USE OF RADIOGRAPHS FOR SECTION IMAGING

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Summary

Basic mathematical procedures are summarised for reconstructing a section image of an object from its measured projections at multiple angles. An analog device has been constructed which uses an X-ray film as recorder, and realises an optical backprojection. A technique is developed where traditional radiographs from multiple angles are used for section imaging. This allows a simple extension of existing technique and use of existing apparatus for section imaging. To test and evaluate the performance of the reconstruction method developed, a test object is constructed and imaged. Radiograph at one angle, section image from 15 radiographs and from a sinogram are demonstrated and compared.

Introduction

A radiograph is a two dimensional projection of a three dimensional object. To obtain information about the spatial locations of different structures within the object a number of radiographs can be taken from different angles. These radiographs can give useful information concerning simple internal constructions.

Modern structures are, however, becoming more and more complex and the evaluation of a three dimensional object by a two dimensional shadow image is nearly impossible. There is therefore increasing demand to use a suitable three dimensional imaging technique. This difficulty has been solved by a technique called "Computed Tomography," where a single section of the object is displayed thus allowing for complete exclusion of sections not under study.

Special advantages may be gained, if traditional radiographs from different angles are used for section imaging. This would allow a simple extension of an existing technique and the use of available apparatus for section imaging.

Principles of the technique

A tomogram is a picture of a slice. The image is reconstructed as if it had been possible to cut and view the object to be tested over a plane. A full, three-dimensional picture can be obtained by stacking a sequence of such layers. Tomography is a technique for making such an image.

All radiographic imaging techniques aim at determining the spatial distribution of the absorption coefficient μ, within an object. In Computed Tomography this problem is solved by mapping the variation of μ within a sequence of selected planes or thin layers of an object (Fig. 1).

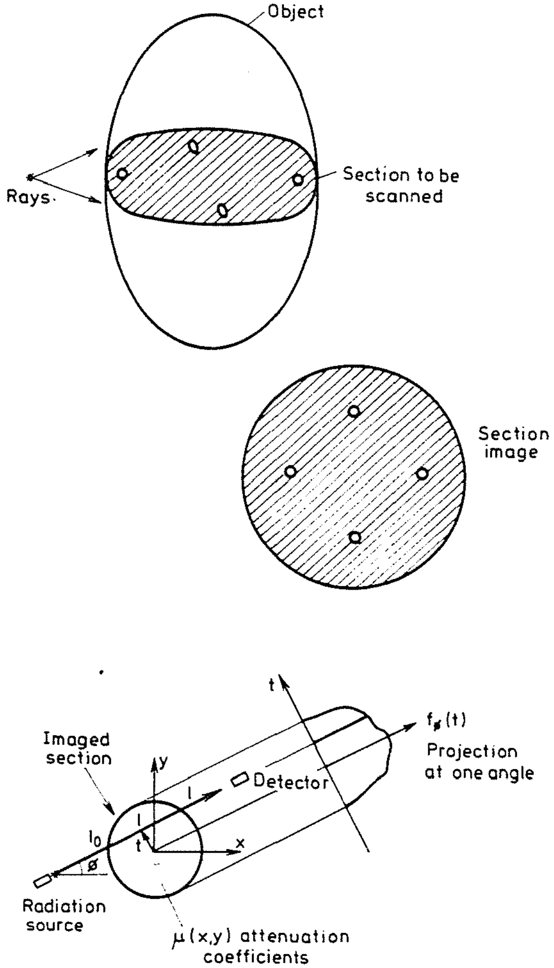

Fig. 1. Principles of tomographic imaging

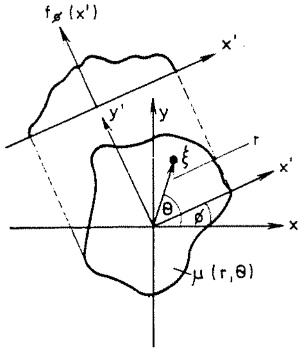

Fig. 2. Projection

The basic principle can be formulated quantitatively as follows: a two dimensional section of an object is characterised by a linear absorption coefficient distribution $\mu(x, y)$. Supposing a thin beam of gamma or X-rays traverses the plane along a straight line 1, and the intensities of the incident beam and the beam emerging from the plane are I_0 and I, respectively (Fig. 2), then

