

# THREE-FLUID HEAT EXCHANGERS OF TWO AND THREE SURFACES\*

By

C. D. HORVÁTH

Department of Heat Energetics, Technical University, Budapest

Received March 2, 1977

Presented by Prof. Dr. I. SZABÓ

## Introduction

A simple implementation of three-fluid heat exchanger may be produced so that three tubes are welded together lengthwise and the outside of this tube bank is insulated. By introducing fluids of different temperatures into the tubes, a three-fluid heat exchanger (or recuperator) is obtained. For the calculation of such a simple heat exchanger, informations are found in literature. For example, D. D. AULDS and R. F. BARRON [1] present a method for this purpose. Nevertheless, the method is a special one as it cannot be generalized for cases where the flow direction of at least one of the fluids crosses that of the others, that is, where the temperature distribution is not unidimensional. A method of more universal validity is composed by the author [2], aiming besides at a proposition for a mode of treatment of three-fluid heat exchangers.

In the discussion of one-pass three-fluid heat exchangers, a system of differential equations, independent of the arrangement and flow directions of the recuperator, is taken as starting point. This system of equations, which can be solved, of course, only if the arrangement, the flow directions and the boundary conditions are known, includes as a special case the differential equations of any one-pass two-fluid heat exchanger too. At the same time, equations of similar structure can be applied for the case of an arbitrary number of fluids.

It seems useful to distinguish two main types of three-fluid recuperators. Those belonging to the first type have three heat transfer surfaces, while those of the second type have only two ones. The latter is the less general case and it can be derived from the other by regarding the thermal conductance of one of the three surfaces as zero. The distinction between the two types is justified, on the one hand, by the calculating method suggested by the author and, on the other, by a difference in practical importance.

If the geometrical arrangement and the type of recuperator as well as the flow directions and boundary conditions are given, then the temperature

\* Abridged text of the Dr. Thesis by the author.

distribution can be calculated by solving the mentioned system of differential equations. The basic idea is the following [2]. The method employed by W. NUSSELT [3] for the first time, on purpose to calculate cross-flow two-fluid recuperators, and improved by M. JACOB [4] can be generalized for the calculation of three-fluid heat exchangers with fluids of two-dimensional temperature distribution. By means of this generalization, a calculating process is elaborated in the Thesis.

#### Letter Symbols

$k$	Unit overall thermal conductance
$t$	Temperature
$x$	Flow co-ordinate
$y$	Flow co-ordinate
$C$	Integrating constant
$F$	Exchanger total heat transfer area on one side
$\dot{W}$	Flow-stream capacity rate
$X$	Length of one side of a rectangular plane exchanger
$Y$	Length of one side of a rectangular plane exchanger
$\eta$	Flow co-ordinate, dimensionless
$\theta$	Temperature, dimensionless
$\xi$	Flow co-ordinate, dimensionless
$\Phi$	Exchanger effectiveness
$\psi$	Term of a sum

#### Subscripts

1	Heat transfer surface between fluids I and II
2	Heat transfer surface between fluids I and III
3	Heat transfer surface between fluids II and III
I	Fluid with the highest inlet temperature
II	Fluid with an intermediate inlet temperature
III	Fluid with the lowest inlet temperature
0, 1, 2, 3, $n$	Numbers of terms of sum

#### Superscripts

' and "	Inlet and outlet of fluids
---------	----------------------------

### The treated heat exchanger types

The following conditions are stipulated:

- a) The fluids flow parallel with and/or at right angles to each other.
- b) The temperature distribution is at most two-dimensional.

Exchangers of the first (three-surface) type satisfy only these conditions while those of the second (two-surface) type satisfy, moreover, the following restriction:

- c) Two of the three fluids are not in heat transfer with each other but only with the third one.

The two-surface type is obviously of more practical significance with the three fluids streaming usually as continuous layers and so the middle one separating the others.

