INVESTIGATION OF THE MECHANICAL BASIC EQUATIONS OF SOLID BODIES BY MEANS OF ACCELERATION WAVE

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Summary

The basic equation of solid bodies experiencing minor deformation can be written after the constitutive equation has been determined. This study outlines a new theory of determining the constitutive equations, permitting new experimental methods to be set up on this basis.

Basic equations are needed for the mechanical tests of solid bodies in motion. The formulation of the basic equations is a problem having a bearing upon the fundamentals of mechanics; in the knowledge of the basic equations and under appropriate supplementary conditions, the problem of determining the motion of the body can be mathematically formulated. In this way, the starting equation system of the mechanical test is obtained.

In case of any medium (body) considered to be continuum, the basic equations are, as follows: kinematic or geometric equation, equation of mass balance, equation of motion, constitutive equations, and other equations expressing physical effects [3]. In investigating the motion of a body, this study takes only the mechanical interactions into consideration; thus, the basic equations are constituted by the first four groups of equations specified. From among the equations considered, the formulation of the constitutive equations can not be considered final in the investigation of, among others, solid bodies, especially of those in motion. This is not only due to the fact that bodies of different material can be described by a constitutive equation of different intrinsic properties each but also to the fact that the experiment, and the variables to be observed during the experiment, by means of which the required constitutive equation can be determined, are unknown [1], [2]. Experimental tests are required to determine the constitutive equation for given material. Theoretical considerations that can be followed also by measurements shall be taken as a basis for the experiment. Constitutive equations that can be taken into consideration on the basis of the theoretical consideration are the possible constitutive equations. First of all, a theory giving information on the structure of the possible constitutive equation shall be set up. In this way, the necessary experiment can be decided and the law applying to given material can be selected from the possible constitutive equation.

In the knowledge of the constitutive equation, we have a complete equation system for given plastic body. Two known groups of this equation system in the Cartesian co-ordinate system are the kinematic equation:

$$2\frac{\partial \varepsilon_{ij}}{\partial t}=\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}$$

and the equation of motion:

$$\rho\frac{\partial v_i}{\partial t}=\frac{\partial \sigma_{ij}}{\partial x_j}+q_i, \qquad \sigma_{ij}=\sigma_{ji},$$

the quantities in the equations being the strain tensor, velocity, density, stress tensor, body force, and the time and space co-ordinates.

Indicated in the known equations specified so far are the first partial derivatives of the strain tensor and stress tensor. The same quantities and basic functions are included in the other six equations. This restriction shall reasonably be completed with the constraint that the body can be inhomogeneous or rheonomous, that is co-ordinate quad $x_{\acute{\imath}}$ also appears explicitly in the constitutive equations (x_1, x_2, x_3 being space co-ordinates while $x_4 \equiv t$ time co-ordinate).

Let the constitutive equation be function system

$$F_\alpha=0 \qquad \alpha=1,2,\ldots,6.$$

Certain requirements can be imposed upon constitutive equations F_α unknown for the time being, such as:

(a) Function F_α is a function of co-ordinates x_i, time $x_4 \equiv t$, basic functions ε_{ij} and σ_{ij}, and of their first partial derivatives.

(b) In spite of any mechanically possible initial conditions, acceleration wave can be induced in the body, propagating at finite speed.

(c) There exist both progressive and return acceleration waves.

(d) F_α is a continuously differentiable function of its variables.

Now, by introducing function $\alpha(ij)$ in subscript and giving partial derivation also in subscript:

$$2\varepsilon_{\alpha(ij),4}=v_{i,j}+v_{j,i} \tag{1}$$

$$\rho v_{i,4}=\sigma_{\alpha(ij),j}+q_i \tag{2}$$

$$F_\alpha(\sigma_{\beta\acute{\imath}}, \varepsilon_{\gamma\acute{\jmath}}, \sigma_\vartheta, \varepsilon_\delta, x_{\acute{k}})=0 \tag{3}$$

where $\sigma_{\beta i}=\frac{\partial\sigma_\beta}{\partial x_i}$ and $\varepsilon_{\gamma j}=\frac{\partial\varepsilon_\gamma}{\partial x_j}$ in function F_α are abbreviations and the Greek letters are again subscript functions, e.g. $\sigma_\beta=\sigma_{\beta(ij)}$ $i,j=1,2,3$; $\beta=1,2,\ldots,6$.