$$I = I_0 \exp\left[- \int_l \mu(x, y) dl \right],$$

or rather

$$\ln \frac{I}{I_0} = - \int_l \mu(x, y) dl = f_\Phi(t)$$

where the line integral (called ray integral or ray sum) is evaluated along the straight line 1.

A set of ray integrals at the corresponding Φ angle forms a projection $f_\Phi(t)$.

The mathematical problem is to find $\mu(x, y)$ when knowing the $f_\Phi(t)$ projections at multiple angles. This problem can be solved approximately only, because the number of ray integrals, which can be determined experimentally by intensity measurements, is necessarily limited. Better approximation can be obtained if more ray integrals are known. Therefore some kind of scanning technique is used.

Image reconstruction from projections

A reconstruction algorithm tells us how to reconstruct a function from its measured projections. Mathematically, the task is the calculation of a two dimensional function from a set of line integrals.

Many such mathematical algorithms have been developed. There is no consensus as to which method is best. It is likely that different methods are better for different applications.

The data for the reconstruction are said to be complete when the number of projections is sufficient to fully determine the image. In some cases it may be necessary to reconstruct an image from fewer projections.

Projection

The data from which we are to reconstruct the object are projections of the object. Let a two-dimensional function $\mu(x, y)$, which is the attenuation coefficient, represent a cross section (Fig. 2). A line running through the cross section is called a ray. The integral of $\mu(x, y)$ along a ray is called a ray integral and a set of ray integrals forms a projection. For any given direction, defined by Φ the projection is:

$$f_{\Phi}(x') = \ln \frac{I}{I_0} = -\int_l \mu(x, y) dy'$$

Back projection

This is an inverse form of the projection operation, since we take a one-dimensional function of $P(x')$ and create from it a two-dimensional distribution by smearing it uniformly over the entire space in the y' direction

$$b(x', y') = f_{\Phi}(x')$$

In the unrotated frame, we would write

$$b(r) = b(r, \Theta) = f_{\Phi}[\cos(\Theta - \Phi)]$$

This operation is illustrated in Fig. 3.

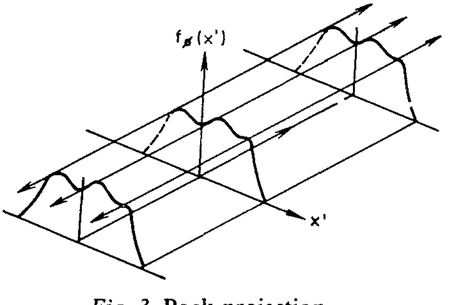

Fig. 3. Back projection

Summation image

The process for combining many back projections created a new two-dimensional distribution, the summation image. With a discrete number of back projections of different Φ projection angles, the summation image would simply be the arithmetic summation of the individual back projections. In our case of continuous data sets

$$G(r) = G(r, \Theta) = \frac{1}{\pi}\int_0^{\pi} f_{\Phi}(x')d\Phi = \frac{1}{\pi}\int_0^{\pi} f_{\Phi}\,[r\cos(\Theta-\Phi)d\Phi]$$

The relation of the reconstructed image by this summation can be characterized by calculating the point spread function. This is done by considering a special case for $\mu(r)$, namely a point located at the origin

$$\mu(r) = \delta(x)\delta(y)$$

The summation image in this case (point spread function) is:

$$g(r) = \frac{1}{\pi}\int_0^{\pi} d\Phi \int_{-\infty}^{\infty} \mu(x', y')dy' = \frac{1}{\pi}\int_0^{\pi} \delta(x')d\Phi$$

$$G(r) = \frac{1}{\pi}\int_0^{\pi} \delta[r\cos(\Theta-\Phi)]d\Phi = \frac{1}{\pi r}$$

