CONDITIONS OF FUEL EQUILIBRIUM OF THE MIXED NUCLEAR ENERGY SYSTEM

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Summary

The interest in mixed nuclear energy systems (MNES) containing both fast and thermal reactors has been increased in the recent 10 years. This work investigates the fuel cycle of such systems. A mathematical model and algorithm are given to determine the conditions of equilibrium. The model presented is suited to determine the conditions of a mixed nuclear energy system at fuel equilibrium (MNESFE) and it can also be used to analyze the characteristics of nuclear energy systems containing fast reactors of unsatisfactory breeding factor.

Introduction

Recently it has been widely believed that only nuclear energy systems built with fast breeder reactors exclusively will economically remedy the energy problem of the years to come. The same is believed also by many experts, of course first of all by those engaged in the development of fast reactors, who say that fast reactor systems utilize natural uranium more efficiently than thermal reactors, and thus a reasonable utilization of the natural uranium resources can be expected from fast reactor systems alone.

However, nuclear energetics is based almost entirely on thermal reactors at present, and, among other réasons, both economical and technological considerations suggest giving the thermal reactors their due role also in the future. Quite likely, the increasing interest in the mixed nuclear energy system (MNES) has been aroused more recently by the intention to preserve the thermal technology.

It was at MEI (Moscow Institute of Energetics) in December 1965 when I reported the first results of my investigations on the mixed system [1] and said for the first time that establishment of MNES containing both thermal and fast reactors was worth considering. Also, I disclosed the approximate conditions of equilibrium of such a mixed system. I personally believed that the mixed nuclear energy system was competitive with the system containing fast reactors only also in respect of economic utilization of natural uranium resources

provided certain conditions were fulfilled. This view has been outlined in detail in our later work [2, 3].

After the mid 1970s, the interest in such a MNES became more and more explicit on an international scale. This paper is intended to sum up the conditions of equilibrium of this mixed system, and to present the mathematical algorithm of the system. The results obtained on the basis of calculations, and the most important conclusions are given in another paper published in this issue [4].

Most important characteristics of the nuclear power plants investigated

Nuclear power plants with thermal and fast reactor in operation or under development have been involved in the investigation. The fuel cycle parameters for nuclear power plants with thermal and fast reactor are tabulated in Tables I and II, respectively. On the basis of the Tables, the following essential conclusions can be drawn:

— Among thermal reactors, the two extremes are represented by HWR/1 and HTGR, some of their parameters differing from each other by more than an order of magnitude. The parameters of PWRs lie close to the mean values of HWR/1 and HTGR.

— As compared with thermal reactors, the divergence between the parameters of fast reactors is less. It is the N_2O_4-cooled fast reactor alone that differs from the other fast reactors considerably first of all in the very low fissile plutonium content of the initial active core. However, as will be seen later, even minor deviations represent quite a significant difference under conditions prevailing in a MNES.

— The amount of uranium to be built into the active core and blanket of fast reactors lies in the same order of magnitude as the initial leading of thermal reactors, or even exceeds it for some types.

— Thermal reactors produce less fissile plutonium than the fissile material they consume. The gap shall be made up by enrichment or, in case of a MNES, by plutonium produced in excess in the fast reactors.

— Fast reactors produce more fissile material than the amount they consume. This excess fissile material can be used as the initial leading of new fast reactors or, in case of a MNES, the amount beyond the demand of fast reactors may be fuelled into thermal reactors in operation or to be put into new reactors entering the system, reducing thus or, in case of a system at fuel equilibrium, meeting entirely the deficit in fissile material in thermal reactors.

The data given in Tables I and II apply to uranium fuelled thermal reactors and to fast reactors fuelled with the plutonium produced in such thermal reactors. This fact shall be emphasized because the plutonium content

Table I

Parameters of fast reactors investigated, important in respect of fuel cycle

Parameter			Fast reactor*			
definition	symbol	unit	LMFBR/1	LMFBR/2	GCFR	N_2O_4-cooled
Initial core fissile Pu content	m_{pi}	kgMW(e)$^{-1}$	3.23	2.488	4.132	1.02
Fissile Pu annual reloading	m_{pr}	kgMW(e)$^{-1}$ year^{-1}	1.292	1.00	1.00	1.25
Fissile Pu annual discharge	m_{pd}	kgMW(e)$^{-1}$ year^{-1}	1.501	1.388	1.250	1.82
Initial core + blanket U content	m_{di}	kgMW(e)$^{-1}$	86.11	86.11	86.0	68.29
Uranium annual reloading into core + blanket	m_{dr}	kgMW(e)$^{-1}$ year^{-1}	26.24	28.265	28.095	20.70
Uranium annual discharge from core + blanket	m_{dd}	kgMW(e)$^{-1}$ year^{-1}	25.29	27.0	27.0	19.06
Capacity factor	L_f	1	0.7	0.7	0.7	0.7

* LMFBR/1 reference LMFBR pre-2000 [7]
LMFBR/2 advanced metal fuel LMFBR-USA [7]
GCFR oxid fuel GCFR [7]
N_2O_4 cooled dissociating nitric tetroxide cooled FBR [6]

Table II

Parameters of thermal reactors investigated, important in respect of fuel cycle

Parameter			Thermal reactors*					
definition	symbol	unit	PWR/1	PWR/2	BWR	HWR/1	HWR/2	HTGR
Initial core enrichment	e_i	%	2.26	2.26	1.66	0.711	1.2	4.53
Initial core loading	m_{ui}	kgMW(e)$^{-1}$	79.7	79.7	121.7	130.8	130.8	21.935
Reloading enrichment	e_r	%	3.2	4.3	2.91	0.711	1.2	10.1
Annual reloading	m_{ur}	kgMW(e)$^{-1}$ year^{-1}	25.13	15.0	27.53	121.1	42.06	5.80
^{235}U residue in spent fuel (HE wt%)	e_s	%	0.89	0.78	0.93	0.23	0.099	1.3
Heavy elements annual discharge	m_{us}	kgMW(e)$^{-1}$ year^{-1}	24.27	14.2	26.68	119.95	41.15	5.15
Fissile Pu annual discharge	m_{pd}	kgMW(e)$^{-1}$ year^{-1}	0.163	0.112	0.169	0.331	0.140	0.045
Capacity factor	L_t	1	0.70	0.70	0.70	0.70	0.70	0.70

* PWR/1 USA reference PWR [5]
PWR/2 USA reference PWR with increased burnup [5]
BWR USA reference BWR [5]
HWR/1 CANDU natural uranium fuelled [8]
HWR/2 CANDU slightly enriched uranium fuelled [9]
HTGR USA reference HTGR [5]

and plutonium composition of spent fuel removed from the reactor highly depend on the type of fuel originally contained in the reactor. On the other hand, also the fissile material production capacity and the critical mass of the reactors depend on the plutonium composition of the fuel built into the reactor, this dependence being different for fast and thermal reactors.

The fissile material production capacity of the reactors decisively depends on the specific neutron yield (η) for the fissile material loaded that is on the number of fast neutrons produced per neutron absorbed in the given fissile material. This is illustrated in Fig. 1 where the values of η are given for four different fissile isotopes as a function of energy of neutrons responsible for fission [10].

The critical mass of fissile material in the reactors depends approximately on the ratio $1/(\eta-1)$. σ_a relating to the fissile material [10]. This quantity is illustrated for four different fissile isotopes as a function of energy of the neutrons responsible for fission in Fig. 2 [10]. Assuming a typical neutron spectrum built up in a gas-cooled fast reactor, the average values of $\bar{\eta}$ and $1/(\bar{\eta}-1)$. $\bar{\sigma}_a$ for four different fissile isotopes are tabulated in Table III [10].

It is not one single plutonium isotope but a mixture of ^{239}Pu, ^{240}Pu, ^{241}Pu and ^{242}Pu isotopes that is produced by thermal and fast reactors and even a very small amount of ^{238}Pu is produced. Therefore, a mixture of the same isotopes shall be recycled to the reactor, a fact that shall be taken into consideration when determining the characteristic values for $\bar{\eta}$ and $1/(\bar{\eta}-1)\cdot\bar{\sigma}_a$.

