

DESIGN OF INDIRECT-TRANSFER (RUN-AROUND-COIL SYSTEM) HEAT EXCHANGERS

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Summary

Heat transfer of gases at atmospheric pressure results in low heat transfer rates per unit of surface area. In addition to some practical considerations, it is the demand on decreasing surface areas that has created the possibility of a wide-spread industrial application for indirect-transfer heat exchangers. In this paper, a heat transfer design theory of indirect-transfer heat exchangers is presented. Comparative evaluation is made for heat engineering parameters of different heat exchanger types with a particular regard to the case of heat exchangers with heat pipes.

It is well known that for gases at atmospheric pressure flowing over a solid surface, the heat transfer coefficients are very small and, consequently, the surface area necessary for the heat transfer is considerably larger to that of heat exchangers with liquid media. If one of the two fluids is a low pressure gas, the latter generally flows over the outside surface of heat transfer tubes and the low heat transfer coefficient is compensated by an increased outside surface area, that is, a finned outside surface.

Increasing the inner surface area of tubes is a difficult task and is never possible to such a degree as in the case of the outside surface. Therefore, heat exchangers with two gas fluids at low pressures have usually larger dimensions than liquid-flow exchangers of the same performance.

In addition to several practical considerations, particularly the layout advantages due to an easy handling of fluids, it is the demand on decreasing surface areas that has created the possibility of a wide-spread industrial application for indirect-transfer heat exchangers.

The indirect-transfer heat exchanger is essentially a system of direct-type heat exchangers and consists of, as shown in Fig. 1, a "hot" heat exchanger with a surface area F_1 and a "cold" one with a surface area F_2 .

In the hot exchanger, the cooling of hot fluid takes place with high heat transfer coefficients by a fluid of flow-stream capacity rate W_p and this fluid

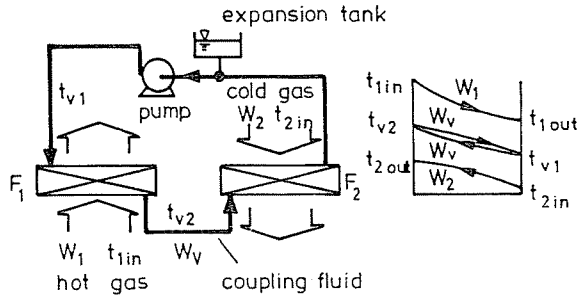


Fig. 1

with the same thermal capacitance W_v is applied for heating the cold fluid in the cold heat exchanger. The coupling fluid is circulated by a pump, however, the overall arrangement in a given case might allow the application of free circulation.

In steady operation, the two exchangers have the same performance thus the coupling fluid is heated up in the hot exchanger in the same degree as it is cooled down in the cold one. Hence, the temperatures t_{v1} and t_{v2} will approach to constant values determined by the values of heat transfer areas, overall thermal conductances and thermal capacitances.

If the thermal conductance W_v is known, the dimensionless temperature change of the coupling fluid can be calculated since it is independent of actual temperature values:

$$\vartheta_{v1} = \frac{t_{v2} - t_{v1}}{t_{1be} - t_{v1}} = \vartheta_{v1} \left(\frac{k_1 F_1}{W_v}, \frac{W_v}{W_1} \right) \quad (1)$$

$$\vartheta_{v2} = \frac{t_{v2} - t_{v1}}{t_{v2} - t_{2be}} = \vartheta_{v2} \left(\frac{k_2 F_2}{W_v}, \frac{W_v}{W_2} \right) \quad (2)$$

Supposing both exchangers are of a simple counterflow arrangement,

$$\vartheta_{vi} = \frac{1 - e^{-\frac{k_i F_i}{W_v} \left(1 - \frac{W_v}{W_i}\right)}}{1 - \frac{W_v}{W_i} e^{-\frac{k_i F_i}{W_v} \left(1 - \frac{W_v}{W_i}\right)}}; \quad W_v \neq W_i$$

and

$$\vartheta_{vi} = \frac{1}{1 + \frac{k_i F_i}{W}}; \quad W_i = W_v = W$$

where $i=1$ refers to the hot exchanger and $i=2$ to the cold one.