The setup of (3) meets the requirements specified under (a) and (d). Because of invariance as against Galilean transformation, a requirement the constitutive equations must by all means meet, F_α can not contain v_i while in accordance with equations (1) and (2), its derivatives are included in $\varepsilon_{\alpha,4}$ and/or $\sigma_{\alpha,4}$. Associated with equation system (1), (2), (3) are furthermore supplementary conditions as well as initial and boundary conditions.

The kinematical and dynamical compatibility condition to meet requirement (b) is known (see e.g. [7]). An additional compatibility condition comes from (3). Let the wave function be $\varphi(x_i)$ with $\mathring{\sigma}_{\beta i}$, $\mathring{\varepsilon}_{\gamma j}$, $\mathring{\sigma}_\vartheta$ and $\mathring{\varepsilon}_\delta$ being given before the wave front. The same quantities after the wave front are $\sigma_{\beta i}$, $\varepsilon_{\gamma j}$, σ_ϑ and ε_δ. There is a relationship of

$$\begin{aligned}&\sigma_{\beta i}=\mathring{\sigma}_{\beta i}+\mu_\beta\varphi_i\\&\varepsilon_{\gamma j}=\mathring{\varepsilon}_{\gamma j}+\kappa_\gamma\varphi_j\\&\mathring{\sigma}_\vartheta=\sigma_\vartheta,\qquad \mathring{\varepsilon}_\delta=\varepsilon_\delta\end{aligned}\tag{4}$$

between both groups of quantities.

Making use of these, the mass compatibility condition of the acceleration wave can be written as

$$F_\alpha(\sigma_{\beta i},\varepsilon_{\gamma j},\sigma_\vartheta,\varepsilon_\delta,x_i)-F_\alpha(\mathring{\sigma}_{\beta i},\mathring{\varepsilon}_{\gamma j},\mathring{\sigma}_\vartheta,\mathring{\varepsilon}_\delta,x_i)=0\tag{5}$$

After utilizing (4), this difference function shall be designated f_α, that is

$$f_\alpha\equiv F_\alpha-\mathring{F}_\alpha,$$

where $\mathring{F}_\alpha$ is a value to substitute for F_α for values distinguished by zero above them.

Thus, the mass compatibility condition:

$$f_\alpha=0.\tag{6}$$

f_α is a function of quantities distinguished by zero above them while x_i that of κ_β, κ_γ and $\varphi_i=\frac{\partial\varphi}{\partial x_i}$.

Let $f_\alpha=0$ be a first-order partial non-linear differential equation for φ with all the other functions in it known. Solution to this equation will be the same function φ if f_α constitutes an involutory function system [4], [8], [6],

[9]. A function system will be involutory if the Poisson bracket is zero. The Poisson bracket is given by the following expression:

$$(f_\alpha, f_\beta) \equiv \frac{\partial f_\alpha}{\partial \varphi_{\hat{p}}} \frac{\partial f_\beta}{\partial x_{\hat{p}}} - \frac{\partial f_\alpha}{\partial x_{\hat{p}}} \frac{\partial f_\beta}{\partial \varphi_{\hat{p}}}.$$

(to be integrated according to subscript $\hat{p}$).