The isotopic composition of plutonium produced in the different thermal and fast reactors is given in Table IV [7, 11, 12]. It can be seen in the Table that

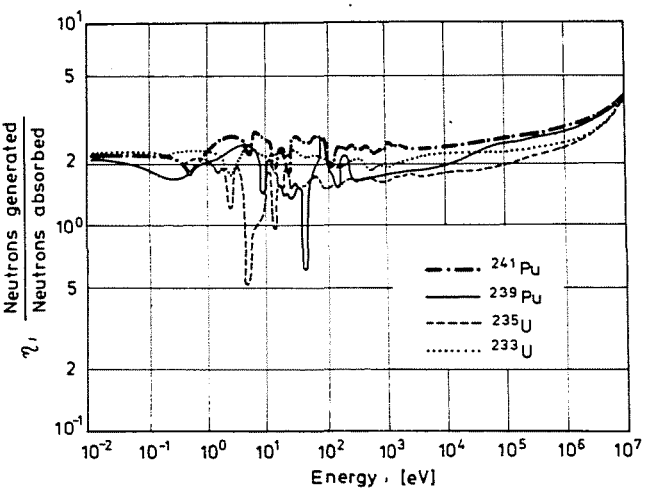

Fig. 1. Values of η for fissile fuels as a function of neutron energy

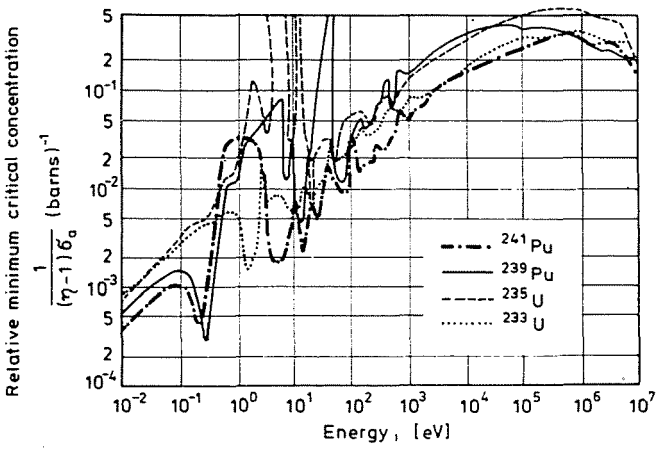

Fig. 2. Relative minimum critical concentration for the fissile fuels as a function of neutron energy

Table III

Values of $\bar{\eta}$ and $[(\bar{\eta}-1)\cdot\bar{\sigma}_a]^{-1}$
for different fissile isotopes assuming
a neutron spectrum developing in gas-cooled fast reactor
(in case of oxide fuel element)

Isotope	$\bar{\eta}$	$\sim\frac{1}{(\bar{\eta}-1)\cdot\bar{\sigma}_a}$
^{233}U	2.31	0.67
^{235}U	1.93	1
^{239}Pu	2.49	0.82
^{241}Pu	2.72	0.49

Table IV

Composition of plutonium produced in thermal and fast reactors (%)

Place of plutonium production		Pu-isotope	^{238}Pu	^{239}Pu	^{240}Pu	^{241}Pu	^{242}Pu
PWR	Uranium fuelled		–	59.8	24.6	11.6	4.0
	Plutonium fuelled		–	40.4	26.3	17.8	15.5
	WWER-440 (after 3 years, initial enrichment: 3.6%)		–	61.8	19.1	14.9	4.2
LMFBR	Core + axial blanket	Plant AI	–	62.1	26.2	7.2	4.5
		Plant GE	–	67.5	24.5	5.2	2.8
		INCFE (2000 to 2025)	0.7	69.7	20.7	6.3	2.6
	Radial blanket	Plant AI	–	97.63	2.33	0.04	–
		Plant GE	–	94.9	4.9	0.2	–
		INCFE (2000 to 2025)	–	95.3	4.4	0.3	–

Table V

Amount of fissile plutonium produced in a 1000 MW_e LMFBR annually ($^{239}Pu + ^{241}Pu$) (Load factor: 0.7)

Fast reactor region	Fissile plutonium	
	kg/year	(%)
In core + axial blanket	1585.3	92.0
In radial blanket	137.6	8.0
Total:	1722.9	100

Table VI

Fissile plutonium produced in excess in a 1000 MW_e LMFBR annually ($^{239}Pu + ^{241}Pu$) (load factor: 0.7)

Fast reactor region	Fissile plutonium	
	kg/year	(%)
In core + axial blanket	119.2	46.5
In radial blanket	137.6	53.5
Total:	256.8	100

as compared with fast reactors, the plutonium produced in thermal reactors is rich in ^{241}Pu isotope; there is a considerable shift towards heavy isotopes in the isotopic composition of plutonium produced in plutonium-fuelled thermal reactors; in LMFBR, the plutonium produced in the radial blanket consists almost entirely of ^{239}Pu isotope; the plutonium produced both in thermal reactors and in the core plus axial blanket of fast reactors contains a considerable amount (~20 to 27%) of ^{240}Pu isotope, a fact affecting unfavourably the value of both $\bar{\eta}$ and $1/(\bar{\eta}-1)\bar{\sigma}_a$.

The spent fuel removed from the radial blanket of the fast reactor can be reprocessed in isolation from the spent fuel removed from the core plus axial blanket. Therefore, in respect of the fuel cycle of the system, the quantitative distribution of plutonium production between these two areas is important. Assuming a LMFBR of a capacity of 1000 MW_e, characteristic data are given for the quantitative distribution of plutonium production in Tables V and VI [7].

Based on the data of Table IV, the values of average neutron yield ($\bar{\eta}_{Pu}$) for plutonium produced in reactors of different type or in different areas of these reactors are summed up for three different neutron energy ranges in Table VII.

Table VII

Thermal neutron yield ($\bar{\eta}_{Pu}$) for plutonium fuel produced in reactors of different type in case of three different neutron energy ranges

Place of plutonium production		Neutron energy range	0—0.215 eV	200—400 keV	0.8—1.4 MeV
PWR	Uranium fuelled		1.930	2.475	2.837
	Plutonium fuelled		1.885	2.403	2.807
	WWER-440 (after 3 years, initial enrichment: 3.6%)		1.980	2.520	2.860
LMFBR	Core + axial blanket	Plant AI	1.908	2.453	2.830
		Plant GE	1.927	2.474	2.840
	Radial blanket	Plant AI	2.098	2.622	2.934
		Plant GE	2.081	2.609	2.924

The range of 0 to 0.215 eV is the typical energy range of the neutron spectrum in a thermal reactor while the energy range of 200 to 400 keV is characteristic of the neutron spectrum developing in an LMFBR, the range of 0.8 to 1.4 MeV includes an important part of the fission spectrum. The difference $\bar{\eta}_{Pu}-1$ approximately expresses the theoretical limit of the breeding ratio. As can be seen in Table VII, this limit lies at 1.4 to 1.5 for LMFBR, leaving the plutonium produced in the radial blanket out of consideration. This value could be appreciably increased only if a harder neutron spectrum were produced in the fast reactor. A change-over from sodium to gas coolant (He, N_2O_4) might be significant also in this respect. It is also seen in the Table that a lower breeding ratio and/or conversion factor can be obtained for plutonium produced in a plutonium-fuelled thermal reactor than in case of plutonium coming from a reactor fuelled with enriched uranium.

The values of $1/(\bar{\eta}_{Pu}-1)$. $\bar{\sigma}_{a,Pu}$ for the cases given in Table VII are indicated in Table VIII. As seen in the Table, the plutonium recovered from the radial blanket is the best also in this respect (obviously because the ratio of ^{240}Pu is lowest in this plutonium). Also, the Table explains why the fast reactor requires considerably more fissile material than the thermal reactor to arrive at criticality.

As is well known, the fuel of new energetic pressurized water reactors is set up of three components, each enriched to a different percentage. E.g. the WWER-440 contains uranium in which 1/3 is enriched to 1.6%, another 1/3 to 2.4% while the last 1/3 to 3.6%, the first component (1.6%) spending 1 year, the second component (2.4%) 2 years, and the third component (3.6%) 3 years in the reactor. After an operation of 1, 2, or year(s), the plutonium content of the burnt up fuels removed from the reactor is therefore different, and also the values of $\bar{\eta}_{Pu}$ and $1/(\bar{\eta}_{Pu}-1)$. $\bar{\sigma}_{a,Pu}$ are slightly different for each component. This is illustrated by Table IX.

Table VIII

Values of $[(\bar{\eta}_{Pu}-1)\cdot\bar{\sigma}_{a,Pu}]^{-1}$ for plutonium fuel produced in reactors of different type in case of three different neutron energy ranges

Place of plutonium production			Neutron energy range 0—0.215 eV	200—400 keV	0.8—1.4 MeV
PWR	Uranium fuelled		0.00128	0.5070	0.3168
	Plutonium fuelled		0.00152	0.6188	0.3369
	WWER-440 (after 3 years, initial enrichment: 3.6%)		0.00114	0.4684	0.3152
LMFBR	Core+axial blanket	Plant AI	0.00137	0.5268	0.3151
		Plant GE	0.00130	0.5020	0.3103
	Radial blanket	Plant AI	0.00092	0.3710	0.2875
		Plant GE	0.00095	0.3814	0.2894

Table IX

Characteristics of spent fuel discharged from WWER-440

Characteristics		Initial enrichment: 1.6% After 1 year of operation	Initial enrichment: 2.4% After 2 years of operation	Initial enrichment: 3.6% After 3 years of operation
Plutonium composition (%)	^{239}Pu	72.3	64.1	61.8
	^{240}Pu	17.7	19.9	19.1
	^{241}Pu	8.7	12.8	14.9
	^{242}Pu	1.3	3.2	4.2
$\bar{\eta}_{Pu}$	E_n=0—0.215 eV	1.992	1.974	1.980
	E_n=200—400keV	2.535	2.516	2.520
	E_n=0.8—1.4MeV	2.871	2.859	2.860
$\frac{1}{(\bar{\eta}_{Pu}-1)\cdot\bar{\sigma}_{a,Pu}}$	E_n=0—0.215 eV	0.00111	0.00116	0.00114
	E_n=200—400keV	0.4465	0.4685	0.4684
	E_n=0.8—1.4MeV	0.3062	0.3131	0.3152

Fuel cycle scheme of the mixed nuclear energy system at fuel equilibrium (MNESFE)

A nuclear energy system is considered to be at fuel equilibrium if neither redundant plutonium nor redundant spent fuel containing uranium are produced in the system, the total enriched uranium demand of the system can be met by enrichment with the plutonium produced so that there is no need for

a so called isotopic enrichment of uranium, and the total amount of natural or depleted uranium introduced to the system enters the fuel cycle. It follows from what has been said that in such a system no stocks are in accumulation in addition to the amount required for technological purposes (e.g. ageing). If the conditions of equilibrium outlined above can be brought about in a system containing also thermal converter reactors besides fast reactors, we speak of a Mixed Nuclear Energy System at Fuel Equilibrium (MNESFE).

