A MATHEMATICAL MODEL FOR LOGISTIC ANALYSIS OF INDUSTRIAL PRODUCTION SYSTEMS

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Summary

It is well known from practical experiences, that a constantly growing part of a product's total production costs falls to purchasing and realization. An attitude is spreading which treats all the processes of production, purchase and realization as a homogeneous system. The paper gives the possibility of setting up a model of material flow processes, taking into account this complex attitude.

The notion of logistics

Due to development of production forces, to increasing division of labour and to the intensification of the competition at the market, the economical conditions for industrial production are becoming more and more complex. One of the consequences is the increasing role of transport within the process of reproduction. Transport and production are separated by storage, which is preceeded and succeeded by loading. Along with the increasing role of these activities, the requirements packing, creating lots, organization and informatics are growing as well. We can learn that the importance of a lot of activities which have always been considered as provision, as the necessary evil belonging to production increased. Then the processes of provision character came into prominence, change of the attitude was needed over almost all areas of it. This new attitude is characterized by the idea of logistics, which—according to Bowersox—can be defined as follows:

The process of strategical managing the movement and storage oi raw materials, parts and finished products from the source of supply through the manufacturer to the customer [1].

It is a new conception, which emphasizes the integration of productional functions, traditionally separated, however the activities of logistics are well known. This fact is one of its attractive features, but at the same time prevents its wide-spread use, since it can be introduced only by a new managerial structure and attitude. The results of applying logistics can easily be studied considering the Japanese car industry. The Japanese cars are very competitive on the market, mainly due to the very low cost of semi-processed goods and material flow. The Japanese recognized the fact, that the costs of production and labour are more or less constant, consequently the possibilities of saving money are to be found in improving the quality of semi-finished goods, bought from cooperating partners and in raising the efficiency of material management. The same conclusion can be drawn from the results of a US-made cost analysis, according to which investments improving the spatial arrangement of processes are often more efficient than those modernizing the production (Fig. 1).

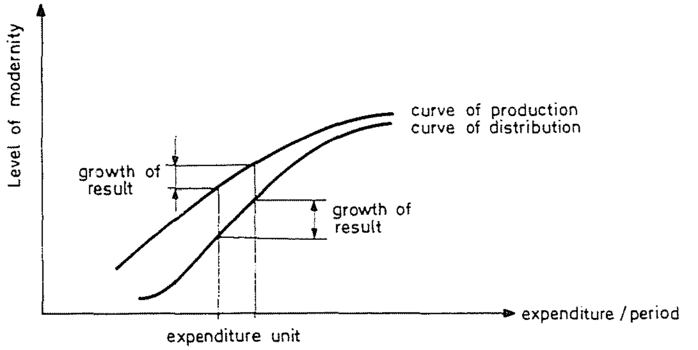

Fig. 1. The efficiency of modernization of production and spatial distribution

Finally a convincing information from Great-Britain: jobs with logistical functions have been made by 29% of inhibitants, and these jobs have achieved the 32,5% of GDP [2].

It can be seen from the last example, that logistics has both micro- and macroeconomic contents, i.e. it can be interpreted on all the hierarchical levels of the economic system. There are three subsystems to be distinguished between:

1. *Macrologistical systems*, i.e. logistical systems of the society, containing the basic LTS (Loading–Transportation–Storage) processes and their elements which control the main socio-economic activities.
2. *Systems of chains of transportation* are structural units of macrologistical systems, consisting of human and technical resources under common control.
3. *Micrologistical systems*, are parts of a macrologistical system, which are to be set up and controlled by the enterprise management.

The main aim of all these systems is to achieve a synchronized material flow, so that the material needed is always

— at the proper place,
— in due course and,
— at the lowest cost.