### General system of equations and the variants of exchangers

The scheme, independent of geometrical relationships, of a three-fluid heat exchanger is shown in Fig. 1. Arrangement of heat transfer surfaces and flow directions is momentarily neglected. In consequence, signs of the flow-

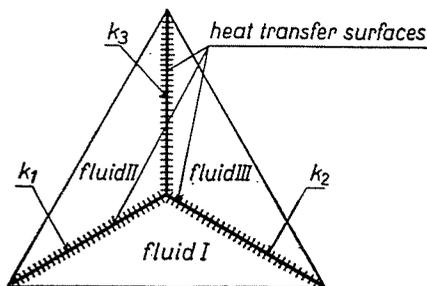


Fig. 1. Scheme of a three-fluid heat exchanger

stream capacity rates are not indicated. Overall thermal conductances  $k_1, k_2, k_3$  refer to the same heat transfer area  $F$ . Considering the introduced heat positive and the released heat negative, a heat balance for each fluid may be set up as follows:

$$W_1 dt_1 = -k_1(t_1 - t_{II}) dF - k_2(t_1 - t_{III}) dF \quad (1)$$

$$W_{II} dt_{II} = k_1(t_1 - t_{II}) dF - k_3(t_{II} - t_{III}) dF \quad (2)$$

$$W_{III} dt_{III} = k_3(t_{II} - t_{III}) dF + k_2(t_1 - t_{III}) dF \quad (3)$$

In accordance with the law of conservation of energy, summation of Eqs (1) to (3) yields

$$W_1 dt_1 + W_{II} dt_{II} + W_{III} dt_{III} = 0 \quad (4)$$

Three-fluid heat exchangers satisfying conditions a) and b) have 10 variants due to varying flow directions as shown in Fig. 2. The ten variants can be derived as follows. Let us assume three fluids at different temperatures. If the purpose is to produce all the possible configurations of temperature distribution of these fluids in a heat exchanger satisfying the above conditions, this can be done by varying the ways of fluid introduction into the exchanger and just the ten variants of Fig. 2 shall be obtained.

The ten variants may be divided into four groups: A, B, C and D in Fig. 2. As it is easily conceivable, the systems of differential equations of the individual variants within a group may differ only by sign. Therefore, a group-by-group determination of the integral equations, necessary for setting up the computer programs, is sufficient.

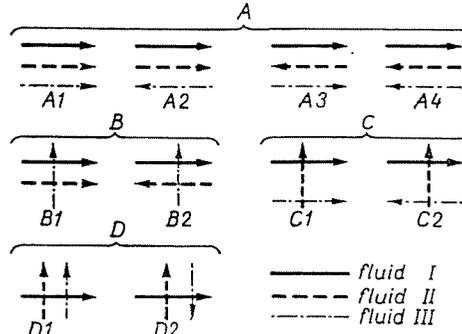


Fig. 2. Variants of three-fluid heat exchangers, depending on the arrangement

**Calculation of two-surface three-fluid heat exchangers**

*Solution of the problem for a given arrangement*

Let us now assume that there is no heat transfer surface between fluids I and II, then, considering that the stream of fluid I and the increase of co-ordinate  $y$  have now opposite directions, the arrangement schematically shown in Fig. 3 leads to the following form of Eqs (1) to (3):

$$-W_I \frac{\partial t_I}{\partial y} dy = -k_1(t_I - t_{II}) X dy \tag{5}$$

$$W_{II} \frac{\partial t_{II}}{\partial y} dy = k_1(t_I - t_{II}) X dy - k_3(t_{II} - t_{III}) X dy \tag{6}$$

$$W_{III} \frac{\partial t_{III}}{\partial x} dx = k_3(t_{II} - t_{III}) Y dx \tag{7}$$

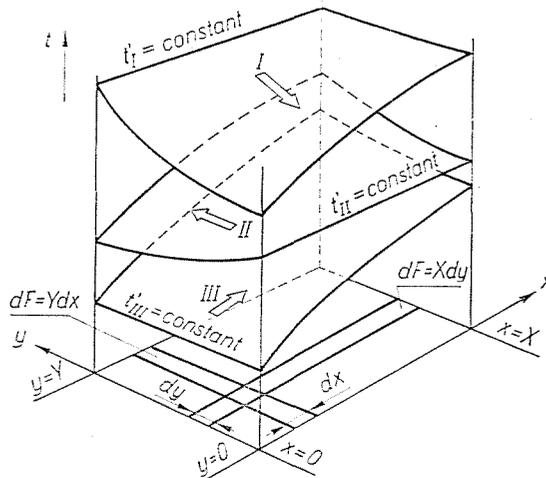


Fig. 3. Scheme of the basic arrangement of a recuperator

Dimensionless variables may be introduced, defined by

$$\vartheta_I = \frac{t_1 - t'_{III}}{t'_I - t'_{III}}, \quad \vartheta_{II} = \frac{t_{II} - t'_{III}}{t'_I - t'_{III}}, \quad \vartheta_{III} = \frac{t_{III} - t'_{III}}{t'_I - t'_{III}} \quad (8)$$

$$\xi = \frac{x}{X}, \quad \eta = \frac{y}{Y}, \quad a = \frac{k_1 F}{W_1}, \quad b_1 = \frac{k_1 F}{W_{II}}, \quad b_2 = \frac{k_3 F}{W_{II}}, \quad c = \frac{k_3 F}{W_{III}}.$$