When planning the fuel cycle of an ideal MNESFE, all what has been said above may be taken into consideration in the following way:

— Spent fuels removed from thermal reactors after an operation of 1, 2, or more year(s), and from the core plus axial blanket, and from the radial blanket of fast reactors shall be reprocessed in isolation each, and also the products of isolated reprocessing shall be stored in isolation.

— The plutonium mixtures obtained in isolation shall be recycled to reactors where the values of $\bar{\eta}_{Pu}$ and $1/(\bar{\eta}_{Pu}-1)\bar{\sigma}_{a,Pu}$ are most promising in this respect. Data in Tables VII thru IX in previous Chapter suggest recycling the total amount of plutonium produced in plutonium fuelled thermal reactors to the thermal reactors, completing it with plutonium produced in the radial blanket of LMFBR to meet the total demand of thermal reactors. In case of fast reactors recycling first the plutonium mixture produced in the core plus axial blanket and then, — if there is still plutonium available after the demand of thermal reactors has been met, — that produced in the radial blanket of fast reactors is suggested.

The fuel cycle so controlled is schematically illustrated in Fig. 3. However, it is obvious on the basis of what has been said in the previous Chapter that the efficiency of fuel utilization of the nuclear energy system is not significantly affected by isolated recycling of plutonium produced in different reactors or reactor parts. Therefore, we are certainly not quite wrong when mixing all the plutonium independently of where it comes from, and recycling this mixture to the system. In this way, the fuel cycle is significantly simplified as illustrated in Fig. 4.

Both fuel cycle flow diagrams consider the fact that there is no isotopic enrichment but only enrichment by plutonium in MNESFE and thus the ^{235}U concentration in spent fuel removed from thermal reactors is lower than the ^{235}U concentration of natural uranium. Therefore, as far as possible, the demand for additional fertile material shall be met by spent uranium fuel removed from the thermal reactors.

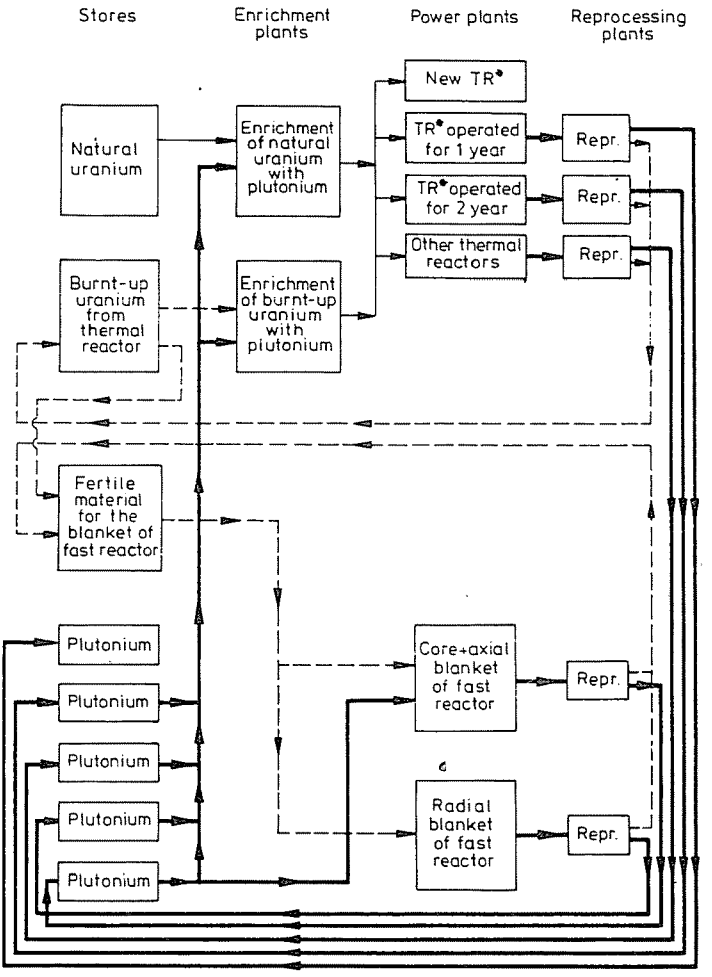

Fig. 3. Fuel cycle scheme of mixed nuclear energy system at fuel equilibrium
* (TR: Thermal Reactors)

Mathematical formulation of the condition of equilibrium

Let us assume that the growth of electric capacity (P) of the nuclear energy system as a function of time (t) is described by the following exponential function:

$$P(t) = P_0 \cdot e^{c(t) \cdot t} \tag{1}$$

where P_0 — electric capacity of the system at the time $t = 0$ (MW_e)
$c(t)$ — exponent characteristic of the growth rate (year^{-1}), usually a function of time t.

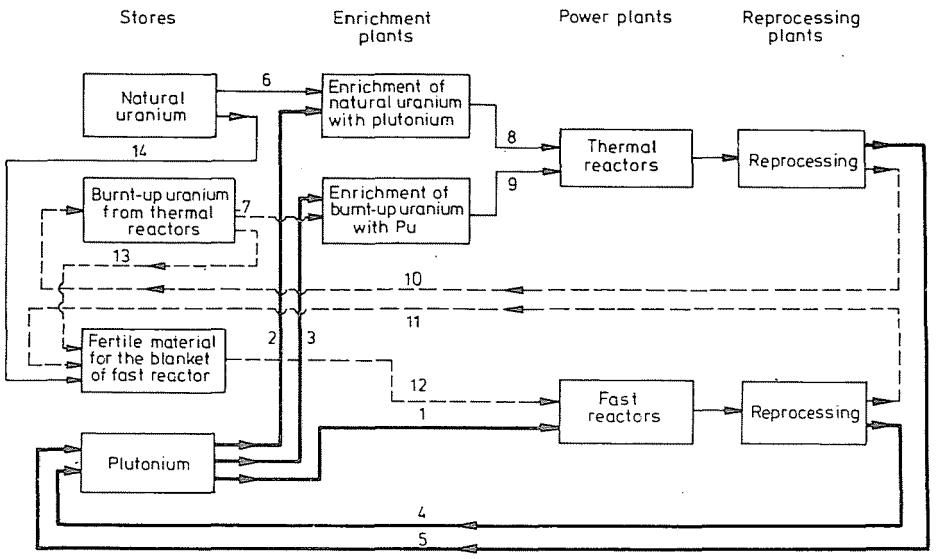

Fig. 4. Simplified fuel cycle scheme of MNESFE

Presumably, exponent $c(t)$ will decrease as a function of time, at least after a certain time period. E.g. in case of the same annual percentage of reduction, it may be written that

$$c(t)=(c_0-c_a)\cdot e^{-\alpha\cdot t}+c_a \tag{2}$$

where c_0 — exponent of growth at time $t=0$
c_a — limit of exponent of growth in case $t\to\infty$
α — value characteristic of the annual reduction of the exponent of growth.

On the basis of (2),

$$\alpha=-\frac{1}{c(t)-c_a}\cdot\frac{d[c(t)-c_a]}{dt} \qquad (\text{year}^{-1}) \tag{3}$$

Hereinafter it is assumed that $\alpha=0$ i.e. $c(t)=c=$constant. With this assumption and considering that the capacity of the system is given by the sum of the capacity of nuclear power plants with thermal and fast reactors we obtain:

$$P_t(t)+P_f(t)=P_0\cdot e^{ct} \tag{4}$$

where $P_t(t)$ and $P_f(t)$ —electric capacity of nuclear power plants with thermal and/or fast reactors (MW_e).

From now on, we calculate with continuous functions for both capacity and fuel flow, an approach feasible if the capacity of the system is sufficiently high and there is a large number of nuclear power generating units in the

system. Since the conditions of equlibrium require a rather long time to take place (see later), this assumption is acceptable.

