APROXIMATIVE METHOD TO CALCULATE STRESS CONCENTRATIONS DUE TO LOCAL FORM DEFLECTION OF LARGE VESSELS

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Summary

In the recent ten-year period, the rapid development of chemical industry has resulted in increasing use of large cylindric or spharical pressure vessels. The geometry of the finished vessels is in most cases other than ideal.

In this paper, an approximate method of suitable accuracy for practical use, describing the stress-strain conditions of the deflected shell section in a local co-ordinate system fixed on the central surface of the ideal shell is presented.

The results of investigations in the environment of a local indentation of a cylindric vessel are given to illustrate the applicability of the method.

Introduction

Large cylindric or spherical pressure vessels have been finding increasing use as a result of modernization of chemical industry and rapid development of technology. The vessels are set up of segments. Depending on the manufacturing technology and on technological discipline, the geometry of the finished vessel is more or less other than ideal. The deflections are of global or local nature. Experiences gained during construction and subsequent testing of large vessels suggest that local 'indentations' the steel plates of originally perfect geometry experience in the course of transportation are predominating.

The question arises, especially in case of vessels with strained operating parameters, whether the deflection from the designed geometry might result in a stress increase that endangers the safe operation of the vessel.

With the up-to-date methods of computer engineering available today—e.g. the method of finite element—the question can be easily answered. However, computations like these are relatively expensive, and the costs would certainly exceed the estimated costs of testing subsequent to construction, especially if deflections of different shape, size, and 'depth' are involved.

All what has been said above necessitates that a calculation method of suitable accuracy for practice be developed by means of which the stress-strain conditions in the environment of deflections can be determined at low costs, taking into consideration the capacity of small computers widely available today.

Principle of the method

The calculation method described here is adjusted to the ideal geometry of the vessel to be tested and to the measurement method to determine the formdeflection. The geometry of the built vessel is tested by determining the deflection from the surface of ideal geometry in points of appropriate number, using a measurement method best complying with the geometry of the vessel and with the required accuracy.

Making use of the points of measurement, a regression or interpolation surface determining the so-called deflection function ($h(x^1, x^2)$) is obtained. The deflection function describes the deviation from the surface of ideal geometry.

Starting assumptions

(a) Investigations relate to so-called thin shell.
(b) The Kirchoff-Love hypothesis is accepted.
(c) Small deformations are investigated.
(d) Shape of displacement field:

$$u = \bar{u}_\alpha a + \bar{u}_3 a^3$$

where

$$\bar{u}_\alpha = v_\alpha(x^\beta) + x^3 \Theta_\alpha(x^\beta)$$

$$\bar{u} = w(x^\beta)$$

(e) $\sigma_{33} \approx 0$
(f) The volumetric forces are negligible.
(g) Internal gas or liquid pressure acts upon the shell.
(h) Disturbance due to support of the vessel, pipe stubs, and other fittings does not affect the shell surface investigated.
(i) Considering its material, the shell can be treated as a homogeneous, isotropic, linearly elastic body. Thus, Hooke's law can be applied as the mass law;

$$\sigma^{ij} = h^{ijkl} \varepsilon_{kl}$$

(j) Thickness b of the shell is constant.

Let us investigate certain part of the central surface of the (deflected) shell, confined by a closed curve s_0 (the geometrical boundary condition along curve s_0 being given).

Taking into consideration the starting assumptions, the total minimum potential energy theorem for the investigated shell surface can be written as

$$\pi(v_\alpha, w) = \int_{\bar{A}} (N^{\alpha\beta}\gamma_{\alpha\beta} + M^{\alpha\beta}\kappa_{\alpha\beta} - f^3 w)\, d\bar{A} \tag{1}$$

where:

$$N^{\alpha\beta} = \frac{b}{2}(\gamma_{\delta\gamma} + \gamma_{\gamma\delta}) h^{\alpha\beta\gamma\delta} \tag{2}$$

$$M^{\alpha\beta} = \frac{b^3}{24}(\kappa_{\gamma\delta} + \kappa_{\delta\gamma}) h^{\alpha\beta\gamma\delta} \tag{3}$$