The performance of each of the exchangers is given by

$$Q = W_1 \vartheta_{v1}(t_{1\text{be}} - t_{1\text{ki}}) = W_v(t_{v2} - t_{v1}) = W_2(t_{2\text{ki}} - t_{2\text{be}}), \quad (3)$$

although, the performance may be written also employing the dimensionless temperature of the coupling fluid as

$$Q = W_v \vartheta_{v1}(t_{1\text{be}} - t_{v1}) \quad (4)$$

$$Q = W_v \vartheta_{v2}(t_{v2} - t_{2\text{be}}) \quad (5)$$

Equations (3), (4) and (5) yield the performance as a function of the known parameters:

$$Q = W_v \frac{t_{1\text{be}} - t_{2\text{be}}}{\frac{1}{\vartheta_{v1}} + \frac{1}{\vartheta_{v2}} - 1}$$

Applying the symbol W_k to the smaller thermal capacitance and introducing this into the performance equation,

$$Q = \frac{W_k \frac{W_v}{W_k}}{\frac{1}{\vartheta_{v1}} + \frac{1}{\vartheta_{v2}} - 1} (t_{1\text{be}} - t_{2\text{be}}) = W_k \Psi (t_{1\text{be}} - t_{2\text{be}}) \quad (6)$$

where Ψ is the effectiveness of the indirect-transfer heat exchanger:

$$\Psi = \begin{cases} \frac{\frac{W_v}{W_1}}{\frac{1}{\vartheta_{v1}} + \frac{1}{\vartheta_{v2}} - 1}; & W_1 \leq W_2 \\ \frac{\frac{W_v}{W_2}}{\frac{1}{\vartheta_{v1}} + \frac{1}{\vartheta_{v2}} - 1}; & W_1 \geq W_2 \end{cases} \quad (7)$$

It is often more convenient to employ, instead of the magnitudes ϑ_{v1} and ϑ_{v2} concerning the coupling fluid, the dimensionless temperature differences ϑ_1 and ϑ_2 concerning the fluid to be cooled and the fluid to be heated, respectively. Since the thermal capacitances are known:

$$\vartheta_1 = \frac{W_v}{W_1} \vartheta_{v1}; \quad \vartheta_2 = \frac{W_v}{W_2} \vartheta_{v2},$$

one obtains for Ψ

$$\Psi = \begin{cases} \frac{1}{\frac{1}{\vartheta_1} + \frac{W_1}{W_2 \vartheta_2} - \frac{W_1}{W_v}}; & W_1 \leq W_2 \\ \frac{1}{\frac{W_2}{W_1 \vartheta_1} + \frac{1}{\vartheta_2} - \frac{W_2}{W_v}}; & W_1 \geq W_2 \end{cases} \quad (8)$$

The outlet temperatures can be calculated from the performance Q as follows:

$$t_{1ki} = t_{1be} - \frac{Q}{W_1} \quad (9)$$

$$t_{2ki} = t_{2be} + \frac{Q}{W_2} \quad (10)$$

where

$$Q = \begin{cases} W_1 \Psi (t_{1be} - t_{2be}); & W_1 \leq W_2 \\ W_2 \Psi (t_{1be} - t_{2be}); & W_1 \geq W_2 \end{cases}$$

The extremes of the coupling fluid temperature are given by Eqs (4) and (5):

$$t_{v1} = t_{1be} - \frac{Q}{W_v \vartheta_{v1}} = t_{1be} - \frac{Q}{W_1 \vartheta_1} \quad (11)$$

$$t_{v2} = t_{2be} + \frac{Q}{W_v \vartheta_{v2}} = t_{2be} + \frac{Q}{W_2 \vartheta_2} \quad (12)$$

