DRYING OF MACROPOROUS SYSTEMS

PART I

By

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1. The mathematical model

Drying of liquid-phase moisture in a macroporous system (i.e. nonhygroscopic system) or drying of solid-phase moisture in an arbitrary system can be examined by the method to de described. The examined model is supposed to have an infinite frontal surface, representing a finite plane of thickness 2L, to be dried from both sides by a gas in constant state, at constant convective transport coefficients.

The thickness ξ of the dried region I of the material grows as the drying time increases, while that one of the wet region II decreases. The examined model is shown in Fig. 1.

One portion of the heat, flowing by convection from the drying gas entering into the dried material, will increase the enthalpy of the dried material, while the other portion will flow towards the wet material. This latter one is also divided in two portions: one of these is the (conductive) portion which is warming up the wet material, while the other one is the portion which serves to evaporate moisture. A small rate of the heat is spent in order to warm up the vapor which is diffusing through the dried material.



Fig. 1. The examined macrocapillary model

According to the above mentioned considerations, the enthalpy balance of the dried front I will be

$$\varrho_{ms}c_{hI}\frac{\partial t^{I}}{\partial \tau} = -\frac{\partial}{\partial z}(q_{hI} - j_{w}c_{pwG}t^{I}), \qquad (1)$$

where the drying velocity is

$$j_w = -rac{1}{F_{\acute e}} rac{dW}{d au} \, .$$

The instantaneous value of the moisture content is:

$$W = W_0 - V_p \frac{Z}{L} \varrho_{wL},$$

whence

$$\frac{dW}{d\tau} = -\frac{V_p}{L} \varrho_{wL} \frac{dZ}{d\tau} \,.$$

On the basis of the foregoing, the drying velocity becomes:

$$j_w = \varrho_{wL} \frac{V_p}{F_e L} \frac{dZ}{d\tau}.$$

Introducing the porosity

$$\psi = \frac{V_p}{F_{\acute{\rm e}}L}\,,$$

the instantaneous value of the drying velocity yields:

 $j_w = \varrho_{wL} \psi \frac{dZ}{d\tau}.$

For a given capillary-porous body

$$\varrho_{wL}\psi = \text{const.}$$

and introducing the term

$$\varrho_{wL} \psi = g$$

furthermore, substituting this term into Eq. (1) gives

$$\varrho_{ms}c_{hI}\frac{\partial t^{I}}{\partial \tau} = -\frac{\partial}{\partial z}\left(q_{hI} - g\frac{dZ}{d\tau}c_{pwG}t^{I}\right)$$

Since

$$q_{hI} = -\lambda_{hI} \frac{\partial t^{I}}{\partial z}, \qquad (2)$$

hence

$$\varrho_{ms}c_{hI}\frac{\partial t^{I}}{\partial \tau}=-\frac{\partial}{\partial z}\left(-\lambda_{hI}\frac{\partial t^{I}}{\partial z}-g\frac{dZ}{d\tau}c_{pwG}t^{I}\right).$$

Hereof

$$\frac{\partial t^{I}}{\partial \tau} = \frac{\lambda_{hI}}{\varrho_{ms}c_{hI}} \frac{\partial^{2}t^{I}}{\partial z^{2}} + \frac{gc_{pwG}}{\varrho_{ms}c_{hI}} \frac{dZ}{d\tau} \frac{\partial t^{I}}{\partial z}$$

can be obtained. Since

$$a_{hI} = \frac{\lambda_{hI}}{\varrho_{ms}c_{hI}}$$

and applying notation

$$\frac{gc_{pwG}}{\varrho_{ms}c_{hI}}=b,$$

Eq. (1) can be rewritten as

$$\frac{\partial t^{I}}{\partial \tau} = a_{hI} \frac{\partial^{2} t^{I}}{\partial z^{2}} + b \frac{dZ}{d\tau} \frac{\partial t^{I}}{\partial z}.$$
(3)

The boundary conditions are:

$$q_h\Big|_{0^-} = \alpha_h(t_G - t_F) = -\lambda_{hI} \frac{\partial t^I}{\partial z}\Big|_{0^+}$$
(4)

at z = 0; and

$$q_{h}\Big|_{\varepsilon} = -\lambda_{hI} \frac{\partial t^{I}}{\partial z}\Big|_{\varepsilon} = -\lambda_{hII} \frac{\partial t^{II}}{\partial z}\Big|_{\varepsilon} + j_{w}r$$
(5)

at $z = \xi$.

