AN APPROXIMATION METHOD OF DESCRIBING FUNCTION FOR SOME HYSTERESIS NONLINEARITIES IN HYDRAULIC SERVOSYSTEMS

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Summary

The describing function method is a more convenient procedure in the investigation of nonlinear feedback control systems. An important condition to use this method is the determination of the describing function of nonlinearity in question. This paper deals with some hysteresis type nonlinearities which are usually encountered in many hydraulic servosystems, and gives a set of suitable approximating formulae in form of very strongly convergent series.

Introduction

Brief summary of describing function method

In the investigation of the nonlinear control systems, for the sake of simplicity, one frequently used to apply several various linearized procedures, when possible. Each of them may possess its advantage or disadvantage. Therefore, if the main goal of this investigation is the statement of the system stability, or of the limit cycle occurrence in the system, it is sufficient to use the describing function method. For the sake of lucidity a brief summary of this method is given. It is assumed that the system satisfies the applicable condition of this method which has been described in [1], [2], [3] and [4], then in output of the linear part a permanent sinusoidal signal is existing affecting the input of the nonlinearity. It is important to remember that in consequence of nature of the nonlinear systems in the engineering domain the linear part behave as a high out-off low-pass filter. Therefore the method gives a relatively faithful picture about the system behaviour. The investigation of the stability and of the limit cycle is not very difficult. For estimating the stability or the limit cycle of the system the conditions are in one side that one has to know the transfer function $Y(s)$ of the linear part and in the other side the describing function $F(B)$

of the nonlinearity (where s is the variable of Laplace transforms and B is the amplitude of the mentioned sinusoidal signal). In other words we can have the Nyquist plot $Y(j\omega)$ (where $j=\sqrt{-1}$ and ω is the fundamental frequency) and the chart of the $F^{-1}(B)$ in the plane of complex variable (the Gaussian plane). From the intersection of these curves and their course the important consequences may be given without worth-while difficulties. (see Fig. 1)

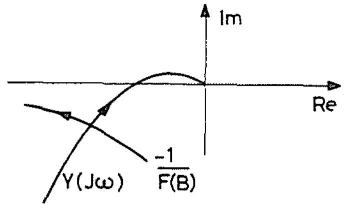


Fig. 1

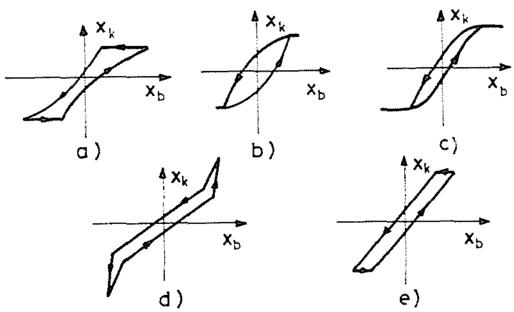


Fig. 2

It is obvious that for solving the above problem the first step is to possess the describing function. This paper is concerned with an approximation method for determining the several double-value nonlinearities in some hydraulic servosystems and gives a set of mathematical formulae of respective describing functions.

The shape of some main hysteresis nonlinearities in hydraulic servosystems

In Fig. 2 we can see the plot of these nonlinearities which express the relation between their input- and output signals (x_b and x_k respectively) [5], [6]. Here we shall deal with the three first cases only (i.e. the a), b) and c) (because the describing function of d) case [1] and e) case (from the author*) are derived exactly. The approximation is only related to drawing near the plot of the characteristics $x_k(x_b)$ by means of the pieces of sinosoidal curve line and straight.

* Since the nonlinearity is double value function, its describing function is a complex variable function and it depends upon the amplitude B of the input signal (the sinusoidal signal) and may be written in the following form

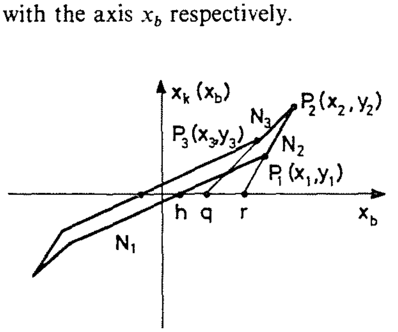
$$F(B)=\frac{C(B)e^{j\varphi(B)}}{B} \tag{I}$$

where $C=(A_1^2+B_1^2)^{1/2}$ is the module, and $\varphi=\operatorname{arc\,tg}\left(\frac{A_1}{B_1}\right)$ is the argument of the describing function. Obviously, these quantities depend upon the independent variable B. The expressions of A_1 and B_1 are:

$$A_1=\frac{2}{\pi}\left\{\frac{B}{2}(N_1-N_2)\left(\frac{x_1}{B}\right)^2+(N_1h-N_2r)\left(\frac{x_1}{B}\right)+N_2\left(\frac{B}{2}+r\right)+\right.$$
$$\left.+\frac{B}{2}(N_3-N_1)\left(\frac{x_3}{B}\right)^2+(N_3q-N_1h)\left(\frac{x_3}{B}\right)-N_3\left(\frac{B}{2}+q\right)\right\} \tag{II}$$

$$B_1=\frac{2}{\pi}\left\{B\frac{N_2-N_1}{2}\left(\frac{x_1}{B}\right)\left[1-\left(\frac{x_1}{B}\right)^2\right]^{1/2}+(N_1h-N_2r)\left[1-\left(\frac{x_1}{B}\right)\right]^{1/2}-\right.$$
$$-B\left(\frac{N_2-N_1}{2}\right)\arcsin\left(\frac{x_1}{B}\right)+\left(\frac{4}{\pi}+1\right)N_1h+$$
$$+B\frac{N_1+N_3}{2}\left(\frac{x_1}{B}\right)\left[1-\left(\frac{x_3}{B}\right)^2\right]^{1/2}+$$
$$\left.+(N_1h-N_3q)\left[1-\left(\frac{x_3}{B}\right)^2\right]^{1/2}-B\left(\frac{N_1+N_3}{2}\right)\arcsin\left(\frac{x_3}{B}\right)\right\} \tag{III}$$

where N_1, N_2, and N_3 express the slope of the straight piece as shown in the figure and h, r, q are intersections of the straight with the axis x_b respectively.



The basis of the approximation and the describing function of the mentioned nonlinearities

The basis of the approximation

Looking the shape of the plots in Fig. 2a, b and c it can be stated that its course seems to consist of the sinusoidal and straight pieces, therefore it will give a result with higher accuracy, and convenient calculation in numerical computer use. First, the mathematical formula of the sinusoidal curve is unknown, more exactly we do not know its parameters and therefore we must

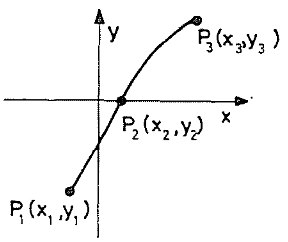


Fig. 3

determine these parameters now. From experimental data at least 3 points, namely two extreme points and such a point which is in the x axis if we describe it in formula as follows, (see the Fig. 3)

$$y = N(\sin x + a) + b \tag{1}$$

where N, a and b are the unknown parameters, namely N is the sinusoidal curve amplitude, a and b are the displacements in direction of x and y axes, respectively, in such coordinate system where the expression of the sinusoidal function is described in canonic form.