The results show that the summation image formed from the projections is closely related to μ itself. For a general absorption distribution

$$G(r) = \mu(r, \Theta) \otimes \otimes\, g(r)$$

$$G(r) = \mu(r, \Theta) \otimes \otimes \frac{1}{\pi r}$$

The symbol $\otimes \otimes$ is used to denote a two-dimensional convolution.

Due to the extended skirts of the $\frac{1}{r}$ function $G(r)$, the reconstructed image will be a blurred version of the object distribution.

Experimental technique

General description

To investigate the feasibility and usefulness of the tomographic procedure for section imaging, experimental apparatuses have been constructed.

A computer assisted tomograph designed for medical applications would be too costly for such a feasibility study. In medicine, for high resolution pictures, the machine would normally require an elaborate and expensive detector array and computer. However, if we are prepared to accept lower accuracy, but still at the higher resolution required, considerable savings could be made. It is possible to use simpler analogue computing methods. Moreover, it is possible to improve spatial resolution cheaply by the use of an X-ray film instead of a detector array.

Techniques theoretically similar to computer assisted tomography are possible by using X-ray film as recording medium and incoherent optical methods for realisation of some special mathematical image reconstruction.

Tomography comprises two processes (Fig. 4.)

a) recording projection: by detecting intensity distribution of radiation transmitted through the object at different directions, which corresponds to the scanning process;

b) reconstruction of the desired tomogram from the recorded projection data. This corresponds to the formation and display of the image matrix in computed tomography.

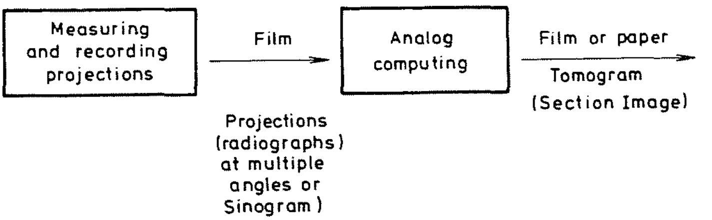

Fig. 4. Making a tomogram (section image)

Recording projections on film

If the film density due to the exposure by the ray intensity I_0 (unattenuated in the object) is D_{max}, any density difference in the strip (see Fig. 5) is:

$$\Delta D = D_{max} - D = \gamma \log \frac{I_0}{I}$$

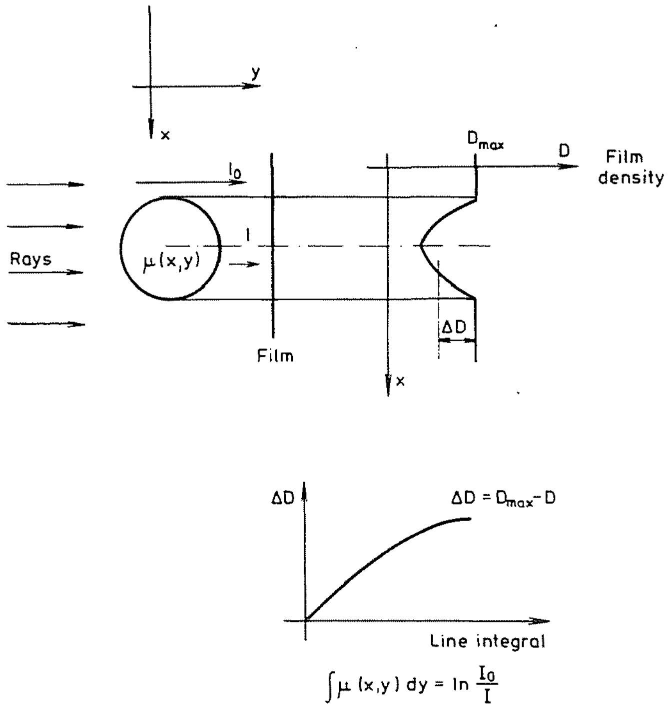

Fig. 5. Recording projection on film

assuming a film characteristic curve of

$$D = \gamma \log \frac{I}{I_{ref}}$$

where γ is the gradient, I the ray intensity I_{ref} is some reference value.