The enthalpy-balance of the wet region II becomes

$$\frac{\partial t^{II}}{\partial \tau} = \frac{\lambda_{hII}}{\varrho_{ms}c_{hII}} \frac{\partial^2 t^{II}}{\partial z^2} = a_{hII} \frac{\partial^2 t^{II}}{\partial z^2}.$$
 (6)

The boundary condition at the median (symmetry) plane is

$$\frac{\partial t^{II}}{\partial z}\Big|_{L-} = 0 \tag{7}$$

at z = L;

and the boundary condition is equal to Eq. (5) at $z = \xi$.

The problem is now to define the moisture content of the material as a function of time.

The first step to the solution would be the determination of the drying velocity $j_{w_2}(\tau)$, depending on time. In principle, this is defined considering given initial conditions by means of the differential equations: Eqs (3) through (7) and the boundary conditions. However, no analytical solution can be presented for this case, since location ξ for the boundary condition changes, depending on time, moreover at this location the temperature is also time-dependent.

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If the solution of $j_{yz}(\tau)$ would be known, then its integral with respect to time would give the rate of the moisture evaporated from the surface unit.

Hereinafter, first the approximate analytical solution of the above mentioned problem will be described. Occurrence of the regular region, obtained by approximate calculus, is illustrated by experimental measurements.

The exact computer aided method of the problem will be described in *Part II* of the paper and followed by a numerical example, where the computer outputs are compared with approximate results.

2. Approximate calculation

Let us first examine a model comprising two layers, in which neither the thickness of the dried region I, nor that one of the wet region II changes [2]

Accordingly: L = const. and $\xi = \text{const.}$

In the case of this model there exists a state of constant drying velocity (denoted by subscript e) where

$$\left(\frac{\partial t^{I}}{\partial \tau}\right)_{e} = 0, \qquad \left(\frac{\partial t^{II}}{\partial \tau}\right)_{e} = 0.$$
 (8)

Consequently

$$\left(\frac{\partial t^{I}}{\partial z}\right)_{e} = \text{const.} = -\frac{(q_{h})_{e}}{\lambda_{hI}} = \frac{t_{\xi e} - t_{F}^{I}}{\xi}$$
(9)

and

$$\left(\frac{\partial t^{II}}{\partial z}\right)_e = \text{const.} = 0 \tag{10}$$

for reasons of symmetry. Owing to these latter reasons, Eqs (4) and (5) yield:

$$q_h \Big|_{0-} = q_h \Big|_{z-} = (q_h)_e = j_{we} r = \text{const.}$$

$$(11)$$

and Eqs (4) and (9)

$$q_{he} = \frac{1}{\frac{1}{\alpha_h} + \frac{\xi}{\lambda_{hI}}} (t_G - t_{\xi_e})$$
(12)

Pursuant to Eq. (11), $j_{we} = \text{const.}$ too. Therefore, the vapor of the evaporating moisture passing through the channels (pores) to reach the surface by restrained, molecular diffusion, convective mass transfer occurs from the surface of the dried material into the main portion of the drying gas. Thus

$$j_{we} = D_e \frac{M_w}{RT} \frac{\partial p_g}{\partial z} \bigg|_{z^-} = D_e \frac{M_w}{RT} \frac{p_{g\xi e} - p_{gF}}{\xi} = \beta (p_{gF} - p_{gG}).$$
(13)

Finally we obtain

$$j_{we} = \frac{1}{\frac{1}{\beta} + \frac{RT}{D_e M_w} \xi} (p_{g\xi e} - p_{gG}) = \frac{1}{\frac{1}{\beta} + e\xi} (p_{g\xi e} - p_{gG})$$
(14)