Remember that, the choosing of these 3 points is totally arbitrary, but if these 3 points are placed very near to each other, the accuracy considerably decreases. In addition, let the point $p_3(x_3, y_3)$ (see Fig. 3) be extremum point, i.e. where the tangent of the curve is horizontal. Using the following symbols

$$\sin a = S,\ \cos a = C,\ \sin x_1 = s_1,\ \sin x_2 = s_2,\ \sin x_3 = s_3$$

and

$$\cos x_1 = c_1,\ \cos x_2 = c_2,\ \cos x_3 = c_3 \tag{1.a}$$

the nonlinear algebraic equation system is given as follows

$$s_1 NC + c_1 NS + b = y_1$$

$$s_2NC+c_2NS+b=0 \tag{2}$$

$$s_3NC+c_3NS+b=y_3$$

for three unknowns N, C and b. The solving of Eq. (2) in this form is hard enough. However, with a convenient variable transformation, namely

$$NC=X; \quad NS=Y \quad \text{and} \quad b=Z \tag{3.a}$$

$$C^2+S^2=1 \quad \text{and} \quad X^2+Y^2=N^2 \tag{3.b}$$

the equation can easily be solved and in the form of vector equation Eq. (2) may be written

$$\mathbf{A}\vec{R}=\vec{y} \tag{4}$$

where

$$\mathbf{A} = \begin{bmatrix} s_1 & c_1 & 1 \\ s_2 & c_2 & 1 \\ s_3 & c_3 & 1 \end{bmatrix}$$

a 3-dimension matrix

$$\vec{R}=[X, Y, Z]^T$$

the unknown column vector, and

$$\vec{y}=[y_1, 0, y_3]^T$$

called as right hand side column vector. The condition of the nontrivial solution of Eq. (4) is

$$\det \mathbf{A} \neq 0. \tag{5}$$

It is convenient to remark in such case, when (but not usually) it does not exist due to "unlucky" choosing of the above three points with experiment, we can use the other P_2 point which has a $y_2 \neq 0$. This is done till condition (5) is satisfied.

After determination of N, a and b parameters we may come to the point, i.e., to the determination of the mentioned describing function.

Determination of the describing function

Describing function of the case 1.*a of Fig.* 2 (*see Fig. 4*)

From the Fig. 4 the output signal is consisting of the following parts:
for the part 1:

$$x_k(t) = N \sin(B \sin \omega t + a) - b; \quad -\alpha \le \omega t \le \frac{\pi}{2}$$

for the part 2:

$$x_k(t) = N \sin(B + a) - b; \quad \frac{\pi}{2} \le \omega t \le \pi - \alpha$$

for the part 3: (6)

$$x_k(t) = N \sin(B \sin \omega t - a) + b; \quad \pi - \alpha \le \omega t \le 3\frac{\pi}{2}$$

for the part 4:

$$x_k(t) = N \sin(B - a) + b; \quad 3\frac{\pi}{2} \le \omega t \le 2\pi$$

where $\alpha = \arcsin \dfrac{B-H}{B}$. (It is evident that H itself is equal to $|x_3| - |x_1|$.

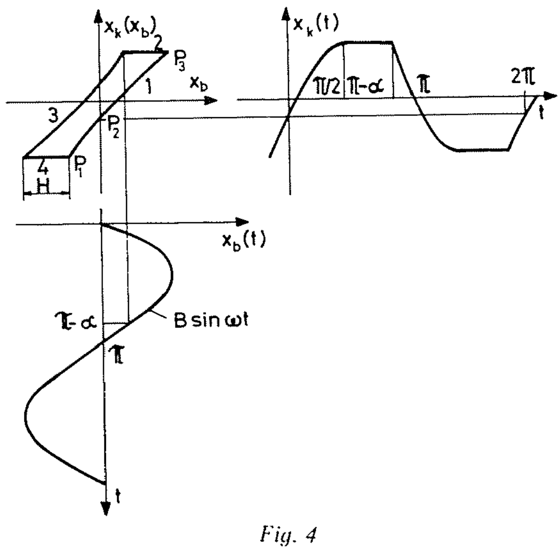


Fig. 4

It is well known that in general the describing function can be written as follows:

$$F(B,\omega)=\frac{1}{B}\left[A_1^2(B,\omega)+B_1^2(B,\omega)\right]^{1/2}\cdot e^{j\varphi_1(B,\omega)} \tag{7}$$

where $A_1(B,\omega)$ and $B_1(B,\omega)$ are the Fourier coefficients of the first harmonic of output signal of the nonlinearity, namely

$$A_1(B,\omega)=\frac{2}{\pi}\int_0^\pi x_k(t)\cos\omega t\,\mathrm{d}\,\omega t$$

and (8)

$$B_1(B,\omega)=\frac{2}{\pi}\int_0^\pi x_k(t)\sin\omega t\,\mathrm{d}\,\omega t$$

as well as

$$\varphi_1(B,\omega)=\operatorname{arc\,tg}\frac{A_1(B,\omega)}{B_1(B,\omega)} \tag{9}$$

In our case it is noteworthy that A_1 and B_1 are only depending upon the amplitude B of the input signal, and the coefficient

$$A_0=\frac{1}{2\pi}\int_{-\pi}^{\pi} x_k(t)\,\mathrm{d}\,\omega t$$

is zero due to central symmetry of the nonlinearity, therefore in the following we shall omit it from our calculations, and—for the sake of simplicity—magnitude B will not appear in every expression of A_1 and B_1 either.