In the special case when changes in phase take place both in the hot and the cold fluid, i.e. both W_1 and W_2 start indefinitely high, then $\vartheta_{v1} = \Phi_1$ and $\vartheta_{v2} = \Phi_2$, therefore

$$\Psi = \frac{1}{\frac{1}{\Phi_1} + \frac{1}{\Phi_2} - 1}; \quad W_1 \rightarrow \infty; \quad W_2 \rightarrow \infty \quad (13)$$

and

$$Q = W_v \Psi (t_{1be} - t_{2be})$$

where

$$\Phi_1 = 1 - e^{-\frac{k_1 F_1}{W_v}} \quad \text{and} \quad \Phi_2 = 1 - e^{-\frac{k_2 F_2}{W_v}}$$

The temperatures of the coupling fluid will be

$$t_{v1} = t_{1be} - \frac{Q}{W_v \Phi_1}; \quad W_1 \rightarrow \infty \quad (14)$$

$$t_{v2} = t_{2be} + \frac{Q}{W_v \Phi_2}; \quad W_2 \rightarrow \infty \quad (15)$$

The value of Ψ depends on the ratio of hot and cold fluid thermal capacitances. In most cases $\partial \vartheta_1 / \partial W_1 < 1$ and $\partial \vartheta_2 / \partial W_2 < 1$, therefore, an investigation of Eq. (8) shows that (with the above restrictions) the function Ψ has a minimum at $W_1/W_2 = 1$, as shown in Fig. 2.

Ψ is a function also of the coupling fluid thermal capacitance. Very low values of W_v result in $\vartheta_{v1} \approx \vartheta_{v2} \approx 1$ and so Eq. (7) leads to

$$\lim_{W_v \rightarrow 0} \Psi = 0 \quad (16)$$

When W_v increases infinitely, the ratios W_v/W_1 and W_v/W_2 start indefinitely high while the dimensionless temperature differences approach the Bošnjaković number Φ (or effectiveness) of heat exchangers with a change of phase in one of the fluids. Consequently, from Eq. (8):

$$\Psi_0 = \lim_{W_v \rightarrow \infty} \Psi = \begin{cases} \frac{1}{\frac{1}{\Phi_1} + \frac{W_1}{W_2 \Phi_2}}; & W_1 \leq W_2 \\ \frac{1}{\frac{W_2}{W_1 \Phi_1} + \frac{1}{\Phi_2}}; & W_1 \geq W_2 \end{cases} \quad (17)$$

The curve $\Psi(W_v)$, characterized by the limits $W_v = 0$ and $W_v \rightarrow \infty$, after an initial raise as far as a maximum value, descends approaching the value given by Eq. (17) as shown in Fig. 3 where Ψ is presented as a function of the non-dimensional parameter $x = W_1/W_2$ for different thermal capacitance ratios $a = W_1/W_2$.

The function has no maximum value at any finite W_v if $W_1 = W_2 \rightarrow \infty$.

The special condition $W_v \rightarrow \infty$ can be established by a heat pipe as shown in Fig. 4.

The temperature of the two-phase fluid in the heat pipe can be calculated from the equality of performances on the two (hot and cold) sides as follows