A necessary and satisfactory condition for the existence of the acceleration wave [4], [6], [9]:

$$(f_\alpha, f_\beta) \equiv 0.$$

In other words, function system $F_\alpha - \mathring{F}_\alpha$ resulting from the constitutive equation constitutes an involutory function system. Thus, the requirement specified under (b) is partially met. Some designations have to be introduced to write the Poisson bracket in detail, such as

$$S_{\alpha\beta\hat{p}} = \frac{\partial f_\alpha}{\partial \sigma_{\beta\hat{p}}} = \frac{\partial F_\alpha}{\partial \sigma_{\beta\hat{p}}}$$

$$E_{\alpha\gamma\hat{p}} = \frac{\partial f_\alpha}{\partial \varepsilon_{\gamma\hat{p}}} = \frac{\partial F_\alpha}{\partial \varepsilon_{\gamma\hat{p}}}, \tag{7}$$

and

$$s_{\alpha\vartheta} = \frac{\partial f_\alpha}{\partial \sigma_\vartheta} = \frac{\partial F_\alpha}{\partial \sigma_\vartheta}, \qquad e_{\alpha\delta} = \frac{\partial f_\alpha}{\partial \varepsilon_\delta} = \frac{\partial F_\alpha}{\partial \varepsilon_\delta}$$

Similarly, the quantities with zero above them can be introduced as well:

$$\mathring{S}_{\alpha\beta\hat{p}} = -\frac{\partial f_\alpha}{\partial \mathring{\sigma}_{\beta\hat{p}}} = \frac{\partial \mathring{F}_\alpha}{\partial \mathring{\sigma}_{\beta\hat{p}}}, \qquad \mathring{E}_{\alpha\gamma\hat{j}} = -\frac{\partial f_\alpha}{\partial \mathring{\varepsilon}_{\gamma\hat{j}}} = \frac{\partial \mathring{F}_\alpha}{\partial \mathring{\varepsilon}_{\gamma\hat{j}}}$$

$$\mathring{s}_{\alpha\vartheta} = -\frac{\partial f_\alpha}{\partial \mathring{\sigma}_\vartheta} = \frac{\partial \mathring{F}_\alpha}{\partial \mathring{\sigma}_\vartheta}, \qquad \mathring{e}_{\alpha\delta} = -\frac{\partial f_\alpha}{\partial \mathring{\varepsilon}_\delta} = \frac{\partial \mathring{F}_\alpha}{\partial \mathring{\varepsilon}_\delta} \tag{8}$$

With these

$$\frac{\partial f_\alpha}{\partial \varphi_{\hat{p}}} = S_{\alpha\beta\hat{p}}\, \mu_\beta + E_{\alpha\gamma\hat{p}}\, \kappa_\delta$$

and

$$\frac{\partial f_\alpha}{\partial x_{\hat{p}}} = (S_{\alpha\beta\hat{\imath}}\, \mu_{\beta\hat{p}} + E_{\gamma\hat{\imath}}\, \kappa_{\gamma\hat{p}})\, \varphi_{\hat{\imath}} +$$

$$+ (s_{\alpha\vartheta}\mu_\vartheta + e_{\alpha\delta}\kappa_\delta)\, \varphi_{\hat{p}} + (S_{\alpha\beta\hat{\imath}} - \mathring{S}_{\alpha\beta\hat{\imath}})\, \mathring{\sigma}_{\beta\hat{\imath}\hat{p}} +$$

$$+ (E_{\alpha\gamma\hat{j}} - \mathring{E}_{\alpha\gamma\hat{j}})\, \mathring{\varepsilon}_{\gamma\hat{j}\hat{p}} + (S_{\alpha\vartheta} - \mathring{S}_{\alpha\vartheta})\, \mathring{\sigma}_{\vartheta\hat{p}} +$$

$$+ (e_{\alpha\delta} - \mathring{e}_{\alpha\delta})\, \mathring{\varepsilon}_{\delta\hat{p}} + f_{\alpha\hat{p}}.$$