1. Determination of A_1. From (8) we have

$$A_1=\frac{2}{\pi}\int_0^\pi x_k(t)\cos\omega t\,\mathrm{d}\,\omega t=\frac{2}{\pi}\int_0^{\pi/2}\left[N\sin(B\sin\omega t+a)-b\right]\cos\omega t\,\mathrm{d}\,\omega t+$$

$$+\frac{2}{\pi}\int_{\pi/2}^{\pi-\alpha}\left[N\sin(B+a)-b\right]\cos\omega t\,\mathrm{d}\,\omega t+$$

$$+\frac{2}{\pi}\int_{\pi-\alpha}^{\pi}\left[N\sin(B\sin\omega t-a)+b\right]\cos\omega t\,\mathrm{d}\,\omega t \tag{10}$$

Considering that the $N \sin(B \sin \omega t) \cos \omega t$, and $N \cos(B \sin \omega t) \cos \omega t$ functions have no finite simple primitive functions, the problem will be solved in form of the convenient series, which are very strongly convergent, therefore in the engineering domain it is satisfactory to use some first terms without a great inaccuracy. For this purpose we expand the expression $N \sin(B \sin \omega t + a)$, and get

$$N \sin(B \sin \omega t + a) = NC \sin(B \sin \omega t) + NS \cos(B \sin \omega t) \tag{11}$$

Expanding the $\sin(B \sin \omega t)$ and $\cos(B \sin \omega t)$ in Taylor's series, the relation (11) may be written as follows:

$$x_k(t) = NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} + NS \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} - b;$$

$$-\alpha \leq t \leq \frac{\pi}{2}$$

$$x_k(t) = NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} - NS \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} + b;$$

$$\pi - \alpha \leq \omega t \leq 3\frac{\pi}{2} \tag{12}$$

It is worth remarking that the above series are absolutely convergent series. This fact is very important, because in this case the summation and the integration is inverted to each other. Therefore

$$A_1 = \frac{2}{\pi} \int_0^{\pi/2} \Big[NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} - NS \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} - b\Big] \cos \omega t \, d\,\omega t$$

$$+ \frac{2}{\pi} \int_{\pi/2}^{\pi-\alpha} [N \sin(B + a) - b] \cos \omega t \, d\,\omega t + \tag{13}$$

$$\frac{2}{\pi} \int_{\pi-\alpha}^{\pi} \Big[NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} + NS \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} + b\Big] \cos \omega t \, d\,\omega t$$

Using the following designations:

$$A_1 = J_1 + J_2 + J_3$$

and

$$J_1 = J_{11} + J_{12} + J_{13} \tag{14}$$

$$J_3 = J_{31} + J_{32} + J_{33}$$

as well as

$$J_2 = \frac{2}{\pi} \int_{\pi/2}^{\pi - \alpha} [N \sin (B + a) - b] \cos \omega t \, d\, \omega t \tag{15}$$

where

$$J_{11} = \frac{2}{\pi} \int_0^{\pi/2} NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} \cos \omega t \, d\, \omega t =$$

$$\frac{2}{\pi} NC \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n+1}}{(2n+1)!} \int_0^{\pi/2} \sin^{2n+1} \omega t \cos \omega t \, d\, \omega t \tag{16}$$

$$J_{12} = \frac{2}{\pi} \int_0^{\pi/2} NS \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} \cos \omega t \, d\, \omega t =$$

$$\frac{2}{\pi} NS \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n)!} \int_0^{\pi/2} \sin^{2n} \omega t \cos \omega t \, d\, \omega t \tag{17}$$

and

$$J_{13} = -\frac{2}{\pi} \int_0^{\pi/2} b \cos \omega t \, d\, \omega t = -\frac{2}{\pi} b \int_0^{\pi/2} \cos \omega t \, d\, \omega t \tag{18}$$

After integration of the relations (16)–(18), we have

$$J_{11} = \frac{2}{\pi} NC \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n+1}}{(2n+2)!} [\sin^{2n+1} \omega t]_0^{\pi/2} =$$

$$\frac{2}{\pi} NC \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n+1}}{(2n+2)!}, \tag{19}$$

$$J_{12} = \frac{2}{\pi} NS \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n+1)!} [\sin^{2n+1} \omega t]_0^{\pi/2} = \frac{2}{\pi} NS \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n+1)!} \tag{20}$$

and

$$J_{13}=\frac{2}{\pi}\,b[\sin\omega t]_0^{\pi/2}=-\frac{2}{\pi}\,b \tag{21}$$

from which

$$J_1=\frac{2}{\pi}\left\{NC\sum_{n=0}^{\infty}(-1)^n\frac{B^{2n+1}}{(2n+2)!}+NS\sum_{n=0}^{\infty}(-1)^n\frac{B^{2n}}{(2n+1)!}-b\right\}$$

$$J_2=\frac{2}{\pi}\,[N\sin(B+a)-b]\,[\sin\omega t]_{\pi/2}^{\pi-\alpha}= \tag{22}$$

$$-\frac{2}{\pi}\,[N\sin(B+a)-b]\,\frac{x_1+x_3}{B} \tag{23}$$

regarding

$$\alpha=\arcsin\left(1-\frac{x_1+x_3}{B}\right).$$

If $x_3=B$ (which is the general case), then

$$J_2=-\frac{2}{\pi}\,[N\sin(B+a)-b]\left(1+\frac{x_1}{B}\right) \tag{23.a}$$

Similarly, the determination of J_3 is easily carried out, the only difference is the limit of integration. The result is as follows:

$$J_{31}=\frac{2}{\pi}\,NC\sum_{n=0}^{\infty}(-1)^n\frac{B^{2n+1}}{(2n+2)!}\,[\sin^{2(n+1)}\omega t]_{\pi-\alpha}^{\pi}=$$

$$=-\frac{2}{\pi}\,NC\sum_{n=0}^{\infty}(-1)^n\frac{B^{2n+1}}{(2n+2)!}\left(1-\frac{x_1+x_3}{B}\right)^{2(n+1)} \tag{24}$$

and when $x_3=B$, we have

$$J_{31}=-\frac{2}{\pi}\,NC\sum_{n=0}^{\infty}(-1)^n\frac{B^{2n+1}}{(2n+2)!}\left(\frac{x_1}{B}\right)^{2(n+1)}. \tag{24.a}$$

$$J_{32}=-\frac{2}{\pi}\,NS\sum_{n=0}^{\infty}(-1)^n\frac{B^{2n}}{(2n+1)!}\,[\sin^{2n+1}\omega t]_{\pi-\alpha}^{\pi}=$$

$$\frac{2}{\pi}\,NS\sum_{n=0}^{\infty}(-1)^n\frac{B^{2n}}{(2n+1)!}\left(1-\frac{x_1-x_3}{B}\right)^{2n+1} \tag{25}$$

if $x_3 = B$, then

$$J_{32} = -\frac{2}{\pi} NS \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n+1)!} \left(\frac{x_1}{B}\right)^{2n+1} \tag{25.a}$$

and

$$J_{33} = -\frac{2}{\pi} b\left(1 - \frac{x_1 + x_3}{B}\right) \tag{26}$$

with $x_3 = B$:

$$J_{33} = \frac{2}{\pi} b \frac{x_1}{B} \quad . \tag{26.a}$$

Now, from the relations (24), (25) and (26) J_3 may be written

$$J_3 = \frac{2}{\pi}\left\{-NC \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n+1}}{(2n+2)!}\left(1 - \frac{x_1+x_3}{B}\right)^{2(n+1)} + \right.$$
$$\left. + NS \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n+1)!}\left(1 - \frac{x_1+x_3}{B}\right)^{2n+1} - b\left(1 - \frac{x_1+x_3}{B}\right)\right\} . \tag{27}$$

