A SOLUTION FOR THE INTERSECTION OF TWO CONVEX POLYHEDRA

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Received June 28, 1984
Presented by Prof. Dr. Gy. STROMMER

Summary

A computer program is presented for determining the polygon of intersection of two convex polyhedra. The algorithm is based upon the trivial construction and uses $O(N^2)$ operations, where N is the sum of the numbers of edges of the two polyhedra. The program is written in Basic for the personal computer Commodore 64.

Finding the intersection of two convex polyhedra in three-dimensional space is a classical problem in computational geometry, and so far a few algorithms have been published already for the solution. In this paper a computer program is presented for determining the polygon of intersection and illustrating the solution.

Let A and B be two convex polyhedra in three-dimensional space. We assume that the coordinates of the vertices are given in Descartes coordinate-system, moreover, the system of the edges of both polyhedra are given, as well [1]. Denote $E_1(A), E_2(A), \ldots, E_{NA}(A)$ the edges of the polyhedron A and $E_1(B)$, $E_2(B), \ldots, E_{NB}(B)$ the edges of the polyhedron B, respectively. In order to find the vertices of the polygon of intersection the edges $E_i(A), i=1, \ldots, NA$ have to be tested against each face of B, then the edges $E_j(B), j=1, \ldots, NB$ against each face of A to see if they intersect. The numbers of the faces of A and of B are $O(NA)$ and $O(NB)$, respectively, because Euler's theorem holds. Thus this investigation for intersection requires $O(N^2)$ operations, where $N=NA+NB$.

The polyhedra and their intersection are illustrated by the methods of descriptive geometry [2]. On the figures orthogonal axonometric projections are shown. The common part of the two polyhedra is omitted.

The program for determining the intersection of the two polyhedra and computing their projections is written in Basic for the personal computer Commodore 64. The plane of the projection is the TV-screen, the direction of the projection is perpendicular to it. The line segments are drawn by the command LINE of the SIMON'S BASIC in the 320×240 resolution of the

screen. The hidden lines are shown in broken lines or they can be omitted. The figures are made by a matrix printer which copies the content of the screen.

It has to be remarked that the algorithm for finding the intersection of two convex polyhedra of Müller and Preparata [1] uses only $O(\log N)$ time, but it is rather complicated for programming in Basic. On the other hand, our algorithm can be extended for the concave case, however, the time requirement may increase up to $O(N^3 \log N)$.