and

$$\gamma_{\alpha\beta} = v_{\alpha||\beta} - w b_{\alpha\beta} \tag{4}$$

$$\kappa_{\alpha\beta} = \Theta_{\alpha||\beta} \tag{5}$$

Since we investigate the stress-strain conditions of a shell deflecting only locally from the surface of ideal geometry, it seems logical not to describe the characteristics of the deflected shell in a co-ordinate system associated with the actual central surface but, instead, in a co-ordinate system fixed on the central surface of the ideal shell. This is at the same time justified by the fact that also the deflection has been described in this system that is deflection function $h(x^\alpha)$ is already available. Hereinafter the characteristics of the co-ordinate system associated with the ideal central surface will be designated 0. The symbols generally accepted in shell theory are used to describe quantities defined on the central surface and in arbitrary points of the shell investigated.

Solution of the varation problem

The Ritz method is used as a numerical solution to the problem. Accordingly, function series

$$\begin{aligned} v_1 &= \mathop{v}_{\circ}{}_1 + \sum_{k=1}^{n_1} \varphi_\kappa d_\kappa \\ v_2 &= \mathop{v}_{\circ}{}_2 + \sum_{k=n_1+1}^{n_2} \varphi_\kappa d_\kappa \\ w &= \mathop{w}_{\circ} + \sum_{k=n_2+1}^{n_3} \varphi_\kappa d_\kappa \end{aligned} \tag{6}$$

is used to approximate the unknown function in functional (1).

Here $\overset{\circ}{v}_1$; $\overset{\circ}{v}_2$; $\overset{\circ}{w}$; $\varphi_k(k=1\ldots n_3)$ are known functions from among which $\overset{\circ}{v}_1$; $\overset{\circ}{v}_2$ and $\overset{\circ}{w}$ fulfil the gegometrical boundary conditions while the value of functions φ_k at the geometrical boundaries amounts equally to zero, $d_k(k=1\ldots n_3)$ being variation constants. Functions $\gamma_{\alpha\beta}$, $\kappa_{\alpha\beta}$ and w in functional (1) can be written as

$$\begin{aligned} \gamma_{\alpha\beta} &= \sum_{\textcircled{i}=1}^{n_3} \gamma_{\textcircled{i}\alpha\beta}\, d_{\textcircled{i}} - \overset{\circ}{\gamma}_{\alpha\beta} \\ \kappa_{\alpha\beta} &= \sum_{\textcircled{i}=1}^{n_3} \kappa_{\textcircled{i}\alpha\beta}\, d_{\textcircled{i}} - \overset{\circ}{\kappa}_{\alpha\beta} \\ w &= \sum_{\textcircled{i}=1}^{n_3} w_{\textcircled{i}}\, d_{\textcircled{i}} + w_0 \end{aligned} \tag{7}$$

On the basis of (4) and (5), using (6) and (7)

$$\overset{\circ}{\gamma}_{\alpha\beta} = \overset{\circ}{w}\, b_{\alpha\beta} + \overset{\circ}{v}_\gamma \bar{\Gamma}^{\gamma}_{\alpha\beta} - \overset{\circ}{v}_{\alpha\beta}$$

$$\overset{\circ}{\kappa}_{\alpha\beta} = \overset{\circ}{v}_\gamma (b^{\cdot\gamma}_{\alpha,\beta} - b^{\cdot\gamma}_{\delta} \bar{\Gamma}^{\delta}_{\alpha\beta}) + b^{\cdot\gamma}_{\alpha} \overset{\circ}{v}_{\gamma,\beta} - \overset{\circ}{w}_{,\gamma} \bar{\Gamma}^{\gamma}_{\alpha\beta} + \overset{\circ}{w}_{,\alpha\beta}$$

In expression (7)

$$\gamma_{\textcircled{i}1\beta} = \begin{cases} \varphi_{\textcircled{i},\beta} - \varphi_{\textcircled{i}} \bar{\Gamma}^{1}_{1\beta} & \text{if } 0 < i \leq n_1 \\ \varphi_{\textcircled{i}} \bar{\Gamma}^{2}_{1\beta} & \text{if } n_1 < i \leq n_2 \\ -\varphi_{\textcircled{i}} b_{1\beta} & \text{if } n_2 < i \leq n_3 \end{cases}$$