In the sense of $A_1 = J_1 + J_2 + J_3$, and after some conversions and reductions we get the formula as follows:

$$A_1 = \frac{2}{\pi}\left\{NC \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n+1}}{(2n+2)!}\left[1 - \left(1 - \frac{x_1+x_3}{B}\right)^{2(n+1)}\right] + \right.$$
$$NS \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n+1)!}\left[1 + \left(1 - \frac{x_1+x_3}{B}\right)^{2n+1}\right] +$$
$$\left.\left[b + 1 - N(\beta_1 C + \beta_2 S)\frac{x_1+x_3}{B}\right]\right\} \tag{28}$$

where $\beta_1 = \sin B$, and $\beta_2 = \cos B$. If $x_3 = B$, then

$$A_1 = \frac{2}{\pi}\left\{NC \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n+1}}{(2n+2)!}\left[1 - \left(\frac{x_1}{B}\right)^{2(n+1)}\right] + \right.$$
$$NS \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n+1)!}\left[1 - \left(\frac{x_1}{B}\right)^{2n+1}\right] +$$
$$\left.\left[b + 1 - N(\beta_1 C + \beta_2 S)\cdot\left(1 + \frac{x_1}{B}\right)\right]\right\} . \tag{29}$$

Now we may deal with the determination of B_1, namely

$$B_1 = \frac{2}{\pi}\int_0^{\pi/2} [NC \sum_{n=0}^{\infty} (-1)^n \frac{(B\sin\omega t)^{2n+1}}{(2n+1)!} +$$

$$NS \sum_{n=0}^{\infty} (-1)^n \frac{(B\sin\omega t)^{2n}}{(2n)!} - b]\sin\omega t \,\mathrm{d}\,\omega t +$$

$$+\frac{2}{\pi}\int_{\pi/2}^{\pi-\alpha} [N\sin(B+a) - b]\sin\omega t\,\mathrm{d}\,\omega t +$$

$$+\frac{2}{\pi}\int_{\pi-\alpha}^{\pi} [NC \sum_{n=0}^{\infty} (-1)^n \frac{(B\sin\omega t)^{2n+1}}{(2n+1)!} -$$

$$-NS \sum_{n=0}^{\infty} (-1)^n \frac{(B\sin\omega t)^{2n}}{(2n)!} + b]\sin\omega t\,\mathrm{d}\,\omega t =$$

$$I_1 + I_2 + I_3\,. \tag{30}$$

From it the first integral (or the first term of B_1):

$$I_1 = \frac{2}{\pi}\int_0^{\pi/2} NC \sum_{n=0}^{\infty} (-1)^n \frac{(B\sin\omega t)^{2n+1}}{(2n+1)!} \sin\omega t\,\mathrm{d}\,\omega t +$$

$$\frac{2}{\pi}\int_0^{\pi/2} NS \sum_{n=0}^{\infty} (-1)^n \frac{(B\sin\omega t)^{2n}}{(2n)!} \sin\omega t\,\mathrm{d}\,\omega t -$$

$$-\frac{2}{\pi}\int_0^{\pi/2} b\sin\omega t\,\mathrm{d}\,\omega t = I_{11} + I_{22} + I_{33} \tag{30.a}$$

where I_{11}, I_{12}, and I_{13} is the first, the second and the third term of the I_1 in form (30.a), respectively. After integrations, we have the result:

$$I_{11} = \frac{2}{\pi} NC \sum_{n=0}^{\infty}\sum_{k=0}^{n} (-1)^{n+1} \frac{B^{2n+1}}{(2n+1)!} \{\mu_k(n)[\sin^{2(n-k)+1}\omega t \cos\omega t]$$

$$-\mu_n(n)[\omega t]\}_0^{\pi/2} \tag{31}$$

$$I_{12} = \frac{2}{\pi} NS \sum_{n=0}^{\infty}\sum_{k=0}^{n} (-1)^{n+1} \frac{B^{2n}}{(2n)!} \{\nu_k(n)\sin^{2(n-k)}\omega t\cos\omega t\}_0^{\pi/2} \tag{32}$$

and

$$I_{13} = \frac{2}{\pi} b[\cos \omega t]_0^{\pi/2} \tag{33}$$

where $\mu_k(n)$ and $\nu_k(n)$ are coefficients, which depend upon positive integer variables n and k, and may be calculated by means of the expressions:

$$\mu_k(n) = \frac{\prod_{i=0}^{k-1} [2(n-i)+1]}{\prod_{i=0}^{k} 2(n-i+1)}; \quad k \leq n; \quad i = 1, 2, 3 \ldots \tag{34}$$

$$\nu_k(n) = \frac{\prod_{i=0}^{k-1} 2(n-1)}{\prod_{i=0}^{k} [2(n-i)+1]}; \quad k \leq n; \quad i = 1, 2, 3 \ldots \tag{35}$$

with the conventional definitions as follows:

$$\prod_{i=0}^{-1} [2(n-i)+1] \triangleq \prod_{i=0}^{-1} 2(n-i) \triangleq 1 \tag{36}$$

consequently

$$\mu_n(n) = \mu_k(n)|_{k=n} \quad \text{and} \quad \nu_n(n) = \nu_k(n)|_{k=n} \tag{37}$$

are independent of k.

Substituting the integral limits to the relations (31), (32) and (33), and after the convenient simplification we have:

$$I_{11} = \frac{2}{\pi} NC \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n+1}}{(2n+1)!} \mu_n(n) \frac{\pi}{2} \tag{31.a}$$

$$I_{12} = \frac{2}{\pi} NS \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n)!} \nu_n(n) \tag{32.a}$$

$$I_{13} = -\frac{2}{\pi} b \tag{33.a}$$

ana therefore

$$I_1 = \frac{2}{\pi} NC \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n+1}}{(2n+1)!} \mu_n(n) \frac{\pi}{2} + \frac{2}{\pi} NS \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n)!} \nu_n(n) - \frac{2}{\pi} b \tag{38}$$

The integral I_2 is easily determined, result of which is:

$$I_2 = \frac{2}{\pi} [N \sin(B+a) - b] \left\{ 1 - \left[1 - \left(1 - \frac{x_1 + x_3}{B} \right)^2 \right]^{1/2} \right\} \tag{39}$$