With these denotations, Eqs (5) to (7) lead to

$$\frac{\partial \vartheta_I}{\partial \eta} = a(\vartheta_I - \vartheta_{II}) \quad (9)$$

$$\frac{\partial \vartheta_{II}}{\partial \eta} = b_1(\vartheta_I - \vartheta_{II}) - b_2(\vartheta_{II} - \vartheta_{III}) \quad (10)$$

$$\frac{\partial \vartheta_{III}}{\partial \xi} = c(\vartheta_{II} - \vartheta_{III}) \quad (11)$$

the boundary conditions being

$$\vartheta_I = 1 \quad \text{at} \quad \eta = 1 \quad (12)$$

$$\vartheta_{II} = \vartheta'_{II} = \frac{t'_{II} - t'_{III}}{t'_I - t'_{III}} \quad \text{at} \quad \eta = 0 \quad (13)$$

$$\vartheta_{III} = 0 \quad \text{at} \quad \xi = 0. \quad (14)$$

Assuming  $\xi$  and  $\eta$  respectively to be constant parameters and substituting from Eqs (12) to (14) into Eqs (9) to (11), the following system of Volterra integral equations with two independent variables is obtained:

$$\vartheta_I(\xi, \eta) = e^{-a(1-\eta)} + ae^{a\eta} \int_{\eta}^1 \vartheta_{II}(\xi, \eta^+) e^{-a\eta^+} d\eta^+ \quad (15)$$

$$\begin{aligned} \vartheta_{II}(\xi, \eta) = e^{-(b_1+b_2)\eta} & \left[ \vartheta'_{II} + b_1 \int_0^{\eta} \vartheta_I(\xi, \eta^+) e^{(b_1+b_2)\eta^+} d\eta^+ + \right. \\ & \left. + b_2 \int_0^{\eta} \vartheta_{III}(\xi, \eta^+) e^{(b_1+b_2)\eta^+} d\eta^+ \right] \quad (16) \end{aligned}$$

$$\vartheta_{III}(\xi, \eta) = ce^{-c\xi} \int_0^{\xi} \vartheta_{II}(\xi^+, \eta) e^{c\xi^+} d\xi^+ \quad (17)$$



tion of fluid II, the program solves Eqs (15) and (17) by substituting values of  $\vartheta_{II}$ . Hence, at the end of the computation, the memory unit of the computer contains the temperature distribution of each of the three fluids. The mean outlet temperatures of the three fluids are obtained through numerical integrations. E.g., the mean outlet temperature of fluid I:

$$\overline{\vartheta}_1^o = \int_0^1 \vartheta_1(\xi, 0) d\xi$$

Computations are performed and the results tabulated for the following values of each of the parameters  $a$ ,  $b_1$ ,  $b_2$  and  $c$ : 0, 0.5, 1, 4.

#### *A conclusion of the investigations*

As is well-known, determination of the dimensionless outlet temperatures of a two-fluid heat exchanger necessitates the knowledge of two dimensionless parameters of the form  $kF/W$ . For two-surface three-fluid heat exchangers, as can be seen from the above explanations, five parameters are necessary. One of the new parameters is the dimensionless inlet temperature  $\vartheta'_{II}$  of the "intermediate" fluid. A brief analysis of the above relationships shows and the computed results prove, that, for the treated heat exchanger, the mean outlet temperatures of the three fluids are linear functions of  $\vartheta'_{II}$ . One can easily ascertain that the arithmetic relationships of two-surface three-fluid recuperators with different arrangements don't differ in this respect. Consequently, in a more general formulation, the following conclusion may be drawn: in two-surface three-fluid one-pass heat exchangers corresponding to conditions a), b) and c), the mean outlet temperature of each fluid is a linear function of the inlet temperatures.

#### *Calculation of two-surface three-fluid heat exchangers in general*

For two-surface heat exchangers, 3 cases of each of the 10 arrangements in Fig. 2 should be distinguished according to the "missing" surface of the three. Hence, the integral equations, constituting the bases of the computer programs, have to be set up for 3 cases of 4 groups, i.e. for 12 cases altogether. Systems of integral equations of all the twelve cases are contained in the Thesis.

