}\quad \acute{X}=\begin{bmatrix}x(t-T)\\ y(t-T)\end{bmatrix}$$

Let us solve the matrix (Eq. 20) by considering the following:

$$x(t)=\alpha e^{\lambda t}$$

and

$$x(t-T)=\alpha e^{\lambda(t-T)}$$

$$y(t)=\beta e^{\lambda t}$$

and

$$y(t-T)=\beta e^{\lambda(t-T)}$$

Substituting these in Eq. (20) for trivial solution of matrix equation, if $\begin{bmatrix}\alpha\\ \beta\end{bmatrix}=0$ and $\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\neq 0$.

The characteristic equation is written as

$$D(\lambda)=\begin{vmatrix}a_{11}\lambda^2+b_{11}\lambda+\mathrm{d}_{11}+e_{11}e^{-\lambda T} & b_{12}\lambda+\mathrm{d}_{12}+e_{12}e^{-\lambda T}\\ b_{21}\lambda+\mathrm{d}_{21}+e_{21}e^{-\lambda T} & a_{22}\lambda^2+b_{22}\lambda+\mathrm{d}_{22}+e_{22}e^{-\lambda T}\end{vmatrix}=0 \tag{21}$$

Equation (21) can be represented in the form

$$D(\lambda)=\mathrm{Re}\,D(\lambda)+i\,\mathrm{Im}\,D(\lambda) \tag{22}$$

where,

$$\lambda=i\Psi,\quad i=\sqrt{-1}$$

and

$$e^{-\lambda T}=e^{-i\Psi T}=\cos(\Psi T)-i\sin(\Psi T)$$

Therefore, the characteristic Eq. (21) takes the new form:

$$D(i\Psi)=\begin{vmatrix} a_{11}\Psi^2+ib_{11}\Psi+d_{11}+e_{11}(\cos(\Psi T)-i\sin(\Psi T)) & ib_{21}\Psi+d_{12}+e_{12}(\cos(\Psi T)-i\sin(\Psi T)) \\ ib_{21}\Psi+d_{21}+e_{21}(\cos(\Psi T)-i\sin(\Psi T)) & -a_{22}\Psi^2+ib_{22}\Psi+d_{22}+e_{22}(\cos(\Psi T)-i\sin(\Psi T)) \end{vmatrix}=0 \tag{23}$$

Let $M(\Psi)=\mathrm{Re}\ D(i\Psi)$ real part

$S(\Psi)=\mathrm{Im}\ D(i\Psi)$ imaginary part

Eq. (22) can be represented as

$$D(i\Psi)=M(\Psi)+iS(\Psi) \tag{24}$$

The solution of Eq. (20) is asymptotically stable, if their roots have negative real parts.

It can be proved [4] that the system is stable if

$$\sum_{K=1}^{m}(-1)^{K+1}\ \mathrm{sign}\ S(\Psi_K)=-2 \tag{25}$$

A computer program was made with the aid of the algorithm resulted from the theorem of G. Stépán for computation zeroes of the real part of Eq. (23) and to check whether Eq. (25) is true or not.

The program has been written in WANG 2200 computer.

The results show that the characteristic equation (21) depends on a large number of parameters. The dynamic cutting force in x-axis dP_x in Eq. (12) depends on the change in chip thickness dS_0, on the change in the feed rate dr, and on the change in the depth of cut dt_0. Therefore, the dynamic instability can occur as a result of dS_0 and dt_0 variations.

Discussion

Figure 3 shows a theoretical and experimental stability chart of cutting process in two-dimensional system. The asymptotic borderline of stability is the principal borderline, since it defines the maximum depth of cut (0.155 mm) which will result in stable cutting at all speeds. The prediction indicates that the stable region increases when the cutting speed increases from a minimum to around 320 rpm.