Similarly, the terms of the integral I_3 can be written without any special difficulty. Namely

$$I_3 = \frac{2}{\pi} \int_{\pi-\alpha}^{\pi} [NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} -$$

$$NS \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} + b] \sin \omega t \, d\, \omega t =$$

$$I_{31} + I_{32} + I_{33} \tag{40}$$

where, similarly to the (31)

$$I_{31} = \frac{2}{\pi} NC \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n+1}}{(2n+1)!} \int_{\pi-\alpha}^{\pi} \sin^{2(n+1)} \omega t \, d\, \omega t =$$

$$\frac{2}{\pi} NC \sum_{n=0}^{\infty} \sum_{k=0}^{n} (-1)^{n+1} \frac{B^{2n+1}}{(2n+1)!} \{\mu_k(n)[\sin^{2(n-k)+1} \omega t \cos \omega t] -$$

$$\mu_n(n)[\omega t]\}_{\pi-\alpha}^{\pi} \tag{41}$$

$$I_{32} = -\frac{2}{\pi} NS \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n)!} \int_{\pi-\alpha}^{\pi} \sin^{2n+1} \omega t \, d\, \omega t =$$

$$\frac{2}{\pi} NS \sum_{n=0}^{\infty} \sum_{k=0}^{n} (-1)^{n+1} \frac{B^{2n}}{(2n)!} \{\nu_k(n) \sin^{2(k+1)} \omega t \cos \omega t\}_{\pi-\alpha}^{\pi} \tag{42}$$

and

$$I_{33} = \frac{2}{\pi} \int_{\pi-\alpha}^{\pi} b \sin \omega t \, d\, \omega t = -\frac{2}{\pi} b[\cos \omega t]_{\pi-\alpha}^{\pi} \tag{43}$$

Substituting the integral limits and carrying out simplifications, the results are

$$I_{31} = \frac{2}{\pi} NC \sum_{n=0}^{\infty} \sum_{k=0}^{n} (-1)^{n+1} \frac{B^{2n+1}}{(2n+1)!} \left\{ \mu_k(n) \cdot \left(1 - \frac{x_1 + x_3}{B}\right)^{2(n-k)+1} \right.$$

$$\left. \cdot \left[1 - \left(1 - \frac{x_1 + x_3}{B}\right)^2\right]^{1/2} - \mu_n(n) \arcsin\left(1 - \frac{x_1 + x_3}{B}\right) \right\} \tag{41.a}$$

$$I_{32} = \frac{2}{\pi} NS \sum_{n=0}^{\infty} \sum_{k=0}^{n} (-1)^{n+1} \frac{B^{2n}}{(2n)!} \{\nu_k(n) \cdot \left(1 - \frac{x_1 + x_3}{B}\right)^{2(n-k)} \cdot$$

$$\cdot\left[1-\left(1-\frac{x_1+x_3}{B}\right)^2\right]^{1/2}+v_n(n)\} \tag{42.a}$$

$$I_{33}=\frac{2}{\pi}b\left\{1-\left[1-\left(1-\frac{x_1+x_3}{B}\right)^2\right]^{1/2}\right\} \tag{43.a}$$

and ultimately:

$$I_3=\frac{2}{\pi}\sum_{n=0}^{\infty}\sum_{k=1}^{n}(-1)^{n+1}\frac{B^{2n}}{(2n)!}\left\{\left(1-\frac{x_1+x_3}{B}\right)^{2(n-k)}\left[1-\left(1-\frac{x_1+x_3}{B}\right)^2\right]^{1/2}\right.$$

$$\cdot\left[NC\frac{B}{2n+1}\mu_k(n)\cdot\left(1-\frac{x_1+x_3}{B}\right)+NSv_k(n)\right]-$$

$$\left.-NC\frac{B}{2n+1}\mu_n(n)\arcsin\left(1-\frac{x_1+x_3}{B}\right)+v_n(n)NS\right\}+$$

$$+\frac{2}{\pi}b\left\{1-[1-\left(1-\frac{x_1+x_3}{B}\right)^2\right]^{1/2}\right\} \tag{44}$$

After this, as known above B_1 is the summation of I_1, I_2 and I_3, and with the convenient conversions and reductions it may be formed as follows

$$B_1=\frac{2}{\pi}\sum_{n=0}^{\infty}\sum_{k=1}^{n}(-1)^{n+1}\frac{B^{2n}}{(2n)!}\left\{\left(1-\frac{x_1+x_3}{B}\right)^{2(n-k)}\left[1-\left(1-\frac{x_1+x_3}{B}\right)\right]^{1/2}\right.$$

$$\left.\cdot\left[NC\frac{B}{2n+1}\mu_k(n)\cdot\left(1-\frac{x_1+x_3}{B}\right)^2+NSv_k(n)\right]\right\}+$$

$$+\frac{2}{\pi}\sum_{n=0}^{\infty}(-1)^n\frac{B^{2n}}{(2n)!}\left\{NC\frac{B}{2n+1}\mu_n(n)\left[\frac{\pi}{2}+arc\sin\left(1-\frac{x_1+x_3}{B}\right)\right]\right\}+$$

$$+\frac{2}{\pi}[N\sin(B-a)-b] \tag{45}$$

Thus, the determination of describing function comes to an end and its final form can be written easily from its definition:

$$F(B)=\frac{1}{B}(A_1^2+B_1^2)^{\frac{1}{2}}\cdot e^{\mathrm{j}\,\mathrm{arctg}\frac{A_1}{B_1}} \tag{46}$$

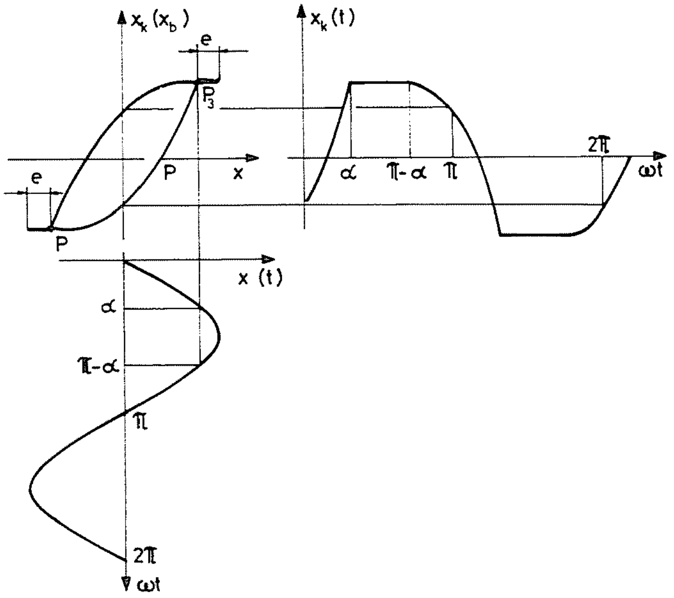


Fig. 5

Describing function of the case 1.*b. of the Fig.* 2 (*see Fig.* 5)

From Fig. 5 we can write

for part 1: $x_k = N\sin(x_b - a) + b,$ $\qquad a - \dfrac{\pi}{2} \le x_b \le a$ (47)

for part 2: $x_k = N\sin(B - e - a) + b,$ $\qquad a \le x_b \le \pi - a$ (48)

for part 3: $x_k = N\sin(x_b + a) - b,$ $\qquad -B + e \le x_b \le B - e$ (49)