The plutonium demand of fast reactors in operation and those entering the system newly for a unit time at time t can be expressed, as follows (the subscripts relating to the corresponding lines in Fig. 4):

$$m_1(t) = m_{pr} \cdot P_f(t+\vartheta_f) + m_{pi} \cdot P'_f(t+\vartheta_f) \quad (\text{kg year}^{-1}) \quad (5)$$

where m_{pi} — specific fissile plutonium content of initial active core ($\text{kg} \cdot \text{MW}_e^{-1}$)

m_{pr} — specific amount of fissile plutonium to be introduced annually to fast reactors in operation ($\text{kg} \cdot \text{MW}_e^{-1}\ \text{year}^{-1}$)

ϑ_f — time until recycle of reprocessed plutonium as part of the fuel element to the fast reactor within the so called external cycle time

$P_f(t+\vartheta_f)$ — electric capacity of nuclear power plants with fast reactors at time $t+\vartheta_f$

$P'_f(t+\vartheta_f)$ — value of the time derivative of P_f assumed at time $t+\vartheta_f$:

$$P'_f(t+\vartheta_f) = \left.\frac{dP_f(t)}{dt}\right|_{t+\vartheta_f} \quad (6)$$

Amount of plutonium produced by the fast reactors within unit time at t:

$$m_4(t) = m_{pf} \cdot P_f(t-\tau_f) \quad (\text{kg year}^{-1}) \quad (7)$$

where m_{pf} — specific amount of fissile plutonium removed from the fast reactors within unit time ($\text{kg} \cdot \text{MW}_e^{-1}\ \text{year}^{-1}$)

τ_f — time between removal of fuel from the reactor and completion of reprocessing within the so called external cycle time (year)

$P_f(t-\tau_f)$ — electric capacity of nuclear power plants with fast reactor at time $t-\tau_f(\text{MW}_e)$.

By external fuel cycle time (Θ_f) for fast reactors we understand the time required for cooling, storage, reprocessing of spent fuel and for fuel element fabrication before the fuel is recycled to the reactor i.e. the time the fuel actually spends out of the reactor, expressed as

$$\Theta_f = \vartheta_f + \tau_f \quad (\text{year}) \quad (8)$$

Amount of plutonium produced by the thermal reactors within unit time at t:

$$m_5(t) = m_{pt} \cdot P_t(t-\tau_t) \quad (\text{kg year}^{-1}) \quad (9)$$

where m_{pt} — specific amount of fissile plutonium removed from the thermal reactors within unit time ($\text{kg} \cdot \text{MW}_e^{-1}\text{year}^{-1}$)

τ_t — value corresponding to τ_f but for thermal reactors

$P_t(t-\tau_t)$ — electric capacity of nuclear power plants with thermal reactors at time $t-\tau_t(\text{MW}_e)$.

Amount of plutonium produced in excess of the demand of fast reactors per unit time:

$$m_p^{(1)}(t)=m_4(t)+m_5(t)-m_1(t) \tag{10}$$

that is

$$m_p^{(1)}=m_{pf}\cdot P_f(t-\tau_f)+m_{pt}\cdot P_t(t-\tau_t)-m_{pr}\cdot P_f(t+\vartheta_f)- \\ -m_{pi}\cdot P'_f(t+\vartheta_f) \qquad (\text{kg year}^{-1}) \tag{11}$$

Amount of spent uranium removed from the reactor within unit time at about time t:

$$m_{11}(t)=m_{dd}\cdot P_f(t-\tau_f) \qquad (\text{kg year}^{-1}) \tag{12}$$

where m_{dd} — amount of spent uranium removed from the core and breeding blanket of the fast reactor annually per unit electric power ($\text{kg}\cdot\text{MW}_e^{-1}\text{year}^{-1}$)

Demand for depleted uranium of fast reactors within unit time at t:

$$m_{12}(t)=m_{dr}\cdot P_f(t+\vartheta_f)+m_{di}\cdot P'_f(t+\vartheta_f) \qquad (\text{kg year}^{-1}) \tag{13}$$

where m_{di} — depleted uranium loading of the active core and breeding blanket of the new reactor per unit electric power ($\text{kg}\cdot\text{MW}_e^{-1}$)

m_{dr} — amount of depleted uranium to be introduced to fast reactors in operation per unit electric power ($\text{kg}\cdot\text{MW}_e^{-1}\ \text{year}^{-1}$).

Amount of spent uranium of an ^{235}U concentration of e_s, removed from the thermal reactor within unit time at t:

$$m_{10}(t)=m_{us}\cdot P_t(t-\tau_t) \qquad (\text{kg year}^{-1}) \tag{14}$$

where m_{us} — amount of spent uranium removed from the thermal reactor annually per unit electric power ($\text{kg}\cdot\text{MW}_e^{-1}\ \text{year}^{-1}$).

Let us now calculate for two different cases:

(a) $m_{10}(t)\geqq m_{12}(t)-m_{11}(t)$ i.e. the case where

$$m_{us}\cdot P_t(t-\tau_t)+m_{dd}\cdot P_f(t-\tau_f)\geqq m_{dr}\cdot P_f(t+\vartheta_f)+m_{di}\cdot P'_f(t+\vartheta_f) \tag{15}$$

(b) $m_{10}(t)<m_{12}(t)-m_{11}(t)$ i.e. the case where

$$m_{us}\cdot P_t(t-\tau_t)+m_{dd}\cdot P_f(t-\tau_f)<m_{dr}\cdot P_f(t+\vartheta_f)+m_{di}\cdot P'_f(t+\vartheta_f) \tag{16}$$

On the basis of (15) and (16), the ratio of $P_t(t)$ to $P_f(t)$ above and below which case (a) and (b) applies, respectively, can be determined.

Ad (a)

depleted uranium demand of fast reactors to be met by spent uranium removed from the thermal reactors:

$$m_{13}(t)=m_{12}(t)-m_{11}(t) \tag{17}$$

that is

$$m_{13}(t)=m_{dr}\cdot P_f(t+\vartheta_f)+m_{di}\cdot P'_f(t+\vartheta_f)-m_{dd}\cdot P_f(t-\tau_f) \quad (\text{kg year}^{-1}) \quad (18)$$

Amount of spent fuel removed from thermal reactors that can be recycled after enrichment with plutonium:

$$m_7(t)=m_{10}(t)-m_{13}(t)$$

that is

$$m_7(t)=m_{us}\cdot P_t(t-\tau_t)+m_{dd}\cdot P_f(t-\tau_f)-m_{dr}\cdot P_f(t+\vartheta_f)- \\ -m_{di}\cdot P'_f(t+\vartheta_f) \quad (\text{kg year}^{-1}) \quad (19)$$

With the plutonium produced in excess, this amount is enriched to fissile material concentration e_r required for refuelling of thermal reactors in operation. The amount of fissile plutonium required for this purpose:

$$m_3(t)=\frac{e_r-e_s}{1-e_r}\cdot m_7(t) \quad (20)$$

that is

$$m_3(t)=\frac{e_r-e_s}{1-e_r}\cdot[m_{us}\cdot P_t(t-\tau_t)+m_{dd}\cdot P_f(t-\tau_f)-m_{dr}\cdot P_f(t+\vartheta_f)- \\ -m_{di}\cdot P'_f(t+\vartheta_f)] \quad (\text{kg year}^{-1}) \quad (21)$$

Amount of fuel of a fissile material concentration of e_s produced by enrichment as mentioned above:

$$m_9(t)=\frac{1-e_s}{1-e_r}\cdot m_7(t) \quad (22)$$

that is

$$m_9(t)=\frac{1-e_s}{1-e_r}\cdot[m_{us}\cdot P_t(t-\tau_t)+m_{dd}\cdot P_f(t-\tau_f)-m_{dr}\cdot P_f(t+\vartheta_f)- \\ -m_{di}\cdot P'_f(t+\vartheta_f)] \quad (\text{kg year}^{-1}) \quad (23)$$

Total annual demand of thermal reactors for fuel of a fissile material concentration of e_r:

$$m_{t,r}(t)=m_{ur}\cdot P_t(t+\vartheta_t) \quad (\text{kg year}^{-1}) \quad (24)$$

where m_{ur} — amount of fuel of a fissile material concentration of e_r to be introduced to thermal reactors in operation annually per unit electric power (kg MW_e^{-1} year^{-1})

ϑ_t — time between reprocessing and recycle of plutonium as part of the fuel element to the thermal reactor (year) within the so called external cycle time

$P_t(t+\vartheta_t)$ — electric power of nuclear power plants with thermal reactor at time $t+\vartheta_t$.

External cycle time for the fuel of thermal reactors in accordance with (8):

$$\Theta_t = \tau_t + \vartheta_t \qquad \text{(year)} \quad (25)$$

Fuel demand of thermal reactors in operation that shall be produced by enrichment of natural uranium with plutonium to a concentration of e_r:

$$m_8^{(1)}(t) = m_{t,r}(t) - m_9(t) \qquad (26)$$

where $m_{t,r}(t)$ and $m_9(t)$ are expressions from (24) and (23), respectively.