At $Z = \xi$, i.e. at the dry—wet interphase surface, the partial pressure $p_{g\xi_e}$ of the evaporating moisture can be considered as saturation pressure at temperature t_{ξ_e} . If the saturation curve is substituted by a straight line with slope s, according to Fig. 2,

then

$$\frac{p_{g\xi e} - p_{gG}}{t_{\xi e} - t_{hp}} = s, \qquad (15)$$

hence, Eq. (14) can be rewritten as

$$j_{we} = \frac{1}{\frac{1}{\beta} + \frac{RT}{D_e M_w} \xi} s(t_{\xi e} - t_{hp})$$
(16)

Eqs (11), (12) and (16) yield

$$j_{we}\left[\frac{1}{\alpha_h} + \frac{1}{\beta sr} + \left(\frac{1}{\lambda_{hI}} + \frac{RT}{D_e M_w sr}\right)\xi\right] = \frac{t_G - t_{hp}}{r}.$$
 (17)

In fact, the relationship between the drying velocity and the penetration velocity of the front is:

$$j_{w} = \psi \varrho_{wL} \frac{dZ}{d\tau} = \psi \varrho_{wL} \frac{d\xi}{d\tau}$$
(18)

whence ψ represents the effective porosity, i.e. the ratio of the volume of channels in the wet material being filled up with liquid to that of the wet material itself.

Let us assume

$$j_w \simeq j_{we}$$
 (19)





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then Eq. (17) can be rewritten as

$$\psi_{\varrho_{wL}}\left[\left(\frac{1}{\alpha_h} + \frac{1}{\beta sr}\right)\xi + \left(\frac{1}{\lambda_{hI}} + \frac{RT}{D_e M_w sr}\right)\frac{\xi^2}{2}\right] \simeq \frac{t_G - t_{hp}}{r}\tau \qquad (20)$$

viz.

$$\frac{\tau}{\psi \varrho_{wL}\xi} = \frac{r}{t_G - t_{hp}} \left(\frac{1}{\alpha_h} + \frac{1}{\beta sr} \right) + \frac{r}{t_G - t_{hp}} \frac{1}{2} \left(\frac{1}{\lambda_{hI}} + \frac{RT}{D_e M_w sr} \right) \xi \quad (21)$$

The result shows that τ/ξ vs. ξ gives approximately a straight line, with the axial section proportional to the convective resistances, and the slop proportional to the conductive resistances.

Correlation (21), is of course, inaccurate at the beginning and will further on grow inaccurate again from the time on, where the material is almost dried out, i.e. $\xi \simeq L$. The period, of approximate validity within which for the linearity relationships, developed according to Eq. (21) cover acceptable approximations, can be called the regular region of material types, drying where by wet front penetration occurs during the process.

The relationship between the thickness ξ of the dried front and the moisture loss of the material may be expressed by

$$\frac{m_0 - m}{F_{\phi}} = \Delta \overline{m} = \psi \cdot \varrho_{wL} \cdot \xi \tag{22}$$

i.e.

$$\frac{\tau}{\overline{\Delta m}} = \frac{r}{t_G - t_{hp}} \left(\frac{1}{\alpha_h} + \frac{1}{\beta sr} \right) + \frac{r}{t_G - t_{hp}} \frac{r}{2\psi \varrho_{wL}} \left(\frac{1}{\lambda_{hl}} + \frac{RT}{D_e M_w sr} \right) \Delta \overline{m} \quad (23)$$

The regular region fits to determine approximately the drying time required for the prescribed moisture loss, with knowledge of the transport coefficients, just as for the determination of transport coefficients from measurement results. In principle, Eq. (23) enables the determination of one of the transport coefficients figuring in both terms on the right hand side of the equation. Since α_h and β are interdependent [1], these may be defined in any case, and so can be any of λ_{hI} and D_e too, if the other one is known.

3. Numerical solution

The differential equations (3) and (6) may be solved numerically by the aid of a computer, considering the boundary conditions (4), (5) and (7) and the prescribed initial conditions. To obtain the numerical solution, the method of finite differences, a modified variety of the so-called explicit method will be applied [3]. The grid, chosen for the numerical solution, is shown in Fig. 3.