In time domain:

for case 1:

$$x_k(t) = N\sin(B\sin\omega t - a) + b; \quad 0 \le \omega t \le \alpha \tag{47.a}$$

for case 2:

$$x_k(t) = N\sin(B - e - a) + b = \text{const.}; \quad \alpha \le \omega t \le \pi - \alpha \tag{48.a}$$

and for case 3:

$$x_k(t) = N\sin(B\sin\omega t + a) - b; \quad \pi - \alpha \le \omega t \le \pi\ . \tag{49.a}$$

In series form they are

$$x_k(t) = NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} -$$

$$-NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} + b \tag{47.b}$$

$$x_k(t) = N \sin (B - e - a) + b \tag{48.b}$$

$$x_k(t) = NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} +$$

$$+NS \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} - b \tag{49.b}$$

respectively. After this we may determine the Fourier coefficients of the first harmonic:

$$A_1 = \frac{2}{\pi} \int_0^{\alpha} [NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} - NS \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} +$$

$$+ b] \cos \omega t \, d\, \omega t + \frac{2}{\pi} \int_{\alpha}^{\pi-\alpha} [N \sin (B - e - a) + b] \cos \omega t \, d\, \omega t +$$

$$+ \frac{2}{\pi} \int_{\pi-\alpha}^{\pi} [NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} + NS \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} -$$

$$- b] \cos \omega t \, d\, \omega t \,. \tag{50}$$

The integration technique is similar to the previous case but we can take a special regard when the integrands are substituted into the integral expressions, and therefore the result is

$$A_1 = \frac{4}{\pi} \left[NS \sum_{n=0}^{\infty} (-1)^{n+1} \frac{B^{2n}}{(2n+1)!} \left(\frac{\mathrm{B}-\mathrm{e}}{\mathrm{B}} \right)^{2n+1} + \mathrm{b} \left(\frac{\mathrm{B}-\mathrm{e}}{\mathrm{B}} \right) \right]. \tag{51}$$

The coefficient B_1 may be expressed in the following form

$$B_1 = \frac{2}{\pi} \int\limits_0^\alpha [NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} - NS \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} +$$

$$+ b] \sin \omega t \, \mathrm{d}\, \omega t + \frac{2}{\pi} \int\limits_\alpha^{\pi-\alpha} [N \sin (B-e-a) + b] \sin \omega t \, \mathrm{d}\, \omega t +$$

$$+ \frac{2}{\pi} \int\limits_{\pi-\alpha}^{\pi} [NC \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} + NS \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} -$$

$$- b] \sin \omega t \, \mathrm{d}\, \omega t \,. \tag{52}$$

With a little complicated integrations and conversions we get the following results

$$B_1 = \frac{4}{\pi} \sum_{n=0}^{\infty} \sum_{k=0}^{n} (-1)^n \frac{B^{2n}}{(2n)!} \left(\frac{B-e}{B} \right)^{2(n-k)}$$

$$\cdot \left[1 - \left(\frac{B-e}{B} \right)^2 \right]^{1/2} \cdot [NS v_k(n) - \frac{B-e}{2n+1} \mu_k(n)] +$$

$$+ \frac{4}{\pi} \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n)!} \left[NS v_n(n) + NC \frac{B}{2n+1} \mu_n(n) \arcsin \left(\frac{B-e}{B} \right) \right] +$$

$$+ \frac{4}{\pi} \left\{ N \sin (B-e-a) + b \left[1 - \left(\frac{B-e}{B} \right)^2 \right] \right\} \tag{53}$$

where $\mu_k(n)$ and $v_k(n)$ are also determined by means of the relations (34) and (35) ($k=0, 1, 2, \ldots n$, and $n=0, 1, 2, \ldots n$)

If $e=0$, i.e. there is no prolongation or horizontal straight piece, denoting the saturation of the nonlinearity output signal (i.e. the saturation of the characteristics $x_k(x_b)$), then

$$A_1 = \frac{4}{\pi} [NS \sum_{n=0}^{\infty} (-1)^{n+1} \frac{B^{2n}}{(2n+1)!} + b] \tag{51.a}$$

and

$$B_1 = \frac{4}{\pi} \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n)!} \left[NS v_n(n) + NC \frac{B}{2n+1} \mu_n(n) \frac{\pi}{2} \right] +$$

$$+ \frac{4}{\pi} [N \sin (B-a)] \tag{53.a}$$

Describing function of the case 1.c of the Fig. 2 (see Fig. 6)

From Fig. 6 it can be written that

$$\gamma = \arcsin \frac{B-e}{B}, \quad \text{and} \quad \alpha = \arcsin \frac{B-2\left(h-\frac{e}{2}\right)}{B}$$

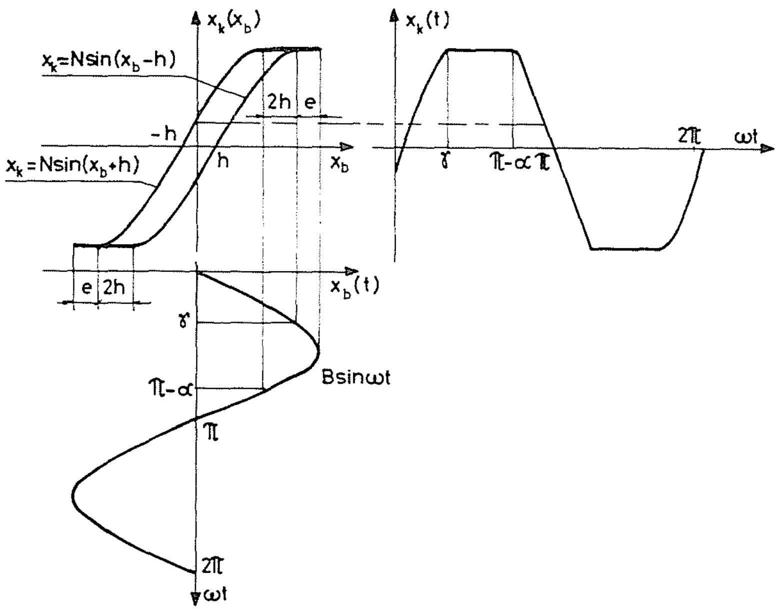


Fig. 6

Similarly to that done in the above case, in the present case for the output signal of the nonlinearity in the $[0, \pi]$ interval the following relations can be formed:

$$\begin{aligned} x_k(t) &= N \sin (B \sin \omega t - h); && 0 \leq \omega t \leq \gamma \\ x_k(t) &= N \sin (B - h - e) = \text{cons}\, t; && \gamma \leq \omega t \leq \pi - \gamma \qquad (54.\text{a}) \\ x_k(t) &= N \sin (B \sin \omega t + h); && \pi - \gamma \leq \omega t \leq \pi \end{aligned}$$