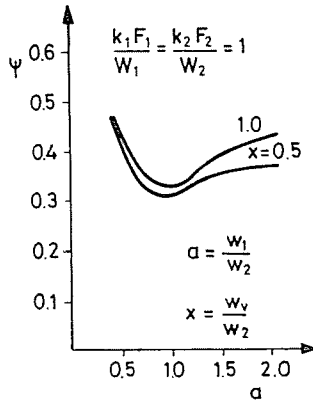


Fig. 2

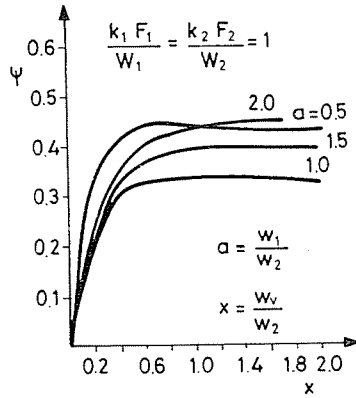


Fig. 3

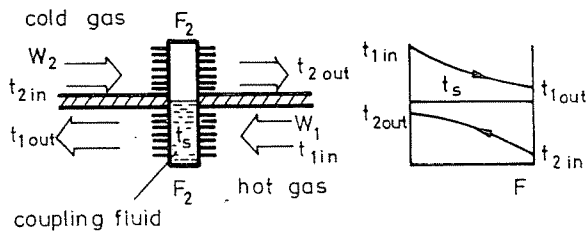


Fig. 4

$$Q = W_1 \Phi_1 (t_{1\text{be}} - t_s) = W_2 \Phi_2 (t_s - t_{2\text{be}}) \quad (18)$$

from which t_s can be expressed as

$$t_s = \frac{\frac{W_1}{W_2} \frac{\Phi_1}{\Phi_2} t_{1\text{be}} + t_{2\text{be}}}{\frac{W_1}{W_2} \frac{\Phi_1}{\Phi_2} + 1} \quad (19)$$

Let us suppose that $W_1 \leq W_2$ and substitute Eq. (19) into (18), then

$$\begin{aligned} Q &= W_1 \Phi_1 (t_{1\text{be}} - t_s) = \frac{1}{\frac{1}{\Phi_1} + \frac{W_1}{W_2 \Phi_2}} (t_{1\text{be}} - t_{2\text{be}}) = \\ &= W_1 \Psi_0 (t_{1\text{be}} - t_{2\text{be}}); \quad W_1 \leq W_2. \end{aligned} \quad (20)$$

Similarly, in the case of $W_2 \leq W_1$,

$$Q = W_2 \Psi_0 (t_{1\text{be}} - t_{2\text{be}}); \quad W_1 \geq W_2. \quad (21)$$

All this goes to show that *heat pipe actually is a special case of indirect-transfer heat exchangers; it is obtained by the restriction $W_v \rightarrow \infty$.*

A single heat pipe or a series of heat pipes¹ can be used only if the outlet temperature of the hot fluid is higher to that of the cold one. However, even in this case, the performance of a heat pipe is smaller to that obtainable by an indirect-transfer heat exchanger with the same value of (kF) . In order to eliminate this disadvantage, a set of heat pipes is usually applied in an equipment which are coupled in series.

A heat exchanger consisting of two heat pipes is shown in Fig. 5. Outlet temperatures and temperature values of the coupling fluid can be obtained, after setting up energy balances and heat transfer formulas, from the following system of linear equations with six unknowns:

$$\begin{aligned} W_1 (t_{1\text{be}} - t_1^*) &= W_2 (t_{2\text{ki}} - t_2^*) \\ t_{1\text{be}} - t_1^* &= \Phi_{11} (t_{1\text{be}} - t_{s1}) \\ t_{2\text{ki}} - t_2^* &= \Phi_{12} (t_{s1} - t_2^*) \\ W_1 (t_1^* - t_{1\text{ki}}) &= W_2 (t_2 - t_{2\text{be}}) \\ t_1^* - t_{1\text{ki}} &= \Phi_{21} (t_1^* - t_{s2}) \\ t_2^* - t_{2\text{be}} &= \Phi_{22} (t_{s2} - t_{2\text{be}}) \end{aligned} \quad (22)$$

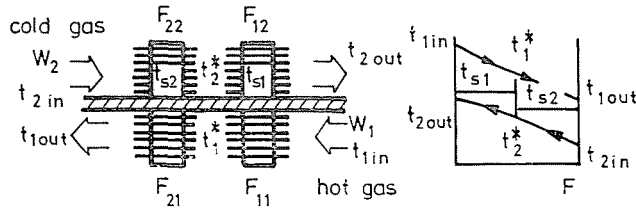


Fig. 5

where

$$\Phi_{ij} = 1 - e^{-\frac{k_{ij} F_{ij}}{W_j}}; \quad i = 1, 2; \quad j = 1, 2.$$