Fig. 3. Grid applied for the numerical solution

Numerical computation, requires an equation system to satisfy the boundary condition (5) at point ξ_{τ} of the penetrating drying front, so that the location of the front should lie in a location-grid point. The penetration velocity of the front is, however, varying (decelerating), therefore either the distances between the front grid points, or the time steps must be variable. For practical computational reasons we have chosen the time steps $\Delta \tau$ to be variable and the front steps ΔZ to be constant.

The number of front steps must correspond to the number of time steps. Accordingly, the grid composed of front steps (auxiliary grid) is not suitable to serve as a location grid at the same time, since in this case no temperature change would be obtained for the wet zone behind the front.

The distances between the points of the location grid (main grid) must be large enough to display the temperature changes within the total wet front, even in case of a relatively poor penetration-rate of the front. The instantaneous location of the above mentioned auxiliary grid, forming one single new grid location will contribute to this main grid. The front grid point divides distribution n between the subsequent and the preceding main grid point in two (generally not equal) portions. Therefore, the derivatives involved in the solution procedure must be expressed in an unequally divided form too.

The plate of half thickness L was divided in k portions, illustrated in Fig. 3, this grid formed with respect to the locations being the so-called main grid. The width of the applied auxiliary grid was ΔZ and one main grid distance was divided in n portions.

According to the preceding considerations the distribution number of the auxiliary grid is:

$$\frac{L}{\Delta Z} = kn.$$

The distribution according to locations, is indicated by the lines i = const., at the main grid. The time distribution of the chosen grid is not constant $(\varDelta \tau \neq \text{const.})$, changing from time level to time level (the time level is indicated by the lines j = const.) in such a way that with the chosen time step $\varDelta \tau$ and with the penetration velocity of the front calculated at the former time level, the drying front will just remove to a distance $\varDelta Z$ at the end of the time step, and thus coincide with a location grid point (lying on an auxiliary grid, and from time to time on a main grid). — With the right choice of the distribution number n of the auxiliary grid, the stability of the explicit solution method might also be guaranteed for changing values of $\varDelta \tau$.

On the basis of the chosen grid, the number of time steps required for complete drying out $(\xi = L)$ will be kn.

According to the foregoing, the time level j = n also means this auxiliary grid location of order n where the drying front is just located at the given instant of time, therefore further on j will denote the number of the auxiliary grids.

Hence, the chosen grid represents:

	main grid locations:	$i=0, n, 2n, \ldots, kn;$
	grid distribution:	$n \Delta Z$
	auxiliary grid locations:	$j=0,1,2,\ldots,kn;$
	grid distribution:	ΔZ
	time grid locations:	$j=0,1,2,\ldots,kn;$
•	grid distribution:	$\Delta \tau \neq \text{const.}$
an	d if $j = n, 2n, \ldots, kn$, then	j=i.

The difference formulas of equal location distributions and unequal location distributions required for the numerical calculations, will be recapitulated in *Part II* of this paper. With knowledge of the difference formulas, the differential equations describing the boundary conditions may be transcribed to difference equations. The calculations are also contained in *Part II* of this paper. Moreover, the detailed description of the difference formulas required for calculation of the internal points of the dried region I and the wet region II, are included in *Part II*, too.

4. Approximations made for the penetration velocity of the front

At a given instant of time the penetration velocity of the front can be written as:



Fig. 4. Numerical computation of the penetration velocity of the front







This velocity is changing as a function of time. In the course of the numerical calculations this velocity is regarded as constant for each elementary time step. Let the penetration velocity of the front be

$$\left(\frac{dZ}{d\tau}\right)_{j=1}$$



Fig. 6b. Detail A2.

at the time level j - 1 and assume the penetration velocity of the front not to change until the time level j, i.e. until reaching the next auxiliary grid point Hence, time $\Delta \tau_j$ required to cover the auxiliary grid distribution ΔZ , can be determined, according to Fig. 4, as:

$$\Delta \tau_j = \Delta Z \frac{1}{\left(\frac{dZ}{d\tau}\right)_{j-1}}$$
(24)



Fig. 6c. Flow-chart of computation

The numerical calculation of the penetration velocity of the front follows in *Part II*.

The course of the numerical calculation is shown in Fig. 5 for the case n = 3 and k = 4, resp.