Now, by means of the customary procedure the Fourier coefficients A_1 and B_1 can be determined without difficulties. Namely, also in the form of convenient series:

$$A_1 = \frac{2}{\pi}\int_0^{\gamma}\left\{NC_h\sum_{n=0}^{\infty}(-1)^n\frac{(B\sin\omega t)^{2n+1}}{(2n+1)!} - NS_h\sum_{n=0}^{\infty}(-1)^n\frac{(B\sin\omega t)^{2n}}{(2n)!}\right\}\cdot$$

$$\cdot\cos\omega t\,\mathrm{d}\,\omega t = \frac{2}{\pi}\int_{\gamma}^{\pi-\alpha} N(B-h-e)\cos\omega t\,\mathrm{d}\,\omega t +$$

$$+\frac{2}{\pi}\int_{\pi-\alpha}^{\pi}\left\{NC\sum_{n=0}^{\infty}(-1)^n\frac{(B\sin\omega t)^{2n+1}}{(2n+1)!} + NS_h\sum_{n=0}^{\infty}(-1)^n\frac{(B\sin\omega t)^{2n}}{(2n)!}\right\}\cdot$$

$$\cdot\cos\omega t\,\mathrm{d}\,\omega t \tag{54}$$

where $C_h=\cos h$ and $S_h=\sin h$. After calculating the definite integrals in the expression (54), The result is

$$A_1 = \frac{2}{\pi}\sum_{n=0}^{\infty}(-1)^n\frac{B^{2n}}{(2n+1)!}\left\{\left(\frac{B-e}{B}\right)^{2n+1}\left(NC_h\frac{B-e}{2(n+1)} - NS_h\right) -\right.$$

$$\left.-\left[\frac{B-2\left(h-\frac{e}{2}\right)}{B}\right]^{2n+1}\left[NC_h\frac{B-2\left(h-\frac{e}{2}\right)}{2(n+1)} - NS_h\right]\right\} -$$

$$-\frac{2}{\pi}N\sin(B-h-e)\cdot\left(\frac{2h}{B}\right). \tag{55}$$

If there is no prolongated piece of the characteristics $x_k(x_b)$ (see Fig. 6), i.e. $e=0$, then

$$A_1 = \frac{2}{\pi}\sum_{n=0}^{\infty}(-1)^n\frac{B^{2n}}{(2n+1)!}\left\{NC_h\frac{B}{2(n+1)}\left[1-\left(\frac{B-2h}{B}\right)^{2n+1}\right]+\right.$$

$$\left.+NC_h\frac{2h}{2(n+1)}\left(\frac{B-h}{B}\right)^{2n+1} - NS_h\left[1+\left(\frac{B-2h}{B}\right)^{2n+1}\right]\right\} -$$

$$-\frac{2}{\pi}N\sin(B-h)\cdot\left(\frac{2h}{B}\right). \tag{55.a}$$

With a similar procedure the B_1 magnitude may be written as follows

$$B_1 = \frac{2}{\pi} \int_0^{\gamma} \left\{ NC_h \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} - NS_h \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} \right\} \cdot$$

$$\cdot \sin \omega t \, \mathrm{d}\, \omega t + \frac{2}{\pi} \int_{\gamma}^{\pi - \alpha} N(B - h - e) \sin \omega t \, \mathrm{d}\, \omega t +$$

$$+ \frac{2}{\pi} \int_{\pi-\alpha}^{\pi} \left\{ NC_h \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n+1}}{(2n+1)!} + NS_h \sum_{n=0}^{\infty} (-1)^n \frac{(B \sin \omega t)^{2n}}{(2n)!} \right\} \cdot$$

$$\cdot \sin \omega t \, \mathrm{d}\, \omega t \, . \tag{56}$$

Carrying out the necessary steps of calculation the next formula will be obtained

$$B_1 = \frac{2}{\pi} \sum_{n=0}^{\infty} \sum_{k=0}^{n} (-1)^{n+1} \frac{B^{2n}}{(2n)!} \{ [NC_h \frac{B}{2n+1} \mu_k(n) \cdot \left(\frac{B-e}{B} \right) - NS_h \nu_k(n)] \cdot$$

$$\cdot \left(\frac{B-e}{B} \right)^{2(n-k)} \left[1 - \left(\frac{B-e}{B} \right)^2 \right]^{1/2} + [NC_h \frac{B}{2n+1} \mu_k(n) \frac{B - 2\left(h + \frac{e}{2}\right)}{B} +$$

$$+ NS_h \nu_k(n)] \left(\frac{B - 2\left(h + \frac{e}{2}\right)}{B} \right)^{2(n-k)} \cdot \left[1 - \left(\frac{B - 2\left(h + \frac{e}{2}\right)}{B} \right)^2 \right]^{1/2} +$$

$$+ \frac{2}{\pi} \sum_{n=0}^{\infty} (-1)^n \frac{B^{2n}}{(2n)!} NC_h \mu_n(n) \frac{B}{2n+1} \left[\arcsin \left(\frac{B-e}{B} \right) + \right.$$

$$\left. + \arcsin \frac{B - 2\left(h - \frac{e}{2}\right)}{B} \right] + 2NS_h \nu_n(n) \} +$$

$$+ \frac{2}{\pi} N \sin (B - h - e) \left\{ \left[1 - \left(\frac{B - 2\left(h + \frac{e}{2}\right)}{B} \right)^2 \right]^{1/2} + \left[1 - \left(\frac{B-e}{B} \right)^2 \right]^{1/2} \right\} . \tag{57}$$

If $e=0$, then

$$B_1=\frac{2}{\pi}\sum_{n=0}^{\infty}\sum_{k=0}^{n}(-1)^{n+1}\frac{B^{2n}}{(2n)!}\left\{[NC_h\frac{B}{2n+1}\mu_k(n)+NS_h\nu_k(n)]\cdot\right.$$

$$\left.\cdot\left(\frac{B-2h}{B}\right)^{2(n-k}\left[1-\left(\frac{B-2h}{B}\right)^2\right]^{1/2}\right\}+\frac{2}{\pi}\sum_{n=0}^{\infty}(-1)^n\frac{B^{2n}}{2n+1}\cdot$$

$$\cdot\{NC_h\mu_n(n)\frac{B}{2n+1}\left[\frac{\pi}{2}+\arcsin\left(\frac{B-2h}{B}\right)\right]+2\nu_n(n)NS_h\}+$$

$$+\frac{2}{\pi}N\sin(B-h)\left[1-\left(\frac{B-2h}{B}\right)^2\right]^{1/2}. \tag{57.a}$$

Conclusions

Practically, the approximation is completed, and the concrete calculations of the three describing functions in question are just mechanical matters by means of formulae (7) and (9). Therefore, it is not worthy to deal with them. As far as the engineering practice and inaccuracy due to the resolving in series form are concerned it is necessary to make some remarks.