For the sake of further abbreviation before the Poisson bracket is written, let

$$M_{\alpha\hat{p}} \equiv S_{\alpha\beta\hat{p}}\,\mu_\beta + E_{\alpha\gamma\hat{p}}\,\kappa_\gamma = \frac{\partial f_\alpha}{\partial \varphi_{\hat{p}}}$$

and (9)

$$m_\alpha \equiv s_{\alpha\vartheta}\,\mu_\vartheta + e_{\alpha\delta}\,\kappa_\delta$$

finally

$$N_{\alpha\hat{p}} = (S_{\alpha\beta\hat{\imath}} - \mathring{S}_{\alpha\beta\hat{\imath}})\,\mathring{\sigma}_{\beta\hat{\imath}\hat{p}} + (E_{\alpha\gamma\hat{\jmath}} - \mathring{E}_{\alpha\gamma\hat{\jmath}})\,\mathring{\varepsilon}_{\gamma\hat{\jmath}\hat{p}} + \\ + (s_{\alpha\vartheta} - \mathring{s}_{\alpha\vartheta})\,\mathring{\sigma}_{\vartheta\hat{p}} + (e_{\alpha\delta} - \mathring{e}_{\alpha\delta})\,\mathring{\varepsilon}_{\gamma\hat{p}} + f_{\alpha\hat{p}}$$

Now the Poisson bracket is equally zero, in particular:

$$(f_\alpha, f_\omega) = M_{\alpha\hat{p}}\,[(S_{\omega\beta\hat{\imath}}\,\mu_{\beta\hat{p}} + E_{\omega\delta\hat{\imath}}\,\kappa_{\delta\hat{p}})\,\varphi_{\hat{\imath}} + m_\omega\varphi_{\hat{p}} + N_{\omega\hat{p}}] - \\ - M_{\omega\hat{p}}\,[(S_{\alpha\beta\hat{\imath}}\,\mu_{\beta\hat{p}} + E_{\alpha\delta\hat{\imath}}\,\kappa_{\delta p})\,\varphi_{\hat{\imath}} + m_\alpha\varphi_{\hat{p}} + N_{\omega\hat{p}}] \equiv 0\,. \qquad (10)$$

After utilizing (4), φ or $\varphi_{\hat{\imath}}$ are not ranging among the variables of matrices $M_{\alpha\hat{p}}\,.\,S_{\omega\beta\hat{\imath}}$, $E_{\omega\delta\hat{\imath}}$, m_ω and $N_{\omega\hat{p}}$, that means they can be calculated from F_α and/or $\mathring{F}_\alpha$ on the basis of (7) and (8), respectively.

Before (10) is further analyzed, the projective equation system falling within the basic range of the characteristics of one of differential equations $f_\alpha = 0$ is worth writing. This, according to [6], [9], can be written as

$$\frac{dx_p}{\dfrac{\partial f_{\underline{\alpha}}}{\partial \varphi_{\underline{p}}}} = \frac{dx_4}{\dfrac{\partial f_{\underline{\alpha}}}{\partial \varphi_4}}$$

that is, since $dx_4 \equiv dt$,

$$\frac{\partial f_{\underline{\alpha}}}{\partial \varphi_4}\,\frac{dx_p}{dt} = \frac{\partial f_{\underline{\alpha}}}{\partial \varphi_{\underline{p}}}\,.$$

α and p can be chosen optionally. With the derivatives of f_α designated as in the first equality of (9):

$$M_{\underline{\alpha}4}\,\frac{dx_p}{dt} = M_{\underline{\alpha}p} \qquad \text{or} \qquad \frac{dx_p}{dt} = \frac{M_{\underline{\alpha}p}}{M_{\underline{\alpha}4}}$$

It is easy to accept that a substitution of $\dfrac{dx_p}{dt}$ into $\varphi_p\dfrac{dx_p}{dt} + \varphi_4 = 0$ and the introduction of normal unit vector $n_p = \dfrac{\varphi_p}{\sqrt{\varphi_k\varphi_k}}$ of the wave front will yield

$$-\frac{\varphi_4}{\sqrt{\varphi_k\varphi_k}} = \frac{\varphi_p}{\sqrt{\varphi_k\varphi_k}}\,\frac{M_{\underline{\alpha}p}}{M_{\underline{\alpha}4}}$$