The flow-chart of the calculation is illustrated in Fig. 6; (Fig. 6a, Fig. 6b and Fig. 6c).

5. Numerical example

A tray dryer is used to dry wet, granular material. The depth of the tray (bed) is 8 cm; the drying time requirement and the drying velocity curve have to be defined for the case of convective drying. The drying air passes over the frontal surface of the bed which can be regarded as of infinite length, the bed is completely insulated everywhere else.

Initial condition:

$$t^{I}(z, 0) = t^{II}(z, 0) = t_{0}.$$

Drying air characteristics:

t_G	$= 45 [^{\circ}C]$
t _{hp}	$= 14.1 [^{\circ}C]$
α_h	$= 9 [kcal/m^2 h ^{\circ}C]$
r	= 570 [kcal/kg]
M_w	= 18 [kg/kmol]
β	= 1.12 [kg/m ² h atm]
c_{pwG}	= 0.46 [kcal/kg °C]
ϱ_{wL}	$= 1000 \ [kg/m^3]$

Characteristics of the material to be dried:

L		0.08 [m]
a_{hI}		0.00094 [m ² /h]
a_{bII}		$0.00152 [m^2/h]$
λ_{hI}		0.8 [kcal/m h °C]
λ_{hII}		1.75 [kcal/m h °C]
ψ		$0.2 \ [m^3/m^3]$
t_0		18 [°C]
\$	'=	0.00158 [atm/°C]
R	=	0.082 [atm $m^3/kmol^{\circ}K$]

For the numerical calculation the distribution numbers of the main grid and of the auxiliary grid, resp., have to be chosen in such a way — with the knowledge of the material characteristics — that the convergence and stability criteria of the calculation should be satisfied.

Since the time step is

$$\varDelta \tau_j = \frac{\varDelta Z}{v_{j-1}}$$

$$v_{j-1} = \left(rac{dZ}{d au}
ight)_{j=1}$$

 $\Delta \tau_1 = \frac{\Delta Z}{v_0}$

and

where

or rather, if only the main grid points are considered — as possible, since at each time level the auxiliary grid is going to be utilized only for calculating the points lying near to the front, we obtain

$$\Delta \tau' \leq \frac{1}{2a_{hII}} (n \Delta Z)^2.$$

After substitution:

$$egin{aligned} & \Delta au^{*} \leq rac{1}{2\ 0.00152}\ (0.01)^{2} = 0.0329\ [h] \ & \Delta Z_{
m max} = v_{0} \Delta au^{*} \ {
m or} \ \ \Delta Z_{
m max} = v_{
m final}\ \Delta au^{*} \ & \Delta Z_{
m max} = rac{1}{g} \left(rac{s}{rac{1}{eta} + Le}
ight) (t_{G} - t_{hp}) = 0.000007\ [m] \end{aligned}$$

and

$$n_{\min} = \frac{n \Delta Z}{\Delta Z_{\max}} = \frac{0.01}{0.000007} = 1428.7 \text{ [pcs]}.$$

Hence, for the number of distributions of the auxiliary grid: n = 2000, and for that one of the main grid: k = 8 has been considered.

The drying characteristics of the bed were determined numerically by a computer, as described in the foregoing. The temperature gradient of the bed as a function of the location at different instants of time is shown in Fig. 7, illustrating that for the completely dried portion I the temperature gradient is linear.

Fig. 8 represents temperature variations at several sites of fixed depth vs. time.

Changes of the penetration velocity of the front (proportional to the drying velocity) are shown in Fig. 9 as a function of time. On the basis of Fig. 9 it can be stated that the drying velocity decreases after the initial section, and that this decrease starts where the diffusion resistance of the dried region gets control over the drying phenomenon.

In the foregoing an approximate relationship has been derived for the determination of the drying time [see Eq. (21)]. The value of the function

$$\frac{\tau}{\xi} = f(\xi)$$

has been defined in the examined case, by both approximation and exact computer analysis. The results are shown in Fig. 10, indicating also the variation of the relative deviation of the approximate solution from the exact one; accordingly it can be stated that approximation causes a great error at the beginning, although offering a fair approximation for the determination of the time required for complete drying up of the bed; the deviation is only about 4°_{0} . The figure clearly demonstrates the development of a so-called regular region following the initial region of formation which is rather short.