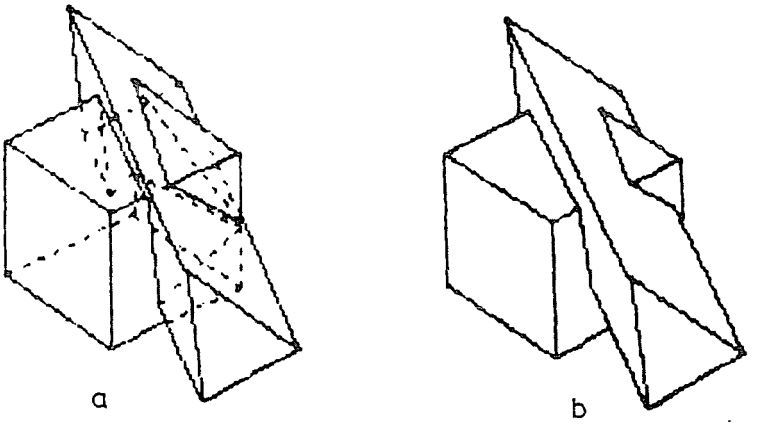

Fig. 1. The intersection of a cube with a prism is a single polygon

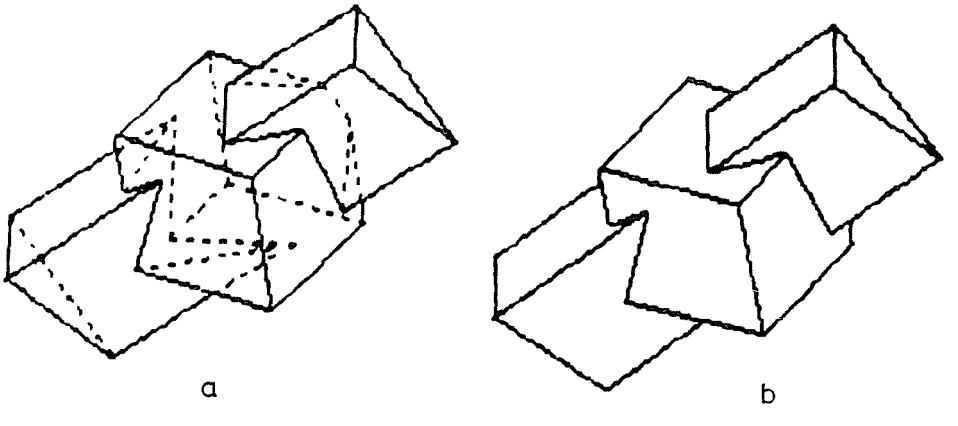

Fig. 2. The intersection of a cube with a prism consists of two separate parts

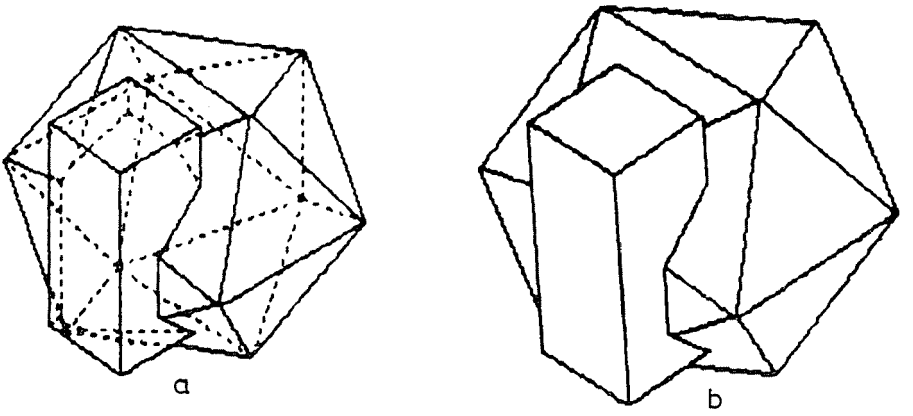

Fig. 3. The intersection of an icosahedron with a prism

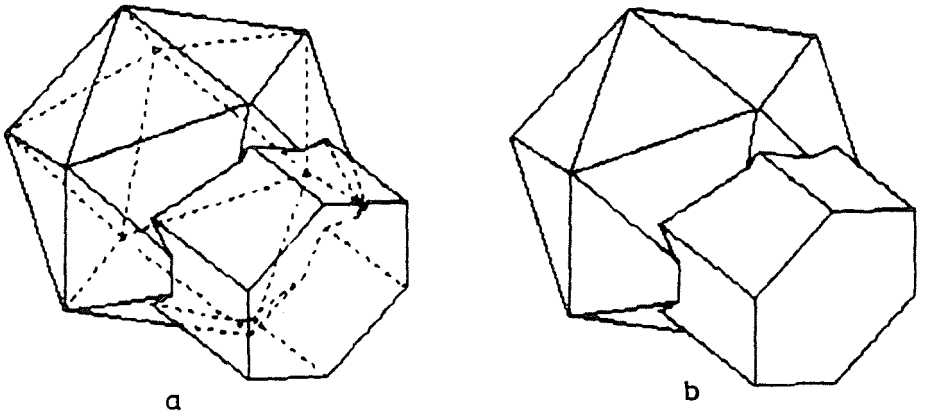

Fig. 4. One face of the prism lies in the interior of the icosahedron

References

1. MÜLLER, D. E.—PREPARATA, F. P.: Finding the intersection of two convex polyhedra. Theoretical Computer Science 7, 217 (1978)
2. STROMMER, Gy.: Ábrázoló geometria. Tankönyvkiadó, Budapest, 1971.

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