Mathematical methods of analyzing material flow

When applying one attitude in the practice, it is always needed to have a well-defined set of criteria. To analyze one element of material flows, there are well-known and proved methods available. One part of them is built on descriptive technics, geometric means, simple mathematical procedures. They are used in general for analyzing material handling processes, (material flowcharts, trading indices) for layout planning (triangle-method, method of fictive order of operations, geometrical method) [4]. There are a lot of restrictive conditions to be considered. Therefore the application of this method is proposed only, if a solution is required for a problem very quickly but not very accurately. Other methods, which apply more sophisticated tools—like operations research and computing—are able to take into consideration more conditions and more complex structures. Accordingly, there are practically well proved methods for analysis e.g. queuing systems [7], for transportation ways, for layout planning [10], for defining inventorial strategy [12] and for optimal loading capacity utilization conditions [3] or solving similar problems. All these methods are optimizing one of the logistical subsystems (i.e. storage, material handling, transportation etc.) However—as mentioned before—it would be our aim to join the traditionally separately treated elements into one integrated system. This can be done by help of the well-known simulation, but simulation is a method based on tests, and therefore it does not lead always to the optimal solution.

Further a model will be introduced which is suitable for co-ordination of logistical activities among themselves and with production processes, as well as for analyzing complex material flow systems. It offeres possibility for describing the connections and relations between micro- and macroeconomic levels.

Verbal definition of the model

The model is based on a directed graph, representing the whole system of material flow. The links of the graph are representing the activities of the subsystems (transportation, storage, production), while the nodes are showing the status of the examined material as a result of the activity in question. When attaching the numerical values of the material processed by the activities to the links of the graph, a network is set up. This idea of a network means a more or less extended version of the transportation problem well-known in operations research.

The transportation problem can be formulated so that if the inventories and the demands are known, the most efficient ways and transported quantities are to be defined. This problem is incorporated in our model which is meeting an aim on a higher level at the same time: it takes the interests of the whole system into consideration, i.e. it optimizes the inventories of the origins and the demands at the destinations, as well.

In the above conception there is a need to modify the interpretation of "material flow". A lot of activities can be found, where material does not "flow" in everyday sense (e.g. storage, heat-treatment). It is better to define material flow as a *change of value*, since the value of the material (product) is increasing when passing the individual activities due to direct and indirect allocations. One of the quantitative characteristics, the economical index attached to the links, shows the measure of this change in value. This index is linked with the economic goal of the whole system, therefore it can be the cost function or the change of break-event point as well. A detailed analysis is needed to choose the most suitable index. There is another quantitative characteristic to be attached to the links: the capacity of the individual subsystems. Now let's define the following question for our model: Supposed, that the given system has to produce a certain amount of products; which are the optimal routes of the material (product) flow?

Let's have two simple examples to illustrate the above conception. There is the route of a product made of plastic shown on Fig. 2. We tried to represent the whole logistical system connected with this product in a simplified way. Using the model introduced, the optimal routing of the product can be defined from the point of view of the optimum input/output ratio. That means, that questions are to be answered, like: how much to transport, how much to store, to which workshop, from the workshops to which store, from the stores to