### **Three-surface three-fluid heat exchangers**

In such recuperators, the three fluids are separated by three surfaces of finite heat resistance and so there are 6 parameters of the form  $kF/W$ . However, the six parameters aren't independent of each other: one of them can be

determined from the others. Dimensionless temperature distributions of a three-surface three-fluid heat exchanger thus, considering also the inlet temperature of fluid II, are functions of 6 parameters. In spite of all these, the use of 7 parameters — 6 of the type  $kF/W$  and  $\vartheta'_{II}$  — is advisable for the sake of more clear and shorter mathematical formulas.

Let us assume once more the arrangement shown in Fig. 3 with the modification that, now, the heat transfer surface with a finite heat resistance between fluids I and III exists. Eqs (1) to (3) may now be written in the form

$$-W_1 \frac{\partial t_1}{\partial y} dy = -k_1(t_1 - t_{II}) X dy - k_2(t_1 - t_{III}) X dy \quad (20)$$

$$W_{II} \frac{\partial t_{II}}{\partial y} dy = k_1(t_1 - t_{II}) X dy - k_3(t_{II} - t_{III}) X dy \quad (21)$$

$$W_{III} \frac{\partial t_{III}}{\partial x} dx = k_3(t_{II} - t_{III}) Y dx + k_2(t_1 - t_{III}) Y dx \quad (22)$$

The definitions of the dimensionless temperatures and co-ordinates are the same as before, while the  $kF/W$  parameters may be defined by

$$a_1 = \frac{k_1 F}{W_1}, \quad a_2 = \frac{k_2 F}{W_1}, \quad b_1 = \frac{k_1 F}{W_{II}}, \quad b_2 = \frac{k_3 F}{W_{II}}, \quad c_1 = \frac{k_3 F}{W_{III}}, \quad c_2 = \frac{k_2 F}{W_{III}} \quad (23)$$

With these, Eqs (20) to (22) yield

$$\frac{\partial \vartheta_1}{\partial \eta} = a_1(\vartheta_1 - \vartheta_{II}) + a_2(\vartheta_1 - \vartheta_{III}) \quad (24)$$

$$\frac{\partial \vartheta_{II}}{\partial \eta} = b_1(\vartheta_1 - \vartheta_{II}) + b_2(\vartheta_{II} - \vartheta_{III}) \quad (25)$$

$$\frac{\partial \vartheta_{III}}{\partial \xi} = c_1(\vartheta_{II} - \vartheta_{III}) + c_2(\vartheta_1 - \vartheta_{III}) \quad (26)$$

The boundary conditions being the same as before, the following integral equations are obtained:

$$\begin{aligned} \vartheta_1(\xi, \eta) = & e^{-(a_1+a_2)(1-\eta)} + a_1 e^{(a_1+a_2)\eta} \int_{\eta}^1 \vartheta_{II} e^{-(a_1+a_2)\eta^+} d\eta^+ + \\ & + a_2 e^{(a_1+a_2)\eta} \int_{\eta}^1 \vartheta_{III} e^{-(a_1+a_2)\eta^+} d\eta^+ \end{aligned} \quad (27)$$

$$\begin{aligned} \vartheta_{11}(\xi, \eta) = & \vartheta'_{11} e^{-(b_1+b_2)\eta} + b_1 e^{-(b_1+b_2)\eta} \int_0^\eta \vartheta_1 e^{(b_1+b_2)\eta^+} d\eta^+ + \\ & + b_2 e^{-(b_1+b_2)\eta} \int_0^\eta \vartheta_{111} e^{(b_1+b_2)\eta^+} d\eta^+ \end{aligned} \quad (28)$$

$$\vartheta_{111}(\xi, \eta) = c_1 e^{-(c_1+c_2)\xi} \int_0^\xi \vartheta_{11} e^{(c_1+c_2)\xi^+} d\xi^+ + c_2 e^{-(c_1+c_2)\xi} \int_0^\xi \vartheta_1 e^{(c_1+c_2)\xi^+} d\xi^+ . \quad (29)$$

In contrast to the case of two-surface exchangers, none of the equations can now be written in a form in which only a single one of the three temperature-functions is involved. For that reason, the method used for the calculation of two-surface exchangers is to be omitted here. Nevertheless, Eqs (27) to (29) can be solved by means of a direct numerical method based on a prediction and successive corrections of temperature values of the fluids. This method is, of course, suitable for computations of two-surface exchangers too. It is more fast, as a rule, than the one previously described but sensitive to the initial temperature values.