After reduction we obtain

$$M_{\alpha\bar{\beta}}\varphi_{\bar{\beta}}=0\,,$$

or in detail with the first equality of (9):

$$S_{\alpha\beta\bar{\beta}}\mu_\beta+E_{\alpha\gamma p}\kappa_\gamma=0\,.$$

Finally, taking into consideration also formula $c=-\dfrac{\varphi_4}{\sqrt{\varphi_k\varphi_k}}$ and the definition of n_p we obtain:

$$(2\rho S_{\alpha\beta 4}c^3-2\rho S_{\alpha\beta p}n_p c^2+E_{\alpha\gamma 4}n_{\gamma\beta}c-E_{\alpha\beta p}n_{\gamma\beta}n_p)\,\mu_\beta=0 \tag{11}$$

and in case $\mu_\beta\neq 0$ we obtain the equation of wave propagation where c is the velocity of wave propagation.

$\mu_\beta\neq 0$ may occur if

$$\det(2\rho S_{\alpha\beta 4}c^3-2\rho S_{\alpha\beta p}n_p c^2+E_{\alpha\gamma 4}n_{\gamma\beta}c-E_{\alpha\gamma p}n_{\gamma\beta}n_p)=0\,, \tag{12}$$

that is if the wave velocity equation is satisfied.

The acceleration wave surface is the characteristic surface of equation system (1), (2) and (3) [9], [10], [12].

The matrix of $n_{\alpha\beta}$ in (11) and (12):

$$(n_{\alpha\beta})=\begin{bmatrix} 2n_1^2 & 0 & 0 & 2n_1n_2 & 2n_1n_3 & 0 \\ 0 & 2n_2^2 & 0 & 2n_1n_2 & 0 & 2n_2n_3 \\ 0 & 0 & 2n_3^2 & 0 & 2n_1n_3 & 2n_2n_3 \\ n_1n_2 & n_1n_2 & 0 & n_1^2+n_2^2 & n_2n_3 & n_1n_3 \\ n_1n_3 & 0 & n_1n_3 & n_2n_3 & n_1^2+n_3^2 & n_1n_2 \\ 0 & n_2n_3 & n_2n_3 & n_1n_3 & n_1n_2 & n_2^2+n_3^2 \end{bmatrix}$$

Thus μ_β substitutes for κ_γ, namely

$$2\rho c^2\kappa_\alpha=n_{\alpha\beta}\mu_\beta\,.$$

(11) is the general formula of the wave propagation equation and an investigation of the equation is usually considered. This investigation shall reasonably be carried out in a possibly most generalized way. Equation (11):

$$(2\rho S_{\alpha\beta 4}c^3-2\rho S_{\alpha\beta p}n_p c^2+E_{\alpha\gamma 4}n_{\gamma\beta}c-E_{\alpha\gamma p}n_{\gamma\beta}n_p)\,\mu_\beta=0\,.$$

General wave amplitude of the stress derivative will differ from zero if wave velocity equation

$$\det\left(2\rho S_{\alpha\beta 4}c^3-2\rho S_{\alpha\beta p}n_p c^2+E_{\alpha\gamma 4}n_{\gamma\beta}c-E_{\alpha\gamma p}n_{\gamma\beta}n_p\right)=0 \tag{13}$$

is satisfied. For c, this equation is of a 18th-order equation. From among the 18 roots, at least one shall be a real positive while another a real negative root in compliance with requirements (b) and (c). Should all the roots in (13) be real roots, then equation system (1), (2) and (3) will be a whole hyperbolic partial differential equation [12].