The total heat transfer area is

$$F = F_{11} + F_{12} + F_{21} + F_{22} \quad (23)$$

If more than two heat pipes are used and coupled in series, a similar system of linear equations is obtained. The resulting Ψ number of the serially-coupled heat pipes can be calculated from the outlet temperatures:

$$\Psi = \frac{t_{k\text{be}} - t_{k\text{ki}}}{t_{k\text{be}} - t_{n\text{be}}} \quad (24)$$

where the index k refers to the fluid with the smaller thermal capacitance while n refers to the other one.

Let us consider now the marginal case where an infinity of heat pipes is coupled in series. Denoting as $F_k dx$ the surface area of an infinitesimal heat pipe in a section dx taken in the flow direction, a thermodynamic balance results in

$$dQ = k_k F_k (t_k - t_s) dx = k_n F_n (t_s - t_n) dx \quad (25)$$

that is to say

$$\frac{k_k F_k}{k_n F_n} (t_k - t_s) = t_s - t_n \quad (26)$$

and the latter yields

$$t_k - t_s = \frac{1}{1 + \frac{k_k F_k}{k_n F_n}} (t_k - t_n) \quad (27)$$

$$t_s - t_n = \frac{1}{1 + \frac{k_n F_n}{k_k F_k}} (t_k - t_n)$$

The transported energy changes the hot and cold fluid temperatures in inverse ratio to the thermal capacitances:

$$dQ = W_k dt_k = W_n dt_n \quad (28)$$

Eqs (25) and (27) yield

$$\left. \begin{aligned} \frac{dt_k}{dx} &= -\frac{k_k F_k}{W_k} (t_k - t_s) \\ \frac{dt_n}{dx} &= -\frac{k_n F_n}{W_n} (t_s - t_n) \end{aligned} \right\} \quad (29)$$

(The negative sign is necessary because, at a counterflow arrangement, both t_k and t_n decrease in the direction of the $+x$ axis.)

Summing up the two equations and substituting Eq. (27), one obtains

$$\begin{aligned} \frac{d}{dx} (t_k - t_n) &= -\frac{k_k F_k}{W_k} \frac{1 + \frac{k_k F_k}{k_n F_n}}{1 + \frac{k_k F_k}{k_n F_n}} \left(1 - \frac{W_k}{W_n}\right) (t_k - t_n) = \\ &= -\beta (t_k - t_n) \end{aligned} \quad (30)$$

Substitution of the boundary conditions gives the results

$$\begin{aligned} t_{k\text{ki}} &= t_{k\text{be}} - \frac{1 - e^{-\beta}}{1 - \frac{W_k}{W_n} e^{-\beta}} (t_{k\text{be}} - t_{n\text{be}}) = \\ &= t_{k\text{be}} - \Psi (t_{k\text{be}} - t_{n\text{be}}) \end{aligned} \quad (31)$$

$$t_{n\text{ki}} = t_{n\text{be}} + \frac{W_k}{W_n} \Psi (t_{k\text{be}} - t_{n\text{be}}) \quad (32)$$

As for the heat exchanger performance,

$$Q = W_k \Psi (t_{k\text{be}} - t_{n\text{be}}) \quad (33)$$

where

$$\Psi = \frac{1 - e^{-\beta}}{1 - \frac{W_k}{W_n} e^{-\beta}} \quad (34)$$

and

$$\beta = \frac{k_k F_k}{W_k} \frac{1 - \frac{W_k}{W_n}}{1 + \frac{k_k F_k}{k_n F_n}} \quad (35)$$

If $k_k F_k = k_n F_n = k \frac{F}{2}$ and F is the total installed heat transfer area, then an evaluation of Eq. (34) shows that, for equal unit overall thermal conductances, a heat pipe heat exchanger requires four times the surface area of a pure counterflow exchanger to obtain the same performance, that is,

$$\Psi(4F) = \Phi_{\text{counterflow}}(F); \quad k_k = k_n = k$$

A summary of indirect-transfer heat exchanger calculating formulas is given in Table 1. One can see that the performance of each of the exchanger types is proportional to some nondimensional factor denoted as Ψ . This factor Ψ can always be calculated from the effectiveness of the given unit and the ratios of thermal capacitances; it is analogous to the Bošnjaković number Φ of direct-transfer heat exchangers.

