ON THE SPECTROSCOPIC AMPLITUDES OF NUCLEAR STATES

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The n-particle — m-hole (np-mh) excitations are considered as one of the most important degree of freedom of nuclei in excited state. The structure of these states can experimentally be investigated mostly by particle transfer reactions. Besides the one-particle transfers used extensively so far for investigating the one-particle structure of states, the up-to-date heavy ion technics makes us possible to perform also several-nucleon transfer reaction and thereby to gather more informations about the several-nucleon clustering phenomena in nuclear states. Among the many-nucleon transfers, the α-transfer reactions seem presently the most important ones since the relatively small number (four) of nucleons transferred enables to treat the process also theoretically, thus offering a novel possibility for testing our knowledge about nuclear reactions and nuclear structure.

It has recently been shown [1] that the α-transfer experiments can successfully be analyzed in terms of α-spectroscopic amplitude (ASA). The theoretical background of such an analysis rests on the supposition that the DWBA transition amplitude for an x-particle transfer reaction:

$$\begin{aligned} T^{DWBA}_{aA\to Bb}(\Theta) &\equiv \langle \Phi^{(-)}_{Bb} \mid V_{Bb} - \overline{V}_{Bb} \mid \Phi^{(+)}_{Aa} \rangle = \\ &= \sum_{\substack{N'L'NL \\ xx'Ab}} A_{N'L'}(a \to \{b + x'\}_{N'L'}) A_{NL}(B \to \{A + x\}_{NL}) B^{N'L'}_{NL}(\Theta) \end{aligned} \tag{1}$$

is a good approximation to the exact problem. In Eq. (1) $\Phi^{(\pm)}_{Yy}$ is the channel wave function with incoming/outgoing (+/—) asymptotical behaviour; the reaction amplitudes $B^{N'L'}_{NL}(\Theta)$ accounting for the dynamics of the process contain the distorted waves of the incoming/outgoing channel as well as the interaction $V_{Bb}-\overline{V}_{Bb}$ producing the transition.

It is the spectroscopic amplitude A_{NL} in Eq. (1) which contains all the information about the nuclei participating in the transfer process. It is defined by the overlap-integral

$$A_{NL}(p \to \{d + x\}_{NL}) \equiv \int [\Phi_d(\zeta_d)\Phi_x(\zeta_x)\varphi_{NLM}(R_{xd})]^*_p \Phi_p(\zeta_p) \mathrm{d}\zeta_p \tag{2}$$

where Φ_i means wave function of the nucleus i depending on the internal coordinates ζ_i. The function φ_{NLM} describes the centre-of-mass (c.m.) motion of the x-nucleon group relative to the daughter nucleus d.

In Eq. (1) the summations are in principle extended over all internal states of the "core" nuclei b and A and of the x-group being in various relative states $N'L'$ or NL with respect to the cores b or A. In the practice, however, it is usual to retain only one term in Eq. (1) thus getting the expression for the differential cross section:

$$\frac{d\sigma}{d\Omega} \propto | T(\Theta) |^2 \sim S_{N'L'}(a) S_{NL}(B) | B_{NL}^{N'L'}(\Theta) |^2 \tag{3}$$

where the quantities $S_{N'L'}(a)$, $S_{NL}(B)$ are defined by the corresponding spectroscopic amplitudes squared and called spectroscopic factor of the projectile (a) or that of the heavy particle (B).

Letting x be equal to an α-particle in Eq. (2), one gets the α-spectroscopic amplitude (ASA). These amplitudes play a very important role in the description of the α-transfer reaction as it is seen in Eq. (1). When one term predominates over the others in the summation of Eq. (1), the approximation of Eq. (3) can be applied. In such a case the α-spectroscopic factor can directly be related to the measurements. In the other cases one has to pre-calculate the necessary ASA's since the coherence effect may cause appreciable error when calculating the differential cross section. The theoretical basis of such a "pre-calculation" is some sort of a model state of the nuclei taking part in the "decays" $a \to \{b + x'\}_{N'L'}$ or $B \to \{A + x\}_{NL}$. The model used is usually the shell model, the wave functions of which are easy to obtain (at least in principle) and to work with. Particularly, oscillator shell model with SU (3) classification has been used [2] in the L—S coupling scheme, and Zuker, Buck, and McGrory's (ZBM) wave function has been chosen [1] in the j—j coupling isotopic-spin formalism.

From Eq. (1) it is clear that reliable ASA's are required if one wants to calculate the α-transfer reactions. Of course, an ASA is the more reliable, the more reliable are the nuclear wave functions in Eq. (2). It has been shown [1] that using ZBM wave functions in Eq. (2), the ASA's obtained reflect many features of the properties of both the nuclear structure and the transfer reaction. Particularly, the population strengths and the *np-nh* structure of states have been investigated. In Fig. 1 of [1], the $4p$—$4h$ and $2p$—$2h$ configurations each are to contribute by about 15 per cent to the ^{16}O ground state. This fact (not mentioned in [1]) was found also by the (^3He, ^7Be) α-pick-up experiment [3].

It is especially worth to emphasize among the results of [1] the connection between the *np-nh* structure of the nucleus being in the J^π state and the c.m. motion $| NJ >$ of the α-particle in the same nuclear state. Writing Eq. (6) of [1] into a form slightly modified:

$$2N + J = 4 + n - \sum_{i=1}^{3} n_i \quad \text{for} \quad \nu_0 < \nu_{0\alpha} \tag{4}$$

we can see that — in addition to relate N to n — this equation clearly tells us that the energy for the c.m. motion of the α-particle inside the nucleus is diminished by the internal motion of the α-particle being in relative s state. This phenomenon can physically be understood in terms of changing degrees of freedom: the degree of freedom resulting from the different sizes $(\nu_0 \neq \nu_{0\alpha})$ is converted into that manifesting in the internal radial excitations $(n_i \neq 0)$. This phenomenon disappears in the case of equal size parameters. In fact,

$$2N + J = 4 + N \quad \text{for} \quad \nu_0 = \nu_{0\alpha} \tag{5}$$

which means that the c.m. motion is unambiguously determined by the np-nh character of the J^π state in which the α-particle is confined.

In summary, the discussion above shows that the spectroscopic amplitudes are very useful quantities which prove to be a sensitive tool for both the characterization of the nuclear states and the interpretation of the nuclear reactions. Thus, the spectroscopic amplitudes can be assumed to provide a suitable connection between the theory and measurement of the transfer reactions.

Summary

The theoretical background of the analysis of transfer reactions in terms of spectroscopic amplitudes is discussed. The sphere of validity of the concept of spectroscopic factor is investigated. Additional interpretation is given to the equation connecting the n-particle — n-hole structure of a state J^π of ^{16}O with the relative motion $| NJ >$ of the α-particle in the same state.

References

1. Apagyi, B.—Fái, G.: α-Spectroscopic Amplitudes of ^{16}O in the Zuker, Buck, and McGrory Shell Model, Journal of Physics **G3** (1977) pp. L163—L167
2. Ichimura, M.—Arima, A.—Halbert, E. C.—Terasawa, T.: Alpha-particle Spectroscopic Amplitudes and the SU(3) Model, Nucl. Phys. **A204** (1973) pp. 225—278
3. Detraz, C.—Duhm, H. H.—Hafner, H.: The (^3He, ^7Be) Reaction on Light nuclei, Nucl. Phys. **A147** (1970) pp. 488—508

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