### Effectiveness of three-fluid heat exchangers

#### Introduction of coefficients $\Phi$

As is well-known, the effectiveness of a two-fluid exchanger is usually characterized by BOŠNJKOVIĆ's [5] coefficient  $\Phi$ , what is the proportion of the temperature change of the fluid with the smaller flow-stream capacity rate ( $W_1$ ) to the difference in fluid inlet temperatures ( $\Delta t_{\max}$ ). The transferred heat amount may be obtained as

$$Q = W_1 \Phi \Delta t_{\max}$$

The reasonability of employing  $\Phi$  as a characteristic of the effectiveness is evident. If the capacity rates and the fluid inlet temperatures are given, then the transferred heat depends only on the coefficient  $\Phi$  of the exchanger.

In the case of a three-fluid recuperator, speaking solely about "the transferred heat" is meaningless, but, in accordance with the number of fluids, the exchanger has always three "heat outputs". To the single value  $\Phi$  at two fluids, the use of two similar quantities corresponds while the third issues from the heat balance. However, a distinction among the fluids is of no significance now. Considering all these, the following definition is proposed for the

effectiveness coefficients of three-fluid exchangers:

$$\begin{aligned}\Phi_I &= \overline{\vartheta}_I'' - 1 \\ \Phi_{II} &= \overline{\vartheta}_{II}'' - \vartheta'_{II} \\ \Phi_{III} &= \overline{\vartheta}_{III}''\end{aligned}\quad (30)$$

With Eqs (8), the coefficients are

$$\begin{aligned}\Phi_I &= \frac{t_1'' - t_1'}{t_1' - t_{III}'}, \\ \Phi_{II} &= \frac{t_{II}'' - t_{II}'}{t_1' - t_{III}'}, \\ \Phi_{III} &= \frac{t_{III}'' - t_{III}'}{t_1' - t_{III}'}.\end{aligned}\quad (31)$$

On the analogy of two-fluid recuperators, the effectiveness coefficient concerning one fluid of a three-fluid heat exchanger is the quotient of the overall temperature change of this fluid and the greatest temperature difference of the exchanger. The value of  $\Phi_{II}$  may thus be negative too, indicating that fluid II might as well be cooled in the recuperator.  $\Phi_I$  is always positive and  $\Phi_{III}$  is always negative.

The heat quantities introduced into and delivered from the fluids:

$$Q_I = W_I \Phi_I (t_1' - t_{III}') \quad (32)$$

$$Q_{II} = W_{II} \Phi_{II} (t_1' - t_{III}') \quad (33)$$

$$Q_{III} = W_{III} \Phi_{III} (t_1' - t_{III}') \quad (34)$$

#### *The problem of the best arrangement of exchanger*

One-pass two-fluid recuperators can practically be arranged in three ways: in unidirectional flow, in counterflow and in pure (rectangular) cross-flow. For given fluid quantities and qualities and unit overall thermal conductances, the counterflow arrangement will have the highest effectiveness while the unidirectional flow the lowest. This fact is not influenced by the proportion of the flow-stream capacity rates. In consequence, unless there are contrasted circumstances, the designer chooses counterflow.

Three-fluid heat exchangers with two-dimensional temperature distributions may have the ten kinds of arrangements seen in Fig. 2, but this time,

$$\begin{aligned}
 k_1 F / W_I &= k_3 F / W_{III} = 1 \\
 k_2 &= 0 \\
 k_1 F / W_{II} &= k_3 F / W_{II} = 0.5 \\
 \psi_I &= 0.1 \\
 \psi_{II} &= 0.6 \\
 \psi_{III} &= 0
 \end{aligned}$$

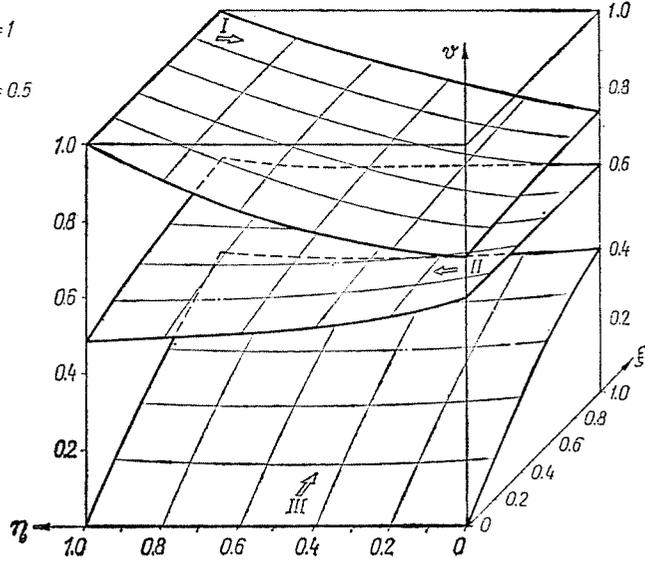


Fig. 4. Computed temperature distributions of a two-surface three-fluid exchanger