The following amount of fissile plutonium is required for enrichment to e_r:

$$m_2^{(1)}(t) = \frac{e_r - e_0}{1 - e_0} \cdot m_8^{(1)}(t) \qquad (27)$$

or, making use of (26), (24) and (23):

$$m_2^{(1)}(t) = \frac{e_r - e_0}{1 - e_0} \cdot m_{ur} \cdot P_t(t + \vartheta_t) - \frac{e_r - e_0}{1 - e_0} \cdot \frac{1 - e_s}{1 - e_r} \cdot [m_{us} \cdot P_t(t - \tau_t) + m_{dd} \cdot P_f(t - \tau_f) - m_{dr} \cdot P_f(t + \vartheta_f) - m_{di} \cdot P'_f(t + \vartheta_f)] \qquad \text{(kg year}^{-1}\text{)} \quad (28)$$

Amount of natural uranium used for enrichment:

$$m_6^{(1)}(t) = \frac{1 - e_r}{1 - e_0} \cdot m_8^{(1)}(t) \qquad (29)$$

that is

$$m_6^{(1)}(t) = \frac{1 - e_r}{1 - e_0} \cdot m_{ur} \cdot P_t(t + \vartheta_t) - \frac{1 - e_s}{1 - e_0} \cdot [m_{us} \cdot P_t(t - \tau_t) + m_{dd} \cdot P_f(t - \tau_f) - m_{dr} \cdot P_f(t + \vartheta_f) - m_{di} \cdot P'_f(t + \vartheta_f)] \qquad \text{(kg year}^{-1}\text{)} \quad (30)$$

Fuel of an average fissile material concentration of e_i shall be used as the initial charge of new thermal reactors (e_i not necessarily equalling e_r, e.g. for PWRs $e_i < e_r$). The amount of fuel required by these reactors can be calculated, as follows:

$$m_8^{(2)}(t) = m_{ui} \cdot P'_t(t + \vartheta_t) \qquad \text{(kg year}^{-1}\text{)} \quad (31)$$

where m_{ui} — specific loading of the new thermal reactor core (kg MW_e^{-1})

$P'_t(t + \vartheta_t)$ — value assumed by the time derivate of P_t at time $t + \vartheta_t$:

$$P'_t(t + \vartheta_t) = \left.\frac{dP_t(t)}{dt}\right|_{t + \vartheta_t} \qquad (32)$$

The following amount of fissile plutonium is required to produce the fuel according to (31):

$$m_2^{(2)}(t) = \frac{e_i - e_0}{1 - e_0} \cdot m_8^{(2)}(t) \qquad (33)$$

that is

$$m_2^{(2)}(t) = \frac{e_i - e_0}{1 - e_0} \cdot m_{ui} \cdot P_t'(t + \vartheta_t) \qquad \text{(kg year}^{-1}\text{)} \quad (34)$$

Amount of natural uranium used for enrichment in this case:

$$m_6^{(2)}(t) = \frac{1 - e_i}{1 - e_0} \cdot m_8^{(2)}(t) \quad (35)$$

that is

$$m_6^{(2)}(t) = \frac{1 - e_i}{1 - e_0} \cdot m_{ui} \cdot P_t'(t + \vartheta_t) \qquad \text{(kg year}^{-1}\text{)} \quad (36)$$

Total amount of natural uranium used for enrichment (representing at the same time the total natural uranium demand of the entire nuclear energy system):

$$m_6(t) = m_6^{(1)}(t) + m_6^{(2)}(t) \quad (37)$$

that is, by the use of (30) and (36):

$$m_6(t) = \frac{1 - e_r}{1 - e_0} \cdot m_{ur} \cdot P_t(t + \vartheta_t) + \frac{1 - e_i}{1 - e_0} \cdot m_{ui} \cdot P_t'(t + \vartheta_t) - \\ - \frac{1 - e_s}{1 - e_0} \cdot [m_{us} \cdot P_t(t - \tau_t) + m_{dd} \cdot P_f(t - \tau_f) - m_{dr} \cdot P_f(t + \vartheta_f) - \\ - m_{di} \cdot P_f'(t + \vartheta_f)] \qquad \text{(kg year}^{-1}\text{)} \quad (38)$$

Amount of fissile plutonium required for enrichment:

$$m_p^{(2)}(t) = m_3(t) + m_2^{(1)}(t) + m_2^{(2)}(t) \quad (39)$$

that is, by the use of (21), (28), and (34) and after reasonable reduction:

$$m_p^{(2)}(t) = \frac{e_r - e_0}{1 - e_0} \cdot m_{ur} \cdot P_t(t + \vartheta_t) + \frac{e_i - e_0}{1 - e_0} \cdot m_{ui} \cdot P_t'(t + \vartheta_t) + \\ + \frac{e_0 - e_s}{1 - e_0} \cdot [m_{us} \cdot P_t(t - \tau_t) + m_{dd} \cdot P_f(t - \tau_f) - m_{dr} \cdot P_f(t + \vartheta_f) - \\ - m_{di} \cdot P_f'(t + \vartheta_f)] \qquad \text{(kg year}^{-1}\text{)} \quad (40)$$

A nuclear energy system will be at fuel equilibrium if the plutonium produced in excess by the fast reactors is just enough to meet the deficit of thermal reactors in fissile materials, i.e. if $m_p^{(1)}(t)$ according to (11) is just equal to $m_p^{(2)}(t)$ according to (40):

$$m_p^{(1)}(t) = m_p^{(2)}(t) \qquad \text{i.e.} \qquad m_p^{(1)}(t) - m_p^{(2)}(t) = 0 \quad (41)$$

Making use of (11) and (40) and after reasonable reduction, the condition of equilibrium can be described by means of the following equation:

$$a \cdot P_t(t - \tau_t) + b \cdot P_t(t + \vartheta_t) + e \cdot P_f(t - \tau_f) + n \cdot P_f(t + \vartheta_f) + r \cdot P_t'(t + \vartheta_t) + \\ + z \cdot P_f'(t + \vartheta_f) = 0 \quad (42)$$

where $a = m_{pt} - \frac{e_0 - e_s}{1 - e_0} \cdot m_{us}$

$$b = -\frac{e_r - e_0}{1 - e_0} \cdot m_{ur}$$

$$e = m_{pf} - \frac{e_0 - e_s}{1 - e_0} \cdot m_{dd}$$

$$n = \frac{e_0 - e_s}{1 - e_0} \cdot m_{dr} - m_{pr} \qquad (43)$$

$$r = -\frac{e_i - e_0}{1 - e_0} \cdot m_{ui}$$

$$z = \frac{e_0 - e_s}{1 - e_0} \cdot m_{di} - m_{pi}$$

Specific natural uranium demand of the system at fuel equilibrium:

$$\mu_e(t) = \frac{m_6(t)}{P^*(t)} \qquad (\text{kg MW}_e^{-1}\ \text{year}^{-1}) \quad (44)$$

where $m_6(t)$ — quantity given in (38),
$P^*(t)$ — power of nuclear energy system at time t:

$$P^*(t) = L_t \cdot P_t(t) + L_f \cdot P_f(t) \qquad (45)$$

where L_t and L_f — load factor of nuclear power plants operating with thermal or fast reactor, respectively.

Efficiency of utilization of natural uranium in MNESFE:

$$\eta_e = \frac{\mu_0}{\mu_e} \cdot 100 \qquad (\%) \quad (46)$$

where μ_0 — uranium mass equivalent of energy of 1 MW year in case of full utilization of uranium (kg MW^{-1} year^{-1}).

Ad (b)

In this case, the breeding material demand of fast reactors that shall be met by natural uranium, or depleted uranium accumulated earlier, is given by the following relationship:

$$m_{14}(t) = m_{12}(t) - m_{10}(t) - m_{11}(t) \qquad (47)$$

that is, by the use of (12), (13) and (14):

$$m_{14}(t) = m_{dr} \cdot P_f(t + \vartheta_f) + m_{di} \cdot P'_f(t + \vartheta_f) - m_{us} \cdot P_t(t - \tau_t) - m_{dd} \cdot P_f(t - \tau_f) \qquad (\text{kg year}^{-1}) \quad (48)$$

Now the total fuel demand of thermal reactors shall be met by enrichment of natural uranium with plutonium, that is $m_7(t)=m_3(t)=m_9(t)=0$.

Utilizing relationship (24), the total annual natural uranium demand of thermal reactors in operation is obtained as

$$m_6^{(1)}(t) = \frac{1-e_r}{1-e_0} \cdot m_{ur} \cdot P_t(t+\vartheta_t) \qquad (\text{kg year}^{-1}) \quad (49)$$

For this enrichment, the annual fissile plutonium demand:

$$m_2^{(1)}(t) = \frac{e_r-e_0}{1-e_0} \cdot m_{ur} \cdot P_t(t+\vartheta_t) \qquad (\text{kg year}^{-1}) \quad (50)$$

The required amount of fuel of an average fissile material concentration of e_i for new thermal reactors entering the system is given by relationship (31), the natural uranium consumption of this fuel production per unit time being given by relationship (36) while the fissile plutonium demand by relationship (34).