Attenuation in the object is

$$\ln \frac{I_0}{I} = \int \mu(x, y)dy$$

which means

$$\Delta D = 0.43\gamma \int \mu(x, y)dy$$
$$\Delta D = \text{const.}\ f_{\Phi}(x')$$

This way X-ray films automatically perform the logarithmic conversion necessary to obtain the projection data from radiation intensity. The developed film has a density that is linear with the effective thickness ($\int \mu dy$) over a limited dynamic range.

The scanner

The arrangement for recording projections continuously can be seen in Fig. 6a. Radiation, after having been passed through and attenuated by the object, is collimated by a slit before it hits the film. During the exposure the object rotates about its own axis (at an angle Φ) perpendicular to the beam and at the same time the film is moved parallel to this axis. This way each projection of a section is recorded as a narrow strip on film immediately below the strip from the preceding projections of the object.

Each element in the film represents a measured line integral. The reduction in density for this element represents the integrated attenuation along this line. Definition, contrast and details are as in an ordinary film.

The recorded transmission data may be plotted in the from of a series of projections where the x' axis represents the position of a particular measurement and the y axis is the measured line integral. The completed projection data set may be plotted as a two dimensional $\Phi - x'$ graph as shown in Fig. 7. Each horizontal line represents a projection measured at the

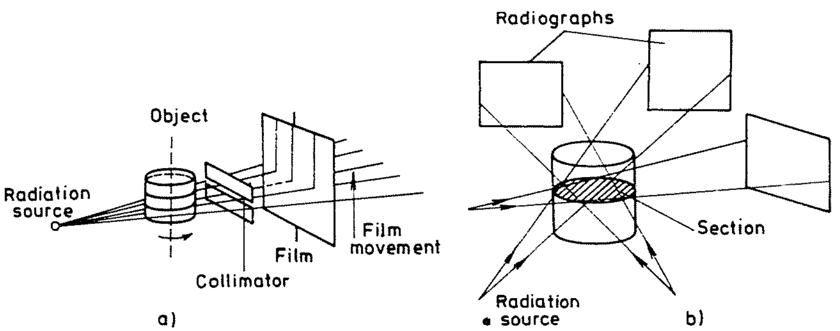

Fig. 6. Recording projections: a) Scanner; b) Radiographs at multiple angles

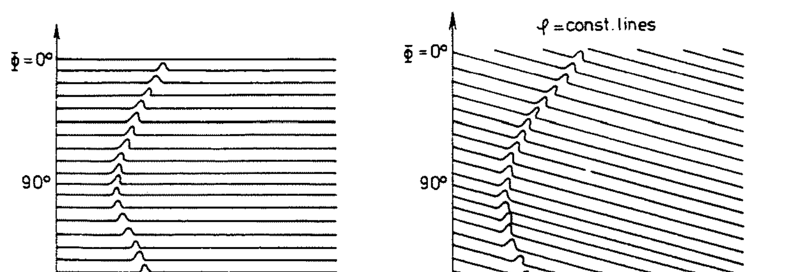

Fig. 7. $f(x')$ projection data arrangement in a sinogram format

corresponding angle. This plot is often called a sinogram as the loci of points inside the object describe a sine curve where amplitude and phase are related to the polar coordinates of the point.

In Fig. 7a projection data scanned by a parallel radiation beam are seen. Peaks in the projections due to a small point are to be seen.