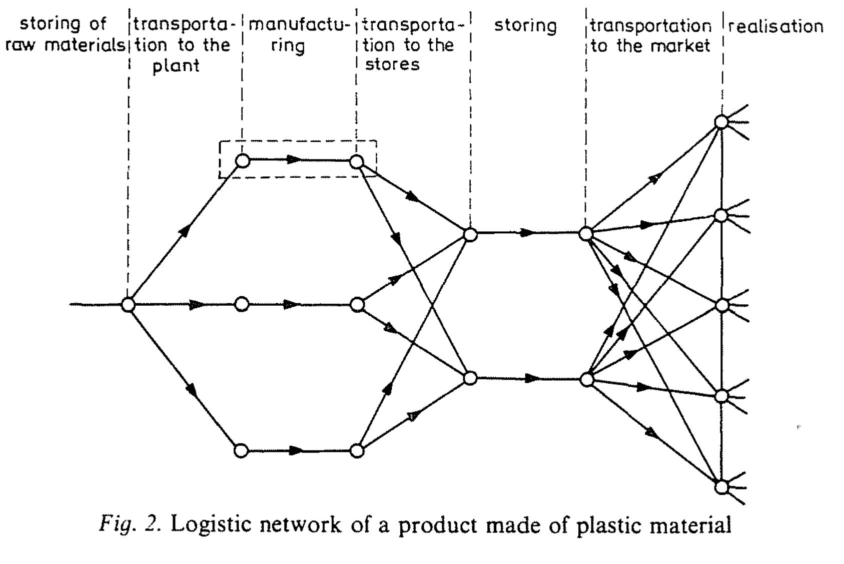

Fig. 2. Logistic network of a product made of plastic material

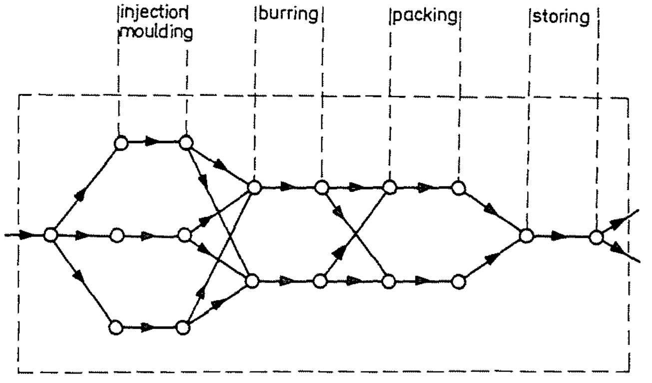

Fig. 3. Symbolic network model of a workshop

which trading department, etc. Let's analyze now the activities carried out by one of the manufacturers in the system. On Fig. 3. the possibilities are shown for applying the model on workshop level. (It's the enlargement of the indicated part in Fig. 2).

In this case, the optimal routing can also be defined from the point of view of input/output. This way, when applying the model of higher hierarchical level, the result obtained defines the tasks of every single subsystem, the operation is to be optimized according to this problem. The example shown is of course a very simplified version of the real situation; it's for illustration only.

Mathematical definition of the model

The following problem is to be defined mathematically:
— the possible routings of material flow are given in a graph;
— the flow value—i.e. the specified output quantity of the system—is known;
— the costs and capacities of the individual activities are also known.
The solution should be an optimal routing from the costs' point of view, which defines the workshops, stores, transport routes, and the quantity of product to be produced, transported, stored.

Let's have the following form of representation:

E — incidency matrix defining the graph;
x_{ik} — material flow between the i-the and k-the node of the graph;
x — vector involving all $x_{ik}-s$;
g — flow value vector; (Its elements are the flow values of the nodes. In the case of a one 0—one I network, reasonably all the elements are zero, except the first and the last one)

c_{ik} — link costs attached to the links;
c — vector involving all the $c_{ik}-s$;
u_{ik} — capacities attached to the links;
u — vector involving the $u_{ik}-s$;
Now the problem can be defined as follows:

$$\mathbf{E}x = g$$
$$O \leq x \leq u \tag{1}$$
$$\min (c'x)$$

It's a linear program, where the coefficient matrix is an incidency matrix representing the graph. To solve linear programs, the simplex algorithm and its version can be used.

When having a large coefficient matrix, the simplex method is not efficient and exact enough, therefore it is not proposed by the literature [11]. If the model has many nodes, the coefficient matrix will surely be large. In this case the out-of-kilter method can be used successfully, which has been developed independently by Minty and Fulkerson [9], [5]. This method is based on duality and complementary theorem.

According to the concept of duality, every linear program has a corresponding program—it's dual. The original and the dual program are in a close relation to such an extent, that if we solve one of them, the other is solved at the same time. Let's have vector y containing the dual variables. In this case the dual problem can be defined in case of (1) as follows:

$$\mathbf{E}'y \leq c$$
$$\max (y'g) \tag{2}$$

The theorem of complementary establishes a relationship between the independent variables (x, y) of both the primal and dual problems. This relationship can be traced with the help of a complementarity diagram (Fig. 4).