It has been observed that the limit of stability of a system is greatly affected by the inequality in rotating speed of the workpiece, depth of cut, chip thickness coefficients K_{Sx} and K_{Sy}, feed rate coefficients, dynamic chip thickness coefficients K_{1x} and K_{1y}, cutting velocity coefficient K_{Vy} and time lag

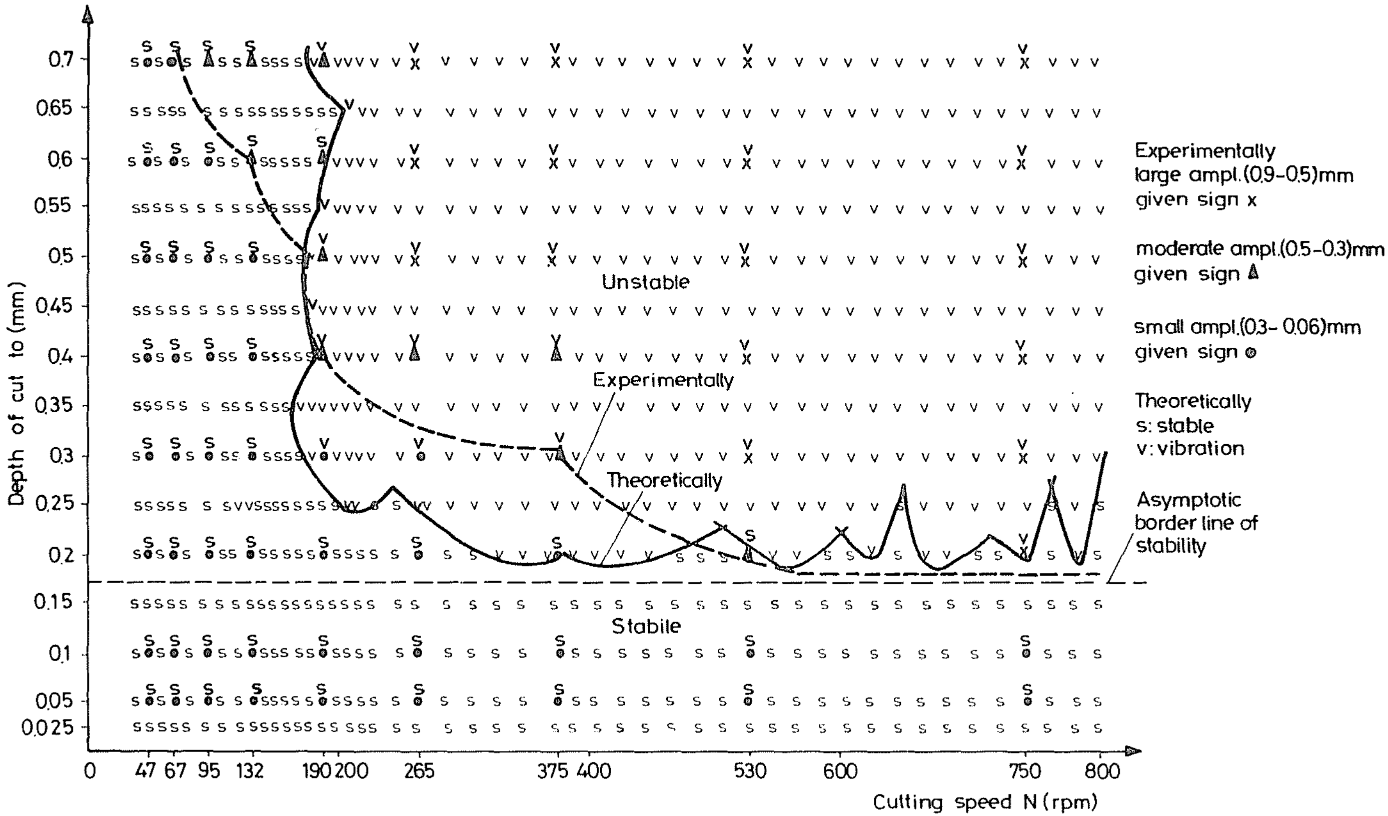

Fig. 3. The stability chart

T. *Figure 4* concluded that the maximum vibration amplitude is directly proportional to the width of cut and the system becomes unstable if it exceeds the critical equivalent width of cut.

Figure 5 shows the linear relation between the cutting velocity and the average main cutting force for each depth of cut in order to determine the cutting velocity coefficient. Similar curves have been got at different depths of cut. The average main cutting force and average thrust force as function of the depth of cut for each cutting speed are measured.