Back projector

The principle of the back projector we realised can be seen in Fig. 8.

The sinogram is illuminated by a linear light source. A given line in the object is projected as a point in the projection and this point is projected back as a corresponding line on a film.

The illuminated line selects from the projection the points representing a complete parallel projection of the object.

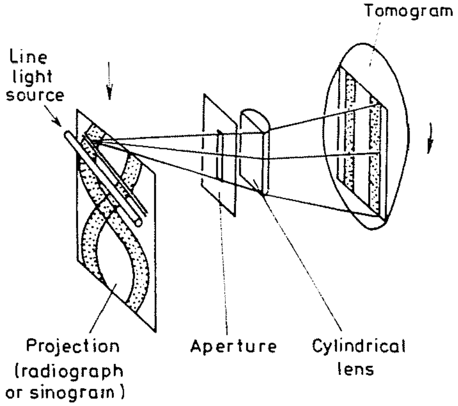

Fig. 8. Back projection with a cylindrical lens

Experimental results obtained with a test object

The test object

To test and evaluate the performance of the reconstruction methods we developed, a test object was constructed and imaged.

Since the objective of the work is aimed at imaging objects of difficult internal constructions, it consists of aluminium tubes (Fig. 9). The internal diameter of the holes inside the tubes are meant for testing resolution capability. The analog method with an X-ray film is expected to give a fine resolution, able to resolve holes of a tenth of a millimeter. No digital computer scanner gives such a resolution.

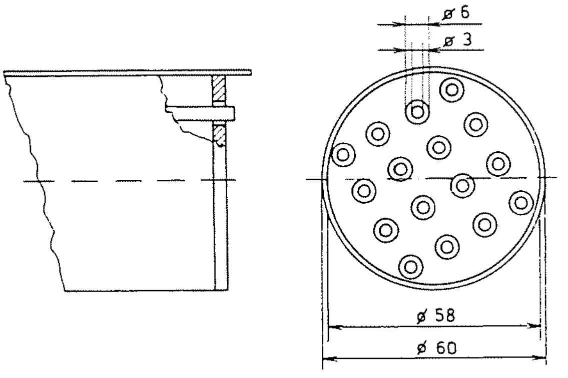

Fig. 9. Test object

Radiograph

Radiograph at one angle, perpendicular to the long axis of the test object can be seen in Fig. 10. Fifteen such radiographs are taken at multiple angles. An internal structure study by evaluating only these radiographs is impossible.

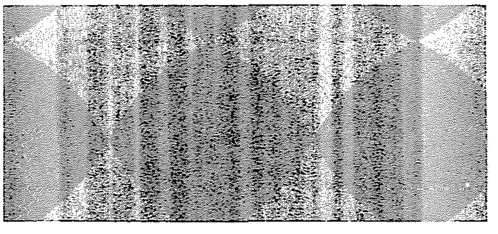

Fig. 10. Radiograph

The sinogram

The sinogram of the test object obtained by continuous scanning can be seen in Fig. 11. The radiation source used is an X-ray tube at 100 kV. For the divergent beam a curved detector is used.

Section image from radiographs

Fifteen radiographs from different angles are used for reconstructing the section image in Fig. 12. The quality of the section image depends on the number of radiographs recorded.