For equal overall conductances, Ψ is the highest possible for direct-transfer, pure counterflow exchangers. ($\Psi = \Phi$) A considerably lower Ψ value can be obtained employing a heat pipe, nevertheless, this value can be improved by serially coupled heat pipes.

For an exchanger operating with liquid coupling fluid, Ψ is a function of the thermal capacitance of the latter but it can be usually adjusted, by a proper choice of W_v , in between the Ψ values of an exchanger with a single heat pipe and that with an infinity of heat pipes (Fig. 6).

However, the above comparison is solely of theoretical importance since unit overall thermal conductances (k) in indirect-transfer exchangers are, in general, higher to those in direct-transfer exchangers. Consider, for example, the case of a counterflow, gas-to-gas exchanger where the heat transfer coefficients inside and outside the tubes are equally $\alpha_b = \alpha_k = 50 \text{ W/m}^2, \text{ K}$. Let the heat transfer tube be finned at a ratio $F_k/F_b = 10$ of surface areas. The overall thermal conductance for this heat exchanger is approximately

$$k = \frac{1}{\frac{10}{\alpha_b} + \frac{1}{\alpha_k}} = 4.545 \text{ W/m}^2, \text{ K}.$$

If, on the other hand, a heat transfer coefficient $\alpha_b = 3000 \text{ W/m}^2, \text{ K}$ is obtainable by using a liquid (e.g. water) coupling fluid, the unit overall thermal

Table 1
Summary of Indirect-Transfer Heat Exchanger Correlations ($W_1 < W_2$)

	Pure counterflow exchanger without coupling fluid	Indirect-transfer exchangers		
		$0 < W_v < \infty$ (liquid coupling fluid)	$W_v \rightarrow \infty$ (heat pipe)	Infinite series of heat pipes
Ψ factor	$\Psi = \vartheta_1 = \frac{1 - e^{-\frac{kF}{W_1} \left(1 - \frac{W_1}{W_2}\right)}}{1 - \frac{W_1}{W_2} e^{-\frac{kF}{W_1} \left(1 - \frac{W_1}{W_2}\right)}}$	$\Psi = \frac{1}{\frac{1}{\vartheta_1} + \frac{W_1}{W_2} \vartheta_2 - \frac{W_1}{W_v}}$ $\vartheta_1 = \vartheta_1 \left(\frac{k_1 F_1}{W_1}, \frac{W_1}{W_v} \right)$ $\vartheta_2 = \vartheta_2 \left(\frac{k_2 F_2}{W_2}, \frac{W_2}{W_v} \right)$	$\Psi = \frac{1}{\frac{1}{\Phi_1} + \frac{W_1}{W_2} \Phi_2}$ $\Phi_1 = 1 - e^{-\frac{k_1 F_1}{W_1}}$ $\Phi_2 = 1 - e^{-\frac{k_2 F_2}{W_2}}$	$\Psi = \frac{1 - e^{-\beta}}{1 - \frac{W_1}{W_2} e^{-\beta}}$ $\beta = \frac{k_1 F_1}{W_1} - \frac{1}{1 + \frac{k_1 F_1}{k_2 F_2}} \left(1 - \frac{W_1}{W_2} \right)$
Total heat transfer area	$F = F$	$F = F_1 + F_2$	$F = F_1 + F_2$	$F = F_1 + F_2$
Exchanger performance		$Q = W_1 \Psi (t_{1\text{ be}} - t_{2\text{ be}})$		
Outlet temperatures		$t_{1\text{ ki}} = t_{1\text{ be}} - \Psi (t_{1\text{ be}} - t_{2\text{ be}})$ $t_{2\text{ ki}} = t_{2\text{ be}} + \frac{W_1}{W_2} \Psi (t_{1\text{ be}} - t_{2\text{ be}})$		