In the present case, the total amount of natural uranium required for enrichment (representing at the same time the total natural uranium demand of the nuclear energy system) is obtained as

$$m_6(t)=m_6^{(1)}(t)+m_6^{(2)}(t)+m_{14}(t) \quad (51)$$

that is, making use of relationships (49), (36) and (43), we obtain

$$m_6(t) = \frac{1-e_r}{1-e_0} \cdot m_{ur} \cdot P_t(t+\vartheta_t) + \frac{1-e_i}{1-e_0} \cdot m_{ui} \cdot P'_t(t+\vartheta_t) + m_{dr} \cdot P_f(t+\vartheta_f) + \\ + m_{di} \cdot P'_f(t+\vartheta_f) - m_{us} \cdot P_t(t-\tau_t) - m_{sd} \cdot P_f(t-\tau_f) \qquad (\text{kg year}^{-1}) \quad (52)$$

Amount of fissile plutonium required for enrichment per unit time:

$$m_2(t)=m_2^{(1)}(t)+m_2^{(2)}(t) \quad (53)$$

or, by the use of relationships (50) and (34),

$$m_2(t) = \frac{e_r-e_0}{1-e_0} \cdot m_{ur} \cdot P_t(t+\vartheta_t) + \frac{e_i-e_0}{1-e_0} \cdot m_{ui} \cdot P'_t(t+\vartheta_t) \qquad (\text{kg year}^{-1}) \quad (54)$$

Now the nuclear power system will be at fuel equilibrium if $m_p^{(1)}(t)$ in accordance with (11) is just equal to $m_2(t)$ in accordance with (54):

$$m_p^{(1)}(t)=m_2(t) \qquad \text{i.e.} \qquad m_p^{(1)}(t)-m_2(t)=0 \quad (55)$$

Now, by the use of (11) and (54), the equilibrium condition equation can be written as

$$A \cdot P_t(t-\tau_t)+B \cdot P_t(t+\vartheta_t)+E \cdot P_f(t-\tau_f)+N \cdot P_f(t+\vartheta_f)+ \\ +R \cdot P'_t(t+\vartheta_t)+Z \cdot P'_f(t+\vartheta_f) \quad (56)$$

where $A = m_{pt}$

$$B = -\frac{e_r - e_0}{1 - e_0} \cdot m_{ur}$$

$$E = m_{pf}$$

$$N = -m_{pr}$$

$$R = -\frac{e_i - e_0}{1 - e_0} \cdot m_{ui} \tag{57}$$

$$Z = -m_{pi}$$

It can be seen that formally relationships (42) and (56) derived for cases (a) and (b), respectively, are identical with only part of the constants in them differing from each other (cf. (43) and (57)).

The relationships derived apply also in case the thermal reactors are fuelled with natural uranium (like one type of HWRs). In this case $e_i = e_r = e_0$, and $b = 0$, $r = 0$ in (43) while $B = 0$, $R = 0$ in (57); that means that there is a difference only in part of the constants of the relationships.

Solving of the differential equation system describing the conditions of equilibrium

On the basis of (4), and of (42) applying to case (a), the differential equation system describing the conditions of equilibrium can be written as

$$a \cdot P_t(t-\tau_t) + b \cdot P_t(t+\vartheta_t) + e \cdot P_f(t-\tau_f) + n \cdot P_f(t+\vartheta_f) + r \cdot P_t'(t+\vartheta_t) + z \cdot P_f'(t+\vartheta_f) = 0 \tag{58}$$

$$P_t(t) + P_f(t) = P_0 \cdot e^{ct} \tag{59}$$

where the wanted functions are $P_t(t)$ and $P_f(t)$. Formally, the equation system in case (*b*) is identical but the constants in (58) are given by (57) instead of (43).

Expressing e.g. $P_t(t)$ from (59) then substituting it into (58) we obtain

$$-r \cdot P_f'(t+\vartheta_t) + z \cdot P_f'(t+\vartheta_f) = a \cdot P_f(t-\tau_t) - e \cdot P_f(t-\tau_f) + b \cdot P_f(t+\vartheta_t) - n \cdot P_f(t+\vartheta_t) - a \cdot P_0 \cdot e^{c(t-\tau_t)} - (b + r \cdot c) \cdot P_0 \cdot e^{c(t+\vartheta_t)} \tag{60}$$

It is possible to assume to a good approximation that $\vartheta_t = \vartheta_f = \vartheta$. Taking this into consideration, after application of transformation $t + \vartheta \to t$ we obtain

$$(z-r) \cdot \frac{dP_f(t)}{dt} = (b-n) \cdot P_f(t) + a \cdot P_f(t-\vartheta-\tau_t) - e \cdot P_f(t-\vartheta-\tau_f) - (b + r \cdot c + a \cdot e^{-c(\vartheta+\tau_t)}) \cdot P_0 \cdot e^{ct} \tag{61}$$

For $P_f(t)$, equation (61) is a so called retarded-argument first-order inhomogeneous differential equation. Solving it for $P_f(t)$, and substituting the result into (59), we obtain the change in time of the capacity of nuclear power plants with thermal reactors, $P_t(t)$.

Retarded-argument differential equations differ both in theory and solving fundamentally from normal differential equations. The problem of initial value becomes here a problem of initial function. This means that, in order to solve the equation, 'initial values' (values associated with $t<0$) of the unknown function shall be given for such a long time interval the magnitude of which is equal to the maximum retardation (most probably to the value $\vartheta+\tau_f$ in the present case).

There exists usually no solution to retarted-argument differential equations that can be formulated in an exact mathematical form even for the simplest initial functions. Analytically, only an asymptotic solution of the equations can be determined, but in most cases the roots of a rather complicated transcendent equation — the so called quasipolynomial — are needed even for an asymptotic solution, and the determination of the roots is a numerical problem again.

Therefore, the following 'strategy' was followed in solving equation (61):

— The first step was to elaborate a numerical process and/or computer programme to calculate the transcendent roots required for the determination of the asymptotic solution.

— In the second step, an approximate method based on series expansion was developed for the total (not only asymptotic) solution: the approximation according to which the retarted argument values of the unknown function were expanded in a series according to retardations that is $\vartheta+\tau_t$ and $\vartheta+\tau_f$ at locus t until second order and thus the retarded equation was reduced to a normal second order differential equation was considered feasible on the basis of physical considerations.

— In the third step, we elaborated the algorithm of numerical integration associated with an arbitrary initial function as well as the corresponding computer programme. Disregarding the inaccuracy of numerical calculations, this computer programme yiels an exact solution to equation (61). For integration of the equation, the programme uses the so called optimized step method.

In the present case, the solution is obtained as the sum of an asymptotic and a transient term, the transient term becoming zero when $t\to\infty$. The ratio of transient and asymptotic terms can be influenced by the initial functions. If the initial functions are reasonably defined, the role of transient terms will become negligible even at low values of t, or these terms will entirely disappear. This is possible also in the actual case as the starting point of the mixed nuclear energy system at fuel equilibrium, can be defined as the time at which the ratio of

capacity of fast and thermal reactors reaches the ratio of values assumed at time $t=0$ by the asymptotic solutions. The period before this time shall be treated as the running-in period and investigated separately.

The use of the process described above makes the third step of the strategy — that is numerical integration in association with the initial function — unnecessary as a correct solution can be obtained either in the first or in the second step. In this case, we obtain the solution in the following way:

$$P_f(t)=P_{f0}\cdot e^{ct} \tag{62}$$

and

$$P_t(t)=P_{t0}\cdot e^{ct} \tag{63}$$

and

$$\frac{P_f(t)}{P_t(t)}=\frac{P_{f0}}{P_{t0}}=\text{constant} \tag{64}$$

independently of time t. Exponent c in relationships (62) and (63) is identical with that in (59). Actual calculation results are reported in our work [4].

Condition of equilibrium of a nuclear energy system containing fast reactors only

This system contains no thermal reactors ($P_t=0$), and the plutonium production by fast reactors is just enough to meet the plutonium demand of fast reactors in operation and new fast reactors entering the system. These conditions will be fulfilled if, with $P_t=0$, $m^{(1)}{}_p(t)$ according to (11) equals zero:

$$m_{pf}\cdot P_f(t-\tau_f)-m_{pr}\cdot P_f(t+\vartheta_f)-m_{pi}\cdot P'_f(t+\vartheta_f)=0 \tag{65}$$

and

$$P_f(t)=P_0\cdot e^{ct} \tag{66}$$

By the use of (66):

$$P_f(t-\tau_f)=\frac{1}{e^{c\tau_f}}\cdot P_0\cdot e^{ct} \tag{67}$$

$$P_f(t+\vartheta_f)=e^{c\vartheta_f}\cdot P_0\cdot e^{ct} \tag{68}$$

$$P'_f(t+\vartheta_f)=c\cdot e^{c\vartheta_f}\cdot P_0\cdot e^{ct} \tag{69}$$

With these equations substituted into (65), then rearranged, we obtain that a nuclear energy system containing fast reactors only, will be at fuel equilibrium, if the following condition is fulfilled:

$$e^{c(\tau_f+\vartheta_f)}=\frac{m_{pf}}{m_{pr}}\cdot\frac{1}{1+\dfrac{m_{pi}}{m_{pr}}\cdot c} \tag{70}$$

The question is in general what a capacity growth rate ie how high an exponent of growth and/or how long a doubling time $T_{2x}=\ln 2/c$ are possible in case of given nuclear power plant parameters and external cycle times. Since (70) is a transcendent equation with respect to c, therefore, except for the case $\tau_f+\vartheta_f=0$, the solution obtained is usually only an approximate one. The exponent so obtained is called "fast alone exponent" while T_{2x} the system fissile material doubling rate.