$$\begin{aligned}
 k_1 F / W_I &= k_2 F / W_I = k_3 F / W_{III} = \\
 &= k_2 F / W_{III} = 1 \\
 k_1 F / W_{II} &= k_3 F / W_{II} = 0.5 \\
 \psi_I &= 1 \\
 \psi_{II} &= 0.6 \\
 \psi_{III} &= 0
 \end{aligned}$$

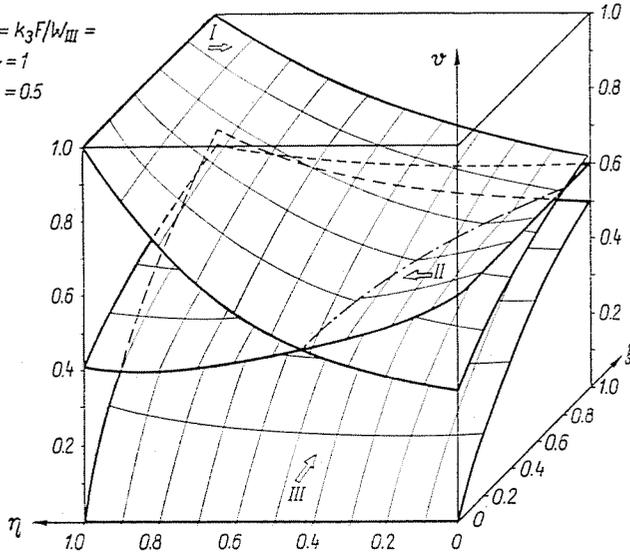


Fig. 5. Computed temperature distributions of a three-surface three-fluid exchanger

supposing to be given the same quantities as before, the arrangement alone does not determine the effectiveness.

This statement may be proved through an investigation, e.g., of arrangements  $A_2$  and  $A_3$  in Fig. 2 assuming that both are employed for the same three-surface exchanger. The purpose is to cool down fluid I as far as possible, i.e., to make  $\Phi_1$  maximum. The following two cases are to be considered:

a) The flow-stream capacity rate of fluid II is negligible as compared with that of both the others. If so, arrangement  $A_2$  practically leads to a counterflow two-fluid exchanger while  $A_3$  leads to a unidirectional one. In consequence,  $A_2$  is of higher effectiveness.

b) The flow-stream capacity rate of fluid III is negligible as compared with that of the others. Now, on the contrary, one may consider  $A_2$  unidirectional flow and  $A_3$  counterflow, so  $A_3$  is more efficient.

Hence, a change merely in the proportion of the capacity rates changes the choice among the arrangements.

### Summary

A general system of differential equations has been set up for one-pass three-fluid heat exchangers with two-dimensional temperature distributions. On the basis of these equations, calculation methods are introduced, the main point being a numerical solution of Volterra integral equations.

Some experiences of computer-aided computations, based on the unfolded method, are made known.

The variants of the mentioned types of heat exchangers follow from the arrangement possibilities.

An interpretation is proposed for three-fluid exchanger effectiveness.

### References

1. AULDS, D. D.—BARRON, R. F.: Three-fluid heat exchanger effectiveness. *Int. J. Heat and Mass Transfer*, No. 10 (1967).
2. HORVÁTH, C. D.: A study of three-fluid heat exchangers of two and three surfaces. Doctor Techn. thesis. Budapest 1974.
3. NUSSELT, W.: *Tech. Mech. Thermodynam.* 1, 417; 1930.
4. JACOB, M.: *Heat Transfer* (1957), Vol. II. Wiley, New York.
5. BOŠNJKOVIĆ, F.—VILIČIĆ, M.—SLIPIČEVIĆ, B.: *VDI-Forschungsheft* No. 432; 1951.

Dr. Csaba Dénes HORVÁTH 1158 Budapest, Frankovics u. 29. Hungary