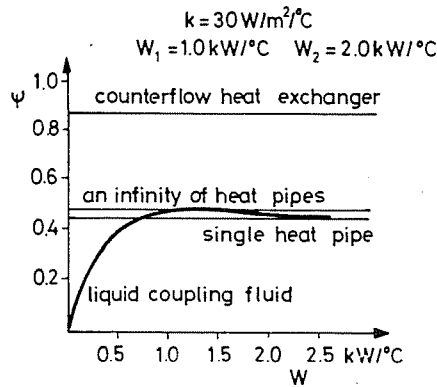


Fig. 6

Table 2

Comparing of Exchanger Type Performances on the strength of a Numerical Example

	Counterflow exchanger without coupling fluid	Indirect-transfer exchanger ($W_c = 1300 \text{ W}$)	Single heat pipe	Infinite series of heat pipes
Unit overall thermal conductance, $\text{W/m}^2, \text{K}$	4.545	42.857	47.619	47.619
Ψ factor	0.3379	0.5863	0.5493	0.6193
Exchanger performance, W	28 724	49 836	46 691	52 641
Related performance, per cent	100	173.5	162.6	183.3

$$F = 100 \text{ m}^2 \quad t_{1bc} = 100 \text{ °C}$$

$$\alpha_k = 50 \text{ W/m}^2, \text{K} \quad t_{2bc} = 15 \text{ °C}$$

$$W_1 = 1000 \text{ W/K}$$

$$W_2 = 2000 \text{ W/K}$$

conductance will be $k = 42.857 \text{ W/m}^2, \text{K}$, furthermore, in case of employing a heat pipe, a heat transfer coefficient as high as $\alpha_b = 10\,000 \text{ W/m}^2, \text{K}$ is attainable resulting in $k = 47.619 \text{ W/m}^2, \text{K}$. These values already yield a better base for comparings.

Using the above conductivity values, Table 2 contains performance data for the different possible realizations of a heat exchanger with 100 m^2 total heat transfer area.

It is clearly indicated by the table that, for the case presented above, the least performance of all is given by the direct-type counterflow exchanger and that a performance higher by 60—80 per cent to this value can be obtained when a coupling fluid is used.

The economic advantage of employing indirect transfer heat exchangers is a conclusion of the above-mentioned train of thoughts. Namely, if the savings deriving from the decrease in surfaces and the layout advantages cover the energy costs connected with assembling and installation, employing of the device is economical.

Nomenclature

Letter Symbols

$a = W_1/W_2$	— Dimensionless parameter
F	— Heat transfer area, m^2
k	— Unit overall thermal conductance, $W/m^2, K$
Q	— Heat transfer rate, W or kW
t	— Temperature, $^{\circ}C$ or K
W	— Flow-stream capacity rate, W/K or kW/K
$x = W_v/W_2$	— Dimensionless parameter
ϑ	— Dimensionless outlet temperature
$\Phi = (t_{kbe} - t_{kki}) / (t_{kbe} - t_{nbe})$	— Bošnjaković number
Ψ	— Effectiveness of indirect-transfer exchanger

Subscripts

be	— Inlet
ki	— Outlet
k	— Refers to the fluid with the smaller thermal capacitance
n	— Refers to the fluid with the larger thermal capacitance
s	— Saturation
v	— Coupling fluid

Note 1

In thermodynamic sense, a single heat pipe or a series of heat pipes in a system is the complex of heat pipes operating at the same temperature and, consequently, containing the fluid at the same saturation temperature.

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