In case of $\tau_f+\vartheta_f=0$ i.e. zero external cycle time, the "fast alone exponent":

$$c_f = \frac{m_{pf}-m_{pr}}{m_{pi}} \qquad (\text{year}^{-1}) \quad (71)$$

and the fissile material doubling time:

$$T_{2x} = \frac{m_{pi}\cdot \ln 2}{m_{pf}-m_{pr}} \qquad (\text{year}) \quad (72)$$

or, since

$$m_{pf} \cong m_{pr} + \frac{L_f}{\eta_f}\cdot f_w \cdot (b_f - 1), \qquad (73)$$

where η_f — efficiency of nuclear power plant with fast reactor
f_w — amount of plutonium consumed per unit energy production in fission and neutron capture without fission (kg MW^{-1} year^{-1})
b_f — breeding factor (breeding ratio) of fast reactor,

we obtain

$$c_f \cong \frac{L_f}{\eta_f \cdot m_{pi}}\cdot f_w \cdot (b_f - 1) \qquad (\text{year}^{-1}) \quad (71a)$$

or

$$T_{2x} \cong \frac{\eta_f \cdot m_{pi}\cdot \ln 2}{L_f \cdot f_w \cdot (b_f - 1)}. \qquad (72a)$$

If $\frac{m_{pi}}{m_{pr}}\cdot c < \varepsilon$, where $\varepsilon \ll 1$ (e.g. $\varepsilon = 0.1$), then $1 + \frac{m_{pi}}{m_{pr}} c \cong e^{\frac{m_{pi}}{m_{pr}}c}$, which is substituted into (70) again to obtain after rearrangement the following approximate solution:

$$c_f \cong \frac{1}{\tau_f + \vartheta_f + \frac{m_{pi}}{m_{pr}}}\cdot \ln \frac{m_{pf}}{m_{pr}} \qquad (\text{year}^{-1}) \quad (74)$$

and

$$T_{2x} \cong \frac{\ln 2}{\ln \frac{m_{pf}}{m_{pr}}}\cdot \left(\tau_f + \vartheta_f + \frac{m_{pi}}{m_{pr}}\right) \qquad (\text{year}) \quad (75)$$

If $c \cdot (\tau_f + \vartheta_f) < \delta$ where $\delta \ll 1$ (e.g. $\delta = 0.1$), then $e^{c(\tau_f + \vartheta_f)} \cong 1 + c \cdot (\tau_f + \vartheta_f)$ which, when substituted into (70) again, yields a second-order equation for c. As one of the solutions to this equation, intelligible also physically, we obtain:

$$c_f \cong \frac{-m_{pr}(\tau_f + \vartheta_f) - m_{pi} + \{[m_{pr}(\tau_f + \vartheta_f) + m_{pi}]^2 - 4(m_{pr} - m_{pf})m_{pi}(\tau_f + \vartheta_f)\}^{\frac{1}{2}}}{2m_{pi}(\tau_f + \vartheta_f)} \tag{76}$$

On the basis of the 'fast alone exponent' specified, there are three variations of the nuclear energy system consisting of fast reactors of given characteristics, such as

(a) $c = c_f$: the exponent of growth of the system and the 'fast alone exponent' are equal i.e. the system capacity doubling time is equal to that of fissile material. In this case, in order to maintain the condition of equilibrium, the system may contain fast reactors only. Hence, *the system is at fuel equilibrium but not mixed.*

(b) $c < c_f$: the exponent of growth of the system lies below the 'fast alone exponent' i.e. the system capacity doubling time is longer than the doubling time of fissile material. In this case, the system may contain also thermal reactors in addition to fast reactors, in a number just enough to maintain the condition of equilibrium. Hence, *the system is mixed and at fuel equilibrium.*

(c) $c > c_f$: the exponent of growth of the system lies above the 'fast alone exponent' i.e. the system capacity doubling time is shorter than the fissile material doubling time. This means that the plutonium production by fast reactors is less than would be required to supply the fast reactors in operation and entering the system newly. In this case, two methods are offering themselves. In the first method,

— the deficit in fissile material of fast reactors shall be met by enriched uranium and no thermal reactors shall be constructed,

while in the second method

— thermal reactors shall be constructed and/or operated in a number that results in a plutonium production just meeting the deficit in fissile material of fast reactors.

The second method seems more practicable, first because here the uranium is enriched only to the degree required by the thermal reactors, and, on the other

hand, because the fast reactors in which the fissile material consists of plutonium exclusively, ensure a better average breeding ability than in case there were also fast reactors containing ^{235}U fissile material in the system. Hence, *the system is mixed but not at fuel equilibrium.*

Mixed nuclear energy system containing fast reactor of unsatisfactory breeding factor

A simplified scheme of the fuel cycle of such a nuclear energy system where $c > c_f$ is shown in Fig. 5. The most important system characteristics are given below:

— The full amount of plutonium produced in both fast and thermal reactors is recycled to the fast reactors.

— This amount of plutonium is just enough to meet the plutonium demand of fast reactors in operation and entering the system newly.

— The total fuel demand of thermal reactors is met by isotopic enrichment where also uranium in spent fuel is enriched in ^{235}U isotope, and only the deficit in fuel of thermal reactors is met by fuel produced by isotopic enrichment of natural uranium.

— Depleted uranium removed from the fast reactors and produced at the isotopic enrichment plant is used to meet the breeding material demand of fast reactors. If there is still a deficit, this shall be met by natural uranium.

Balance equation resulting from the equilibrium of produced and consumed plutonium:

$$m_{pt} \cdot P_t(t-\tau_t) + m_{pf} \cdot P_f(t-\tau_f) - m_{pr} \cdot P_f(t+\vartheta_f) - m_{pi} \cdot P'_f(t+\vartheta_f) = 0 \quad (77)$$

When substituting P_t from (4) into this equation we obtain

$$m_{pi} \cdot P'_f(t+\vartheta_f) = m_{pf} \cdot P_f(t-\tau_f) - m_{pt} \cdot P_f(t-\tau_t) - m_{pr} \cdot P_f(t+\vartheta_f) + m_{pt} \cdot P_0 \cdot e^{c(t-\tau_t)} \quad (78)$$

With transformation $t+\vartheta_f \to t$:

$$m_{pi} \cdot \frac{dP_f(t)}{dt} = m_{pf} \cdot P_f(t-\tau_f-\vartheta_f) - m_{pt} \cdot P_f(t-\tau_t-\vartheta_f) - m_{pr} \cdot P_f(t) + m_{pt} \cdot P_0 \cdot e^{c(t-\tau_t-\vartheta_f)} \quad (79)$$

This equation is a retarded-argument first-order inhomogeneous differential equation similar to (61) to be solved in the way outlined in Chapter 4.

Amount of spent uranium ^{235}U of a concentration of e_s removed from thermal reactors per unit time:

$$m_8(t) = m_{us} \cdot P_t(t-\tau_t) \qquad (\text{kg year}^{-1}) \quad (80)$$

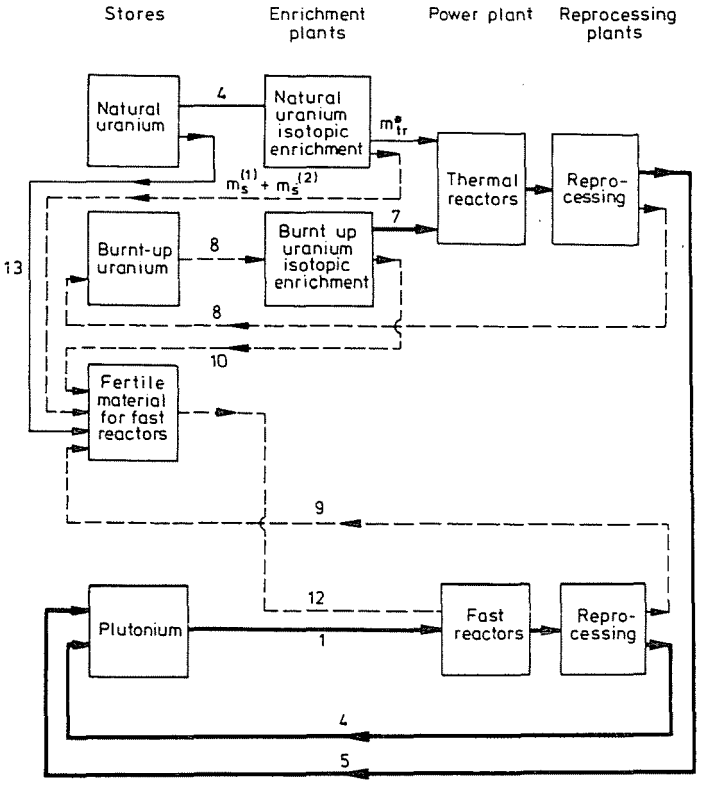

Fig. 5. Simplified fuel cycle scheme of a mixed nuclear energy system containing fast reactors of unsatisfactory breeding factor

Amount of fuel of enrichment e_r that can be produced from the above amount of spent uranium:

$$m_7(t) = \frac{e_s - e_d}{e_r - e_d} \cdot m_{ud} \cdot P_t(t - \tau_t) \qquad \text{(kg year}^{-1}\text{)} \quad (81)$$

where e_d — ^{235}U concentration of depleted uranium produced in isotopic enrichment.