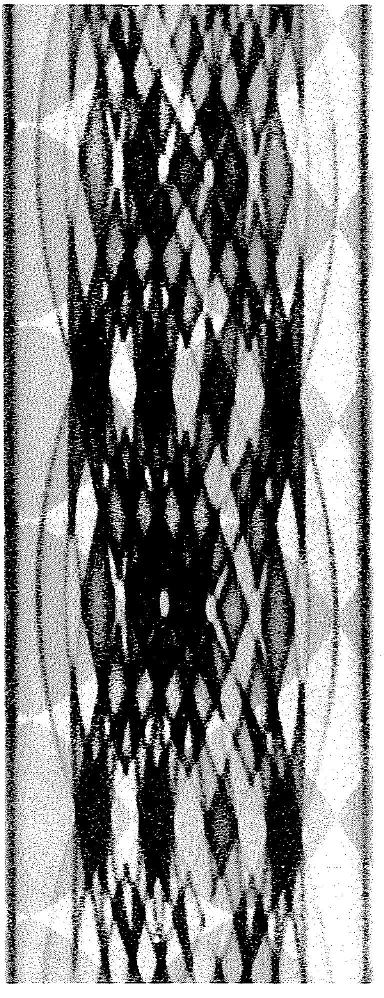

Fig. 11. The sinogram

Section image from a sinogram

A section image reconstructed from a complete sinogram can be seen in Fig. 13. It corresponds to an infinite number of radiographs recorded.

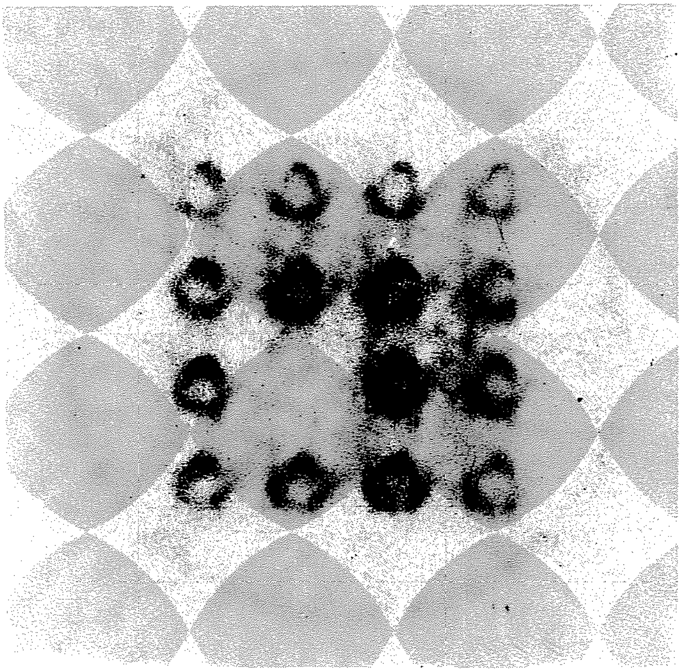

Fig. 12. Section image from 15 radiographs

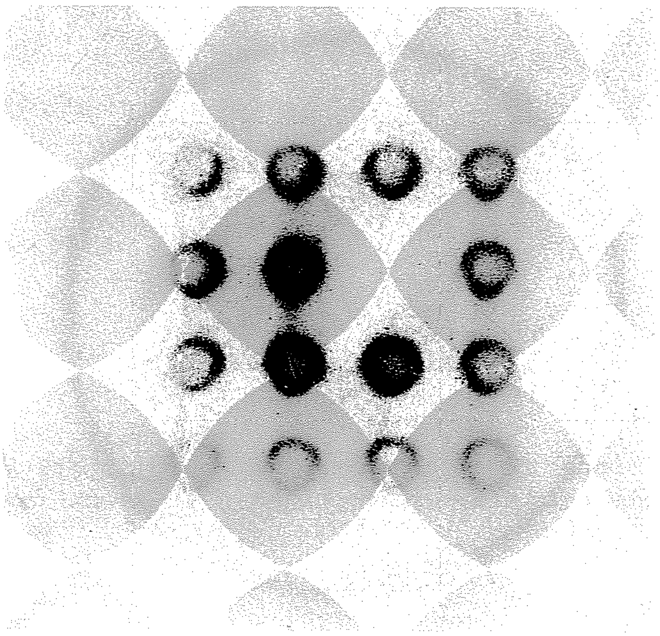

Fig. 13. Section image from a sinogram

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