Assuming that c and the associated μ_α are known, an expression quadratic in μ shall reasonably assigned to (11) by multiplying (11) by μ_α. The following designation can be introduced to abbreviate the expression:

$$\begin{aligned}
S_1 &\equiv 2\rho S_{\alpha\beta 4}\mu_\alpha\mu_\beta=2\rho S_{(\alpha\beta)4}\mu_\alpha\mu_\beta\\
S_2 &\equiv -2\rho S_{\alpha\beta p}n_p\mu_\alpha\mu_\beta=-2\rho S_{(\alpha\beta)p}n_p\mu_\alpha\mu_\beta\\
E_1 &\equiv E_{\alpha\gamma 4}n_{\gamma\beta}\mu_\alpha\mu_\beta=E_{(\alpha\gamma p}n_p n_{\gamma\beta)}\mu_\alpha\mu_\beta\\
E_2 &\equiv -E_{\alpha\gamma p}n_p n_{\gamma\beta}\mu_\alpha\mu_\beta=-E_{(\alpha\gamma p}n_p n_{\gamma\beta)}\mu_\alpha\mu_\beta\,.
\end{aligned} \tag{14}$$

In the zero case of the quadratic form assigned to (11), c associated with μ_α, according to the above assumption, satisfies equation

$$S_1c^3+S_2c^2+E_1c+E_2=0\,. \tag{15}$$

$S_{(\alpha\beta)4}$ in (14) means the symmetric part of $S_{\alpha\beta 4}$. $\mu_\alpha\mu_\beta=\mu_\beta\mu_\alpha$ is a symmetrical 6×6 matrix the twice contracted product of which after multiplication by an obliquely symmetrical matrix yielding zero.

The roots in (15) are real roots with both positive and negative roots ranging among them. This can be checked by means of the Sturm sequence.

For (15), the Sturm sequence is, as follows:

$$S_1c^3+S_2c^2+E_1c+E_2,\qquad 3S_1c^2+2S_2c+E_1,$$

$$2\frac{S_2^2-3S_1E_1}{3S_1}c+\frac{S_2E_1-9S_1E_2}{3S_1}$$

and

$$\frac{S_1S_2^2E_1^2+18S_1^2S_2E_1E_2-4S_1S_2^3E_2+S_1^2E_1^3-27S_1^3E_2^2}{S_2^2-3S_1E_1^2}\equiv r_2$$

The signs of the sequence are sign S_1, sign S_1, sign $(S_2^2-3S_1E_1)$ and sign r_2 for ∞, sign E_2, sign E_1, sign $(S_2E_1-9S_1E_2)$ and sign r_2 for 0, and, finally, sign $(-S_1)$, sign S_1, sign $(3S_1E_1-S_2^2)$ and sign r_2 for $-\infty$.

As is well known, if the number of sign reversals is V_∞ for ∞ and V_0 for 0, then the number of positive roots will be $V_0 - V_\infty$, the number of negative root $V_{-\infty} - V_0$ while the number of real roots $V_{-\infty} - V_\infty$.

It is possible to assume if all the roots of (15) are real, without limiting generality, that $S_1 > 0$, and that the term of the sequence has the same sign r_2 for ∞, 0, and $-\infty$ uniquely. Taking into consideration the fact that if the value of V_∞ is other than zero, then there will be no three real roots in (15) and sign $r_2 = +1$ that is

$$S_2^2 E_1^2 + 18 S_1 S_2 E_1 E_2 - 4 S_2^3 E_2 + S_1 E_2^2 - 27 S_1^2 E_2^2 > 0 \tag{16}$$

Possible cases:

$\alpha_1)$ $S_1 > 0, \quad S_2 \neq 0, \quad E_1 < 0, \quad E_2 > 0$

2 positive roots, 1 negative root

$\alpha_2)$ $S_1 > 0, \quad S_2 \neq 0, \quad E_1 < 0, \quad E_2 < 0$

1 positive root, 2 negative roots

$\alpha_3)$ $S_1 > 0, \quad S_2 > 0, \quad E_1 > 0, \quad E_2 > 0, \quad S_2^2 > 3 S_1 E_1$

2 positive roots, 1 negative root

$\alpha_4)$ $S_1 > 0, \quad S_2 > 0, \quad E_1 > 0, \quad E_2 < 0$ and $S_2^2 > 3 S_1 E_1$

2 positive roots, 1 negative root.