$$\gamma_{\textcircled{i}2\beta} = \begin{cases} -\varphi_{\textcircled{i}} \bar{\Gamma}^{1}_{2\beta} & \text{if } 0 < i \leq n_1 \\ \gamma_{\textcircled{i}\beta} - \gamma_{\textcircled{i}} \bar{\Gamma}^{2}_{2\beta} & \text{if } n_1 < i \leq n_2 \\ -\gamma_{\textcircled{i}} b_{2\beta} & \text{if } n_2 < i \leq n_3 \end{cases}$$

$$\kappa_{\textcircled{i}\alpha\beta} = \begin{cases} (b^{\cdot 1}_{\gamma} \bar{\Gamma}^{\gamma}_{\alpha\beta} - b^{\cdot 1}_{\alpha,\beta})\varphi_{\textcircled{i}} - b^{\cdot 1}_{\alpha} \varphi_{\textcircled{i},\beta} & \text{if } 0 < i \leq n_1 \\ (b^{\cdot 2}_{\gamma} \bar{\Gamma}^{\gamma}_{\alpha\beta} - b^{\cdot 2}_{\alpha,\beta})\varphi_{\textcircled{i}} - b^{\cdot 2}_{\alpha} \varphi_{\textcircled{i},\beta} & \text{if } n_1 < i \leq n_2 \\ \varphi_{\textcircled{i},\gamma} \bar{\Gamma}^{\gamma}_{\alpha\beta} - \varphi_{\textcircled{i},\alpha\beta} & \text{if } n_2 < i \leq n_3 \end{cases}$$

$$w_{\textcircled{i}} = \begin{cases} 0 & \text{if } 0 < i \leq n_2 \\ \varphi_{\textcircled{i}} & \text{if } n_2 < i \leq n_3 \end{cases}$$

Substituting (2) and (3) into (1) we obtain

$$\Pi = \int_{\bar{A}} \left\{ h^{\alpha\beta\gamma\delta} \left[\frac{b}{2} (\gamma_{\gamma\delta} + \gamma_{\delta\gamma}) \overset{\circ}{\gamma}_{\alpha\beta} + \frac{b^3}{24} (\kappa_{\gamma\delta} + \kappa_{\delta\gamma}) \kappa_{\alpha\beta} \right] - pw \right\} d\bar{A}$$

Taking (7) into consideration, partial derivatives $\dfrac{\partial \pi}{\partial d_{\textcircled{i}}}$ can be calculated.

From condition $\frac{\partial \pi}{\partial d_{\textcircled{i}}} = 0$ of the existence of extremum, inhomogeneous linear equation system

$$\mathbf{A}d = e$$

is obtained for parameters d_i.

Elements of the matrix of the coefficients:

$$A_{\textcircled{i}\textcircled{j}} = \int_{\bar{A}} \frac{b}{2} h^{\alpha\beta\gamma\delta} \Big[(\gamma_{\textcircled{j}\gamma\delta} + \gamma_{\textcircled{j}\delta\gamma}) \gamma_{\textcircled{i}\alpha\beta} + (\gamma_{\textcircled{i}\gamma\delta} + \gamma_{\textcircled{i}\delta\gamma}) \gamma_{\textcircled{j}\alpha\beta} +$$

$$+ \frac{b^2}{12} \{ \kappa_{\textcircled{i}\gamma\delta} + \kappa_{\textcircled{j}\delta\gamma}) \kappa_{\textcircled{i}\alpha\beta} + (\kappa_{\textcircled{i}\gamma\delta} + \kappa_{\textcircled{i}\delta\gamma}) \kappa_{\textcircled{j}\alpha\beta} \} \Big] d\bar{A}$$

Coefficients of the inhomogeneous term:

$$C_{\textcircled{i}} = \int_{\bar{A}} \Big\{ p w_{\textcircled{i}} + \frac{b}{2} h^{\alpha\beta\gamma\delta} \Big[(\underset{\circ}{\gamma}_{\gamma\delta} + \underset{\circ}{\gamma}_{\delta\gamma}) \gamma_{\textcircled{i}\alpha\beta} + \underset{\circ}{\gamma}_{\alpha\beta} (\gamma_{\textcircled{i}\gamma\delta} + \gamma_{\textcircled{i}\delta\gamma})$$

$$+ \frac{b^2}{12} (\underset{\circ}{\kappa}_{\gamma\delta} + \underset{\circ}{\kappa}_{\delta\gamma}) \kappa_{\textcircled{i}\alpha\beta} + \frac{b^2}{12} (\kappa_{\textcircled{i}\gamma\delta} + \kappa_{\textcircled{i}\delta\gamma}) \underset{\circ}{\kappa}_{\alpha\beta} \Big] \Big\} d\bar{A}$$

Calculation of local deflection of circular cylindrical shell

A computer programme in FORTRAN language has been elaborated for the case of circular cylindrical shell on the basis of the relationships described.