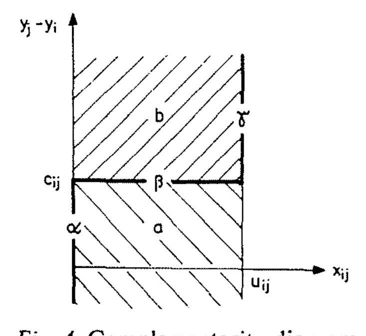

Fig. 4. Complementarity-diagram

In cases denoted by α, β and γ the variables are in "in-kilter" status while in cases a and b, they are "out-of-kilter". So called kilter-numbers are introduced for measuring the values of relationships in a and b status:

$$k_a = x_{ij}(c_{ij} - y_j - y_i)$$

$$k_b = (u_{ij} - x_{ij})(y_j - y_i - c_{ij})$$

All the variables with kilter-number zero are in α, β or γ status, and are optimal. As a result of the algorithm monotonously decreasing the kilter-numbers, the variables are reaching the α, β, γ status—thus the optimal solution. In the literature mentioned above the method of decreasing the kilter-numbers is presented in detail.

The kilter-method has remarkable advantages over the simplex algorithm: it is capable of handling many variables, and it does not need an allowed starting-solution. (It is known, that the simplex algorithm has two phases: the first one is to search for a solution to start with.) [5].

Note, that the problem raised is the problem of finding a route with minimal amount of costs. If we interprete this problem as a linear program, then simplex and kilter algorithms are suitable for finding a solution. There are however other methods for defining a cost-minimal route, which can also be used [8].

Some remarks on the model

To have a product of good quality, you first of all need a good design. The quality of the design is, however, only one of the factors determining the output (profit) of this product: the costs of production and marketing are relevant, too.

Therefore it is very useful to set up models, which analyze the "realization" of a given product with given technical parameters. Realization can be understood as the whole logistical process, related to the product.

The main benefit of the above described model is, that it raises the efficiency of a given process from the whole system's point of view. Most of the previously known methods optimized the elements, but not the system itself. Having an enterprise with a very good inventory management does not mean having high efficiency at the same time: the material consumption in the production may be too high, transportation out of data and therefore unable to move the finished goods to the users in time, etc. The operation of the individual subsystems—elements—should be optimized together, but of course it can occur that the optimal solution of the whole system does not equal with the sum of the optimal solutions on subsystem level. It's task of a well-suited managerial strategy to solve these contradictions or to develop systems, which already take into consideration this problem in the design phase.

The model shown is suitable for simulating the changing environment as well. In case of some changes in costs (e.g. energy costs are getting higher) the model determines a new optimal flow of material, which can be compared with the previous solution to analyze the effects of the change.

In order to realize all the possibilities offered by the model, it should fit well into the operating mechanism of a given enterprise. There should also be a powerful manager, who pushes the results through.

It's always a fundamental prerequisite to have reliable data when using computer based models. Having non real data leads to irreal plans. All the financial and intellectual efforts invested by using up-to-date calculating means are only then worthwhile if the solutions offer a practicable surplus of the output (or decrease of allocation). There could be data needful, which are not in disposition because the former methods used did not require their determination. May be that no methods exist to gain another data. It is very important therefore, that users of models have economic and budgeting knowledge, which establishes link between mathematical model based on abstractions and the everyday life.

There are certain drawbacks of the model, which should be mentioned. One of them is the application of stochastical variables. The other appears only when dealing with materials of different types; simultaneously, i.e. the model cannot solve a multi-commodity (space sharing) network. Tests are in progress to develop methods in these fields, too.

Among the benefits of using the model in the practice two aspects are to be underlined:

1. The model stimulates the management of organizations towards such a concept, the realization of which is one of the basic preconditions of successful operation and management in our economic environment today.

2. The network model of the system examined offers not only quantitive results, but can be also a source for several important qualitative considerations.

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