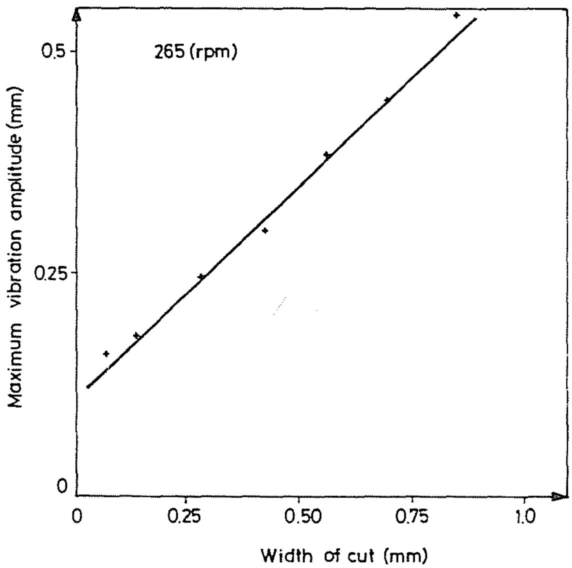

Fig. 4

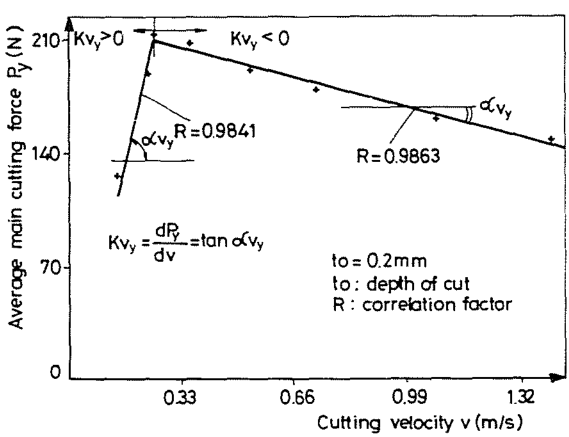

Fig. 5. The cutting velocity coefficient Kv_y

A typical graph of these forces is shown in *Fig. 6*. All the curves follow this expected form. The forces versus the depth of cut are linear.

The dynamic chip thickness coefficients K_{1y} and K_{1x} in Eq. (14) and (15) are more affected by $\left(\frac{t_0}{S_0}\right)$ than the chip thickness coefficients K_{S_x} and K_{S_y}. Therefore, the ratio of t_0 and S_0 have a high effect on the stability condition of the cutting process.

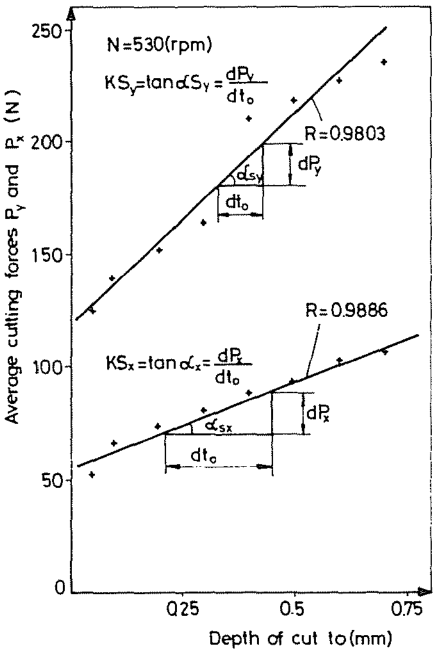

Fig. 6. The thickness coefficients

Conclusions and directions for further research

Conclusions could be summarized as follows:

1. The level of stability is increased as the depth of cut is decreased because the $\left(\frac{t_0}{S_0}\right)$ reduces the values of the dynamic chip thickness coefficients K_{1x} and K_{1y}.
2. The predicted borderline of stability shows good qualitative agreement with experimental results and indicates stability in low speed range due to the damping effects of the cutting process.
3. The dynamic cutting coefficients are properly evaluated from the steady-state cutting parameters. The feed rate variation dr has a stabilizing effect.

4. The following factors are noticed to influence the cutting stability: The cutting velocity coefficient, stiffness, chip thickness coefficients, feed rate factor, damping coefficients, dynamic chip thickness coefficients, displacement coefficient and time lag.
5. In future an on-line data processing system could be made by connecting the control system of CNC or NC-lathe with computer to use the stability chart and to stabilize the system by changing the cutting conditions (feed, cutting speed and depth of cut) automatically.

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