Fig. 7. Temperature gradient of the bed at various instants of time



Fig. 8. Temperature changes of several planes of the bed

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Fig. 9. Changes of the penetration velocity of the front





6. Results of experimental measurements

Experiments were carried out too, in the described manner, for the drying of wet granular materials. In the pharmaceutical industry, drying of granular materials occur as a frequent task, subsequent to centrifugation when the materials are yet in a centrifuge-wet state. Chamber tray dryers are often used for this purpose in such a way that a drying air stream passes over the trays. Potassium asparaginate was applied for our experiments, the height of the bed being 33 [mm]. The average particle size of the wet granular material was 0.6 [mm]. The drying experiment was performed in the experimental dryer demonstrated in Fig. 11.



Fig. 11. Sketch of the experimental dryer

In the course of our investigations the state variables of the drying gas were measured and the weight loss of the bed too, as a function of time. The aim of our examinations was to determine the existence of a regular region obtained as the result of the described mathematical model. This conception was verified by plotting the function:

$$\frac{\tau}{\Delta \overline{m}} = f(\Delta \overline{m})$$

and it was seen to deviate from the function

$$\frac{\tau}{\xi} = f(\xi)$$

only by a single constant. The result is shown in Fig. 12.

Fig. 12 demonstrates that after the short region of formation, the function will turn linear. Hence, the regular region does factually exist so that the previous — numerical or approximate — analysis methods suit to determine the drying time requirement of such beds. The calculations impose to know some of the material characteristics of the granular material to be dried, and



Fig. 12. Results of the experimental measurements

the transfer coefficients of heat and mass transport, respectively. Also the unknown transport coefficients or material characteristics can be determine from the slope and axial section, resp., of the regular region, according to Eq. (23).

Notations

\boldsymbol{a}	[m²/h]	temperature conductivity
C	[kcal/kg °C]	specific heat
D	$[m^2/h]$	diffusion coefficient
F		surface
i	[pcs]	number of location steps
j	[pcs]	number of time steps
j_{w}	$[kg/m^2 h]$	drying velocity
k	[pcs]	number of location steps on the main grid
\boldsymbol{L}	[m]	thickness
m	[kg]	mass
n	[pcs]	distribution number of the auxiliary grid
P, p	[atm]	pressure
q	[kcal/m ² h]	heat flux density
r	[kcal/kg]	evaporation heat
R	[atmm ³ /kmol°K]	universal gas constant
s	[atm/°C]	slope of the tension curve
t	[°C]	temperature
v	[m/h]	penetration velocity of the front
V	[m ³]	volume
W	[kg]	mass of moisture
z, Z	[m]	distance

Greek letters

α	[kcal/m ² h °C]	heat transfer coefficient, specific
β	[kg/m² h atm]	mass transfer coefficient, specific
e	$[kg/m^3]$	density
λ	[kcal/m h °C]	thermal conductivity

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 $\begin{array}{l} \xi & [m] \\ \varphi & [m^3/m^3] \\ \tau & [h] \end{array}$

Subscripts

e	equilibrium state
ê	interface (intercontacting surface)
F	boundary surface of air and drying material
G	(essential) mass of air
h	temperature
hp	dew-point value
p^{-}	constant pressure
ms	material, solid
W, WL	water
WG	water vapor
0	(original) initial value
z	at location z
I	dried region (layer)
II	wet region (layer)

thickness

porosity time

Summary

Drying of wet (centrifuge-wet) granular materials, is a frequent task in industry. The bed established by such particles is macroporous. In the case of drying macroporous beds, the evaporating "plane" tends to penetrate into the bed, thus forming a double layer (i.e. where one of both layers is already dried up, while the other one is wet). In the course of the drying process the thickness of the layers is changing. In *Part I* of this paper an approximate, analytical relationship has been derived for the determination of the drying time of macroporous beds. — *Part II* will deal with the exact computer aided calculation method of the problem. Approximation and computer results are compared by means of a numerical example.

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