The problem was to investigate a local indentation of a circular cylindrical shell where the deflection could be described by a function

$$h = -\underset{\circ}{h} \left(\cos \frac{\pi}{S_*} S_1 + 1 \right) \left(\cos \frac{\pi}{\varphi_*} \varphi + 1 \right)$$

to a good approximation.

The indentation was located between values—S_* and S_* in axial direction and between values—φ_* and φ_* in tangential direction on the cylinder, its maximum deflection from the ideal central surface being $\underset{\circ}{h}$.

For the particular problem, the shape of functions $d_{\textcircled{i}}$ was selected as

$$\cos\left(i_1 \frac{\pi}{S_0} S_1 \right) \left[\cos\left(j_1 \frac{\pi}{\varphi_0} \varphi \right) + 1 \right] \quad \text{if} \quad 0 < i \leq n_1$$

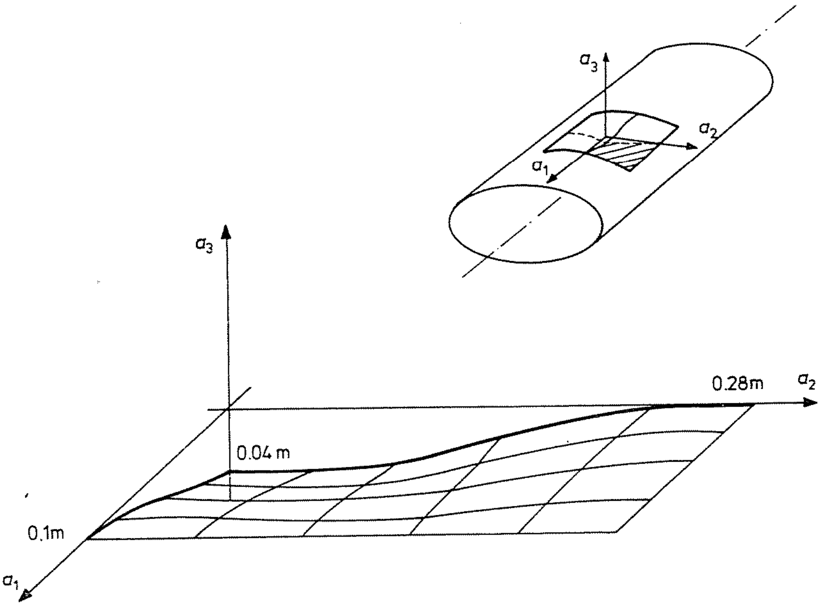

Fig. 1

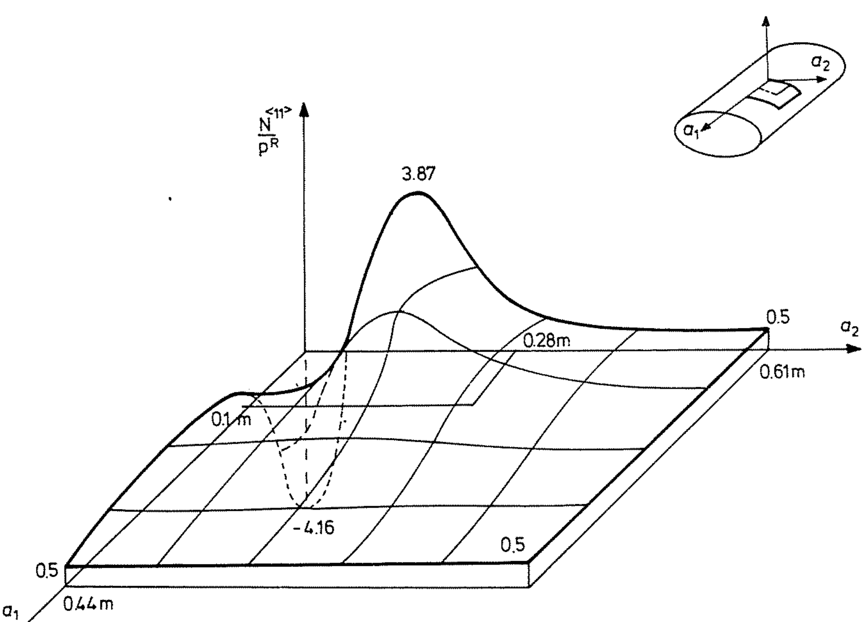

Fig. 2

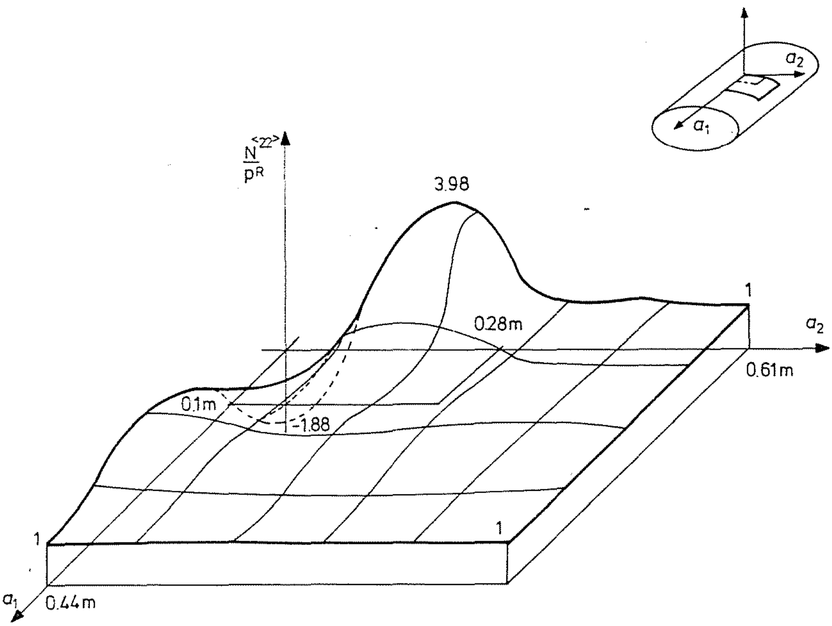

Fig. 3

$$\sin\left(i_2 \frac{\pi}{S_0} S_1\right)\left[\cos\left(j_2 \frac{\pi}{\varphi_0}\varphi\right)+1\right] \quad \text{if} \quad n_1 < i \le n_2$$

$$\left[\cos\left(i_3 \frac{\pi}{S_0} S_1\right)+1\right]\sin\left(j_3 \frac{\pi}{\varphi_0}\varphi\right) \quad \text{if} \quad n_2 < i \le n_3$$

where $i_1, j_1, i_2, j_2, i_3, j_3$ may be positive odd whole numbers.

Numerically, the programme was run for a cylinder of a radius of $R = 2$ m and a wall thickness of $b = 1$ cm, subjected to an internal overpressure of 10 bar. The maximum extent of the deflection investigated was 20 cm in the axial direction, while 56 cm in tangential direction.

Maximum deflection of the deformed surface from the ideal geometry was 4 cm.

As a result of the calculations made, an axonometric presentation of the deflection function is given in Fig. 1, and of the distribution of nondimensional axial and tangential forces per unit lengths in Figs. 2 and 3, respectively, the value of other stresses and momentum being negligible as compared with axial and tangential forces.

Conclusion

The results of numerical calculations based on the derived relationships comply with the measurements and practical experience. According to investigations made so far, the method seems to be suited for the investigation of the effect of different deflections from the ideal geometry.

The method is advantageous in that calculations can be made by means of small computers available for the manufacturer and users of the vessels and thus the stress concentration due to deflection from the designed geometry can be quickly and easily detected in the course of after-construction measurements, and the necessary measures to eliminate the effect of deflections can be taken on the basis of the results of calculations.

The results of investigations in the accuracy of the approximations, the comparison of the numerical calculations with the measurements will be published in the next paper.

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