The amount of depleted uranium is given by

$$m_{10}(t) = \frac{e_r - e_s}{e_r - e_d} \cdot m_{us} \cdot P_t(t - \tau_t) \qquad \text{(kg year}^{-1}\text{)} \quad (82)$$

The enriched uranium demand of thermal reactors in operation ($m_{t,r}$) is given by relationship (24). Within this, the amount that can be produced by isotopic enrichment of natural uranium:

$$m^*_{t,r}(t) = m_{t,r}(t) - m_7(t)$$

that is

$$m^*_{t,r}(t) = m_{ur} \cdot P_t(t+\vartheta_t) - \frac{e_s - e_d}{e_r - e_d} \cdot m_{us} \cdot P_t(t-\tau_t) \text{ (kg year}^{-1}\text{)} \quad (83)$$

Amount of natural uranium required to produce this amount:

$$m_4^{(1)}(t) = \frac{e_r - e_d}{e_0 - e_d} \cdot m^*_{t,r}$$

that is

$$m_4^{(1)}(t) = \frac{e_r - e_d}{e_0 - e_d} \cdot m_{ur} \cdot P_t(t+\vartheta_t) - \frac{e_s - e_d}{e_0 - e_d} \cdot m_{us} \cdot P_t(t-\vartheta_t) \quad \text{(kg year}^{-1}\text{)} \quad (84)$$

Amount of depleted uranium produced in this isotopic enrichment:

$$m_d^{(1)}(t) = \frac{e_r - e_0}{e_r - e_d} m_4^{(1)}(t)$$

that is

$$m_d^{(1)}(t) = \frac{e_r - e_0}{e_0 - e_d} \cdot m_{ur} \cdot P_t(t+\vartheta_t) - \frac{e_r - e_0}{e_r - e_d} \cdot \frac{e_s - e_d}{e_0 - e_d} \cdot m_{us} \cdot P_t(t-\tau_t) \quad \text{(kg year}^{-1}\text{)} \quad (85)$$

The amount of uranium of an enrichment e_i required in the first loading of thermal reactors entering the system newly is given by relationship (31). To produce this enriched uranium, the following amount of natural uranium is required:

$$m_4^{(2)}(t) = \frac{e_i - e_d}{e_0 - e_d} \cdot m_{ui} \cdot P'_t(t+\vartheta_t) \quad \text{(kg year}^{-1}\text{)} \quad (86)$$

Amount of depleted uranium produced in this enrichment:

$$m_d^{(2)}(t) = \frac{e_i - e_0}{e_i - e_d} m_4^{(2)}(t)$$

that is

$$m_d^{(2)}(t) = \frac{e_i - e_0}{e_0 - e_d} \cdot m_{ui} \cdot P'_t(t+\vartheta_t) \quad \text{(kg year}^{-1}\text{)} \quad (87)$$

Total natural uranium demand of thermal reactors:

$$m_4(t) = m_4^{(1)}(t) + m_4^{(2)}(t)$$

that is

$$m_4(t) = \frac{e_r - e_d}{e_0 - e_d} \cdot m_{ur} \cdot P_t(t+\vartheta_t) - \frac{e_s - e_d}{e_0 - e_d} \cdot m_{us} \cdot P_t(t-\tau_t) + \frac{e_i - e_0}{e_0 - e_d} \cdot m_{ui} \cdot P'_t(t+\vartheta_t) \quad \text{(kg year}^{-1}\text{)} \quad (88)$$

Total amount of depleted uranium produced in the isotopic enrichment processes:

$$m_d(t)=m_{10}(t)+m_d^{(1)}(t)+m_d^{(2)}(t)$$

i.e. after reasonable reduction:

$$m_d(t)=\frac{e_0-e_s}{e_0-e_d}\cdot m_{us}\cdot P_t(t-\tau_t)+\frac{e_r-e_0}{e_0-e_d}\cdot m_{ur}\cdot P_t(t+\vartheta_t)+$$
$$+\frac{e_i-e_0}{e_0-e_d}\cdot m_{ui}\cdot P_t'(t+\vartheta_t) \qquad (\text{kg year}^{-1}) \quad (89)$$

The amount of fertile material removed from the fast reactors per unit time (m_{11}) is given by relationship (12). Therefore, the amount entering the fertile material store is, as follows:

$$m_f(t)=m_d(t)+m_{11}(t)$$

that is

$$m_f(t)=\frac{e_r-e_0}{e_0-e_d}\cdot m_{ur}\cdot P_t(t+\vartheta_t)+\frac{e_0-e_s}{e_0-e_d}\cdot m_{us}\cdot P_t(t-\tau_t)+m_{dd}\cdot P_f(t-\tau_f)+$$
$$+\frac{e_i-e_0}{e_0-e_s}\cdot m_{ui}\cdot P_t'(t+\vartheta_t) \qquad (\text{kg year}^{-1}) \quad (90)$$

Fertile material demand of fast reactors per unit time (m_{12}) is given by relationship (13).

For given fast and thermal reactor parameters, there exist a growth exponent $c_1>c_f$ (where c_f — 'fast alone exponent' discussed in Chapter 5) where $m_f(t)=m_{12}(t)$.

If $c>c_1$, then $m_f(t)>m_{12}(t)$, that is, the total fertile material demand of fast reactors can be met by depleted uranium. In this case, the total natural uranium consumption of the nuclear energy system per unit time is described by $m_4(t)$ in (88). The specific natural uranium demand of such a mixed system can be expressed as

$$\mu_n(t)=\frac{m_4(t)}{P^*(t)} \qquad (\text{kg MW}_e^{-1}\ \text{year}^{-1}) \quad (91)$$

where $P^*(t)$ — quantity given in (45).

In case $c_f\leqq c\leqq c_1$, $m_f(t)\leqq m_{12}(t)$, that is the amount of depleted uranium continuously produced in the system is not enough to meet the total fertile material demand of fast reactors. The deficit shall be met by natural uranium provided no depleted uranium accumulated earlier is available. Due to this additional supply, the natural uranium consumption will be

$$m_{13}(t)=m_{12}(t)-m_f(t)$$

that is

$$m_{13}(t)=m_{di}\cdot P'_f(t+\vartheta_f)+m_{dr}\cdot P_f(t+\vartheta_f)-\frac{e_r-e_0}{e_0-e_d}\cdot m_{ur}\cdot P_t(t+\vartheta_t)-$$
$$-\frac{e_0-e_s}{e_0-e_d}\cdot m_{us}\cdot P_t(t-\tau_t)-m_{dd}\cdot P_f(t-\tau_f)-$$
$$-\frac{e_i-e_0}{e_0-e_d}\cdot m_{ui}\cdot P'_t(t+\vartheta_t) \qquad \text{(kg year}^{-1}\text{)} \quad (92)$$

In this case, the total natural uranium consumption of the nuclear energy system can be described as

$$m_n(t)=m_4(t)+m_{13}(t)$$

that is, after reasonable reduction,

$$m_n(t)=m_{ur}\cdot P_t(t+\vartheta_t)+m_{dr}\cdot P_f(t+\vartheta_f)-m_{us}\cdot P_t(t-\tau_t)-m_{dd}\cdot P_f(t-\tau_f)+$$
$$+m_{di}\cdot P'_f(t+\vartheta_f) \qquad \text{(kg year}^{-1}\text{)} \quad (93)$$

In such a mixed system the specific natural uranium demand:

$$\mu_n^*=\frac{m_n(t)}{P^*(t)} \qquad \text{(kg MW}_e^{-1}\text{ year}^{-1}\text{)} \quad (94)$$

where $P^*(t)$ — quantity given in (45).

In a system like this, i.e. mixed but not at fuel equilibrium, the efficiency of utilization of natural uranium will be (similarly to that given in 46):

$$\eta_m=\frac{\mu_0}{\mu_n}\cdot 100 \qquad (\%) \quad (95)$$

in case $c_f\leqq c\leqq c_1$,
where μ_n — quantity given in (91),
and

$$\eta_m^*=\frac{\mu_0}{\mu_n^*}\cdot 100 \qquad (\%) \quad (96)$$

in case $c>c_1$,
where μ_n^* — quantity given in (94).

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