It is obvious, that every formulae, derived above, are a little complicated, on this account in reality we shall only use the first few terms of them knowing that these series are very strongly convergent. The inaccuracy, in fact, is the residue series, which may easily be estimated, because it is the sign of alternative series and absolutely convergent, therefore the maximal error is not greater than the absolute magnitude of the first term of the residue series. If in practice we use only the first three (i.e. $n=3$) terms of the series of A_1 and B_1, then the order of error is already some percentages due to the factorial of $2n$ and $(2n+1)$, and in such a case $2n!=6!=720$ and $(2n+1)!=7!=5040$. In addition the numerator magnitude in the series is always smaller than unit.

Illustrative examples

For presentation of preceding method in the following we shall calculate some examples, namely we shall deal with the determination of the nonlinearity parameters and the describing function of case c) (see Fig. 6).

Example 1.

If in a testing the following data are received

$$x_1 = -1.372; \quad y_1 = -2$$
$$x_2 = 0.2; \quad y_2 = 0$$
$$x_3 = 1.772; \quad y_3 = 2$$

By application of the relation (1.a) and the Eq. (4) we have

$$\mathbf{A} = \begin{bmatrix} -0.9800666 & 0.1986693 & 1 \\ 0.1986693 & 0.9800666 & 1 \\ 0.9800666 & -0.1986693 & 1 \end{bmatrix}$$

$$\vec{y}^T = [-2 \quad 0 \quad 2]$$

It is evident that

$$\text{Det } \mathbf{A} = -2 \neq 0$$

So, the method of Cramer can be used and the results are

$$X = 1.96013315568$$
$$Y = -0.39733866159$$
$$Z = 0$$

from which the parameters of interest are

$$a = -0.2; \quad b = 0; \quad N = 2.$$

(Here it must be stated that the so-called testing data are taken down by the author with purpose for controlling, therefore the received values are real or integer.)

Example 2.

Describing function of case c) with $e=0$: In this case we shall use the expression (55.a) and (57.a). If $n=3$ then

$$A_1=\frac{2}{\pi}N\left\{\left[C_h\frac{B}{2}-S_h-\frac{B-2h}{B}\left(C_h\frac{B-2h}{B}-S_h\right)\right]\right.$$

$$-\frac{B^2}{3!}\left[C_h\frac{B}{4}-S_h-\left(\frac{B-2h}{B}\right)^3\left(C_h\frac{B-2h}{4}-S_h\right)\right]+$$

$$+\frac{B^4}{5!}\left[C_h\frac{B}{6}-S_h-\left(\frac{B-2h}{B}\right)^5\left(C_h\frac{B-2h}{6}-S_h\right)\right]$$

$$\left.-\frac{B^6}{7!}\left[C_h\frac{B}{8}-S_h-\left(\frac{B-2h}{B}\right)^7\left(C_h\frac{B-2h}{8}-S_h\right)\right]-\sin(B-h)\frac{2h}{B}\right\}$$

$$B_1=\frac{2}{\pi}N\left\langle\left\{-\left(C_h\frac{B}{2}+S_h\right)\left[1-\left(\frac{B-2h}{B}\right)^2\right]^{1/2}+\frac{B^2}{2!}\left[1-\left(\frac{B-2h}{B}\right)^2\right]^{1/2}\right.\right.$$

$$\left[\left(C_h\frac{B}{12}+S_h\frac{1}{3}\right)\left(\frac{B-2h}{B}\right)^2+C_h\frac{B}{8}+S_h\frac{2}{3}\right]-\frac{B^4}{4!}\left[1-\left(\frac{B-2h}{B}\right)^2\right]^{1/2}$$

$$\left[\left(C_h\frac{B}{30}+\frac{S_h}{5}\right)\left(\frac{B-2h}{B}\right)^4+\left(C_h\frac{B}{24}+S_h\frac{4}{15}\right)\left(\frac{B-2h}{B}\right)^2+C_h\frac{B}{16}+S_h\frac{8}{15}\right]+$$

$$+\frac{B^6}{6!}\left[1-\left(\frac{B-2h}{B}\right)^2\right]^{1/2}\left[\left(C_h\frac{B}{56}+S_h\frac{1}{7}\right)\left(\frac{B-2h}{B}\right)^6+\right.$$

$$+\left(C_h\frac{B}{48}+S_h\frac{6}{35}\right)\left(\frac{B-2h}{B}\right)^4$$

$$\left.\left.+\left(C_h\frac{5B}{192}+S_h\frac{8}{35}\right)\left(\frac{B-2h}{B}\right)^2+C_h\frac{5B}{128}+S_h\frac{16}{35}\right]\right\}+$$

$$+\frac{1}{2}C_hB\left[\frac{\pi}{2}+\arcsin\left(\frac{B-2h}{B}\right)\right]+2S_h-\frac{B^2}{2}\left\{C_h\frac{B}{8}\left[\frac{\pi}{2}+\arcsin\left(\frac{B-2h}{B}\right)\right.\right.$$

$$\left.+S_h\frac{4}{3}\right\}+\frac{B^4}{4!}\left\{C_h\frac{B}{16}\left[\frac{\pi}{2}+\arcsin\left(\frac{B-2h}{B}\right)\right]+S_h\frac{16}{15}\right\}-\frac{B^6}{6!}$$

$$\left.\left\{C_h\frac{5B}{128}\left[\frac{\pi}{2}+\arcsin\left(\frac{B-2h}{B}\right)\right]+S_h\frac{32}{35}\right\}+\sin(B-h)\left[1-\left(\frac{B-2h}{B}\right)^2\right]^{1/2}\right\rangle.$$

With a convenient computer program we received the following numerical and graphical results (see the table 1 and Fig. 7). After this the next step of the investigation of a system of interest may be carried out with the same procedure presented in [1]. Therefore, it is not dealt here, because the determination of the describing function of nonlinearity in question comes to an end.

Table 1

H/B	$MOD \cdot N(B)/N$	$ARGUM \cdot N(B) = (\varphi^\circ)$
0.059851	1.140653	—0.02573
0.112943	1.119944	—0.19447
0.160359	1.078643	—0.63763
0.202963	1.018513	—1.49751
0.241453	0.939837	—2.96216
0.276396	0.842636	—5.32940
0.308261	0.727660	—9.14614
0.337439	0.597956	—15.5513
0.364255	0.462843	—27.1791

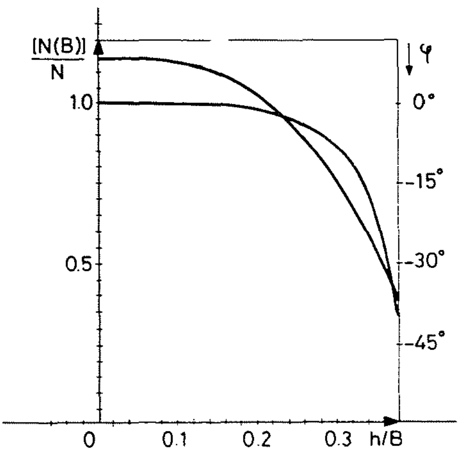


Fig. 7

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