With the investigation presented carried out also in case $S_2 = 0$ in (16), two additional cases are worthy of consideration, such as

$\beta_1)$ $S_1 > 0, \quad S_2 = 0, \quad E_1 < 0, \quad E_2 > 0$

2 positive roots, 1 negative root

$\beta_2)$ $S_1 > 0, \quad S_2 = 0, \quad E_1 < 0, \quad E_2 < 0$

1 positive root, 2 negative roots.

Additional cases are also

$\gamma_1)$ $S_1 > 0, \quad S_2 \neq 0, \quad E_1 < 0, \quad E_2 = 0$

1 positive root, 1 zero root, 1 negative root

$\gamma_2)$ $S_1 > 0, \quad S_2 = 0, \quad E_1 < 0, \quad E_2 = 0$

1 positive root, 1 zero root, 1 negative root.

On the basis of cases α, β and γ specified above, we may say on the bracketed matrices of equation (11) that $S_{(\alpha\beta)4}$ is positive definite in every case. In cases α_1), α_2), $S_{(\alpha\beta)p}=n_p$ is indefinite but $S_{\alpha\beta p}n_p\mu_\alpha\mu_\beta$ can never be zero once μ_β has been realized. According to α_3), $S_{(\alpha\beta)p}n_p$ is a positive definite and according to α_4) negative definite while $S_{\alpha\beta p}n_p=0$ according to β_1), β_2). This latter shall reasonably be understood as

$$\frac{\partial F_\alpha}{\partial \sigma_{\beta p}}=0\,.$$

$E_{(\alpha\gamma 4}n_{\gamma\beta)}$ can be definite and semi-definite because of $n_{\gamma\beta}$. Neither $E_{\alpha\gamma 4}n_{\gamma\beta}\mu_\alpha\mu_\beta$ nor$E_{\alpha\gamma p}n_p n_{\gamma\beta}\mu_\alpha\mu_\beta$ can equal zero in the second case either. $E_{(\alpha\gamma 4}n_{\gamma\beta)}$ is positive definite or semi-definite in cases α_2) and α_4), respectively while negative definite or semi-definite in other cases. In case of α_1), α_3) and β_1), $E_{(\alpha\gamma p}n_p n_{\gamma\beta)}$ is negative definite or negative semi-definite, respectively. $\dfrac{\partial F_\alpha}{\partial \varepsilon_{\gamma p}}=0$ in case of γ_1) and γ_2), while positive definite or semi-definite in other cases.

Inequality (16) and the inequalities in α_3) and α_4) can be checked and/or realized in the actual case of μ_β.

In a special case, these results resemble the so-called strictly elliptical behaviour specified for the acoustic tensor in [11].

What has been said in relation with equation (15) can be understood as a generalization of the results obtained in [3]. On the basis thereof, the coefficients of formula (11) in [3] comply with the coefficients of equation (15) in the following way:

$$S_1 \text{ complies with } \rho\frac{\partial\Phi}{\partial\sigma_t}, \qquad S_2 \text{ with } -\rho\frac{\partial\Phi}{\partial\sigma_x},$$

$$E_1 \text{ complies with } \frac{\partial\Phi}{\partial\varepsilon_t} \quad\text{and}\quad E_2 \text{ with } -\frac{\partial\Phi}{\partial\varepsilon_x}.$$

On the basis of this similarity, it can be quite obviously assumed that the setup of (15) complies with equation (11) obtained in the experimental test of acceleration wave in a strip. Let this assumption be designated e). Assumption e) suggests one possible way of experimental tests.

Quadratic form (15) assigned to (11) suggests that the coefficient matrices of the powers of c in (11) are symmetric for subscripts α, β and that the entire bracketed expression on the left side of (11) is positive semi-definite. In this case, according to theorem 2.8.19 of [7], this positive semi-definite expression will be zero if (11) is fulfilled. Hence, now the quadratic equation and the roots of (11) are in compliance.

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