DRYING OF MACROPOROUS SYSTEMS — PART II

By

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Introduction

In *Part I* the drying of wet granular materials — an often occurring task in industry — was dealt with [1]. The drying mechanism of such macroporous beds has been described. In the case of drying macroporous beds, the evaporating "plane" penetrates into the bed, thus forming a bouble layer (i.e. two layers, one of which is already dried up, while the other one is wet). In the course of the drying process the thickness of the layers changes. An approximate analytical correlation has been developed for the determination of drying time.

In *Part II* an accurate, numerical, computer-aided calculation method of the problem and the equation systems required for computation have been summarized.

I. Definition of the difference formulas

Throughout the difference formulas, subscript i will be applied to denote changes with respect to main grid distribution. Main grid points i and $(i + 1)$ are at a distance $n\Delta Z$.

In our calculations of the examined time level, symbol ' will denote the new value, obtained at a subsequent instant of time $\Delta\tau_j$; e.g.:

t_i = the value at i, at the examined instant of time;
t'_i = the new value at i, at a subsequent time $\Delta\tau_j$.

1. *Difference formulas of even division with respect to location*

a) The derivative with respect to time at i:

$$\frac{\partial t}{\partial \tau} = \frac{t'_i - t_i}{\Delta\tau} . \tag{I. 1}$$

b) The first derivative with respect to location at i:

$$\frac{\partial t}{\partial z} = \frac{t_{i+1} - t_i}{n\Delta Z} . \tag{I. 2}$$

c) The second derivative with respect to location at i:

$$\frac{\partial^2 t}{\partial z^2} \approx \frac{t_{i+1} - 2\, t_i + t_{i-1}}{(n\Delta Z)^2} \tag{I. 3}$$

d) Definition of the first derivatives with respect to location considering boundary surfaces.

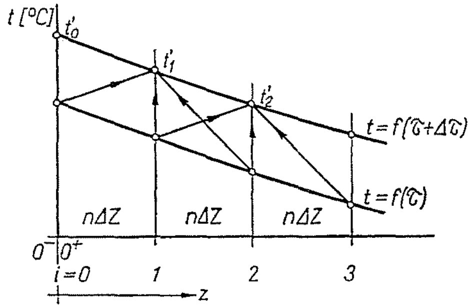

Fig. 1. Approximation of temperature distribution, with respect to location

In the case of boundary surfaces it is indispensable to know the temperature values at grid points both before and after the boundary surface. Assuming that the temperature gradient vs., time is a quadratic curve; on the basis of *Fig. 1.*

$$t = az^2 + bz + c.$$

Thus, at

$$z = 0:$$

$$t'_0 = c,$$

furthermore

$$\left.\frac{\partial t}{\partial z}\right|_{0+} = b$$

i.e.

$$t'_1 = a(n\Delta Z)^2 + b\,(n\Delta Z) + t'_0$$

and

$$t'_2 = a\, 4(n\Delta Z)^2 + b\, 2(n\Delta Z) + t'_0$$

respectively.
Hereof

$$b = \frac{-3t'_0 + 4t'_1 - t'_2}{2(n\Delta Z)} \tag{I. 4}$$

On the basis of Eq. (I. 4) and considering a grid evenly divided for the examined case, the derivatives with respect to the boundary surface will be the following:

α) for the drying surface, at $z = 0$, $i = 0$:

$$\left.\frac{\partial t}{\partial z}\right|_{0+} \approx \frac{-3t_0' + 4t_1' - t_2'}{2(n\Delta Z)}\,; \tag{I. 5}$$

β) for the median (symmetry) plane of the drying plate, at $z = L$, $i = k\,n$:

$$\left.\frac{\partial t}{\partial z}\right|_{L-} \approx \frac{3t_k' - 4t_{k-1}' + t_{k-2}'}{2(n\Delta Z)}\,. \tag{I. 6}$$

2. *Difference formulas of uneven division with respect to location*

At the instant of time $\tau = 0$, the drying front corresponds to the surface ($i = 0$, $z = 0$) and will later on take over the points of the auxiliary grid; the difference formulas vary, depending on the position of the front.

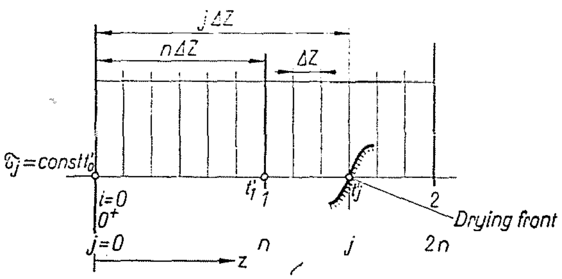

Fig. 2. Penetrating front, between first and second main grid point

α) If the front lies between the surface and the first main grid point i.e. for the case $0 < j < n$, linear approximation has to be applied, namely

$$\left.\frac{\partial t}{\partial z}\right|_{0+} = \frac{t_j' - t_0'}{j\Delta Z}\,. \tag{I. 7}$$

β) If the front lies between the first and the second main grid point and $n < j \leq 2n$, *Fig.* 2 shows that a closer approximation of second degree may be applied for the determination of the derivative with respect to the boundary surface, since the 3 known points required: namely t_0', t_1' and t_j' are provided here. Then, for the case $z = 0$:

$$t_0' = c,$$

and furthermore

$$t_1' = a(n\Delta Z)^2 + b(n\Delta Z) + t_0'$$

and

$$t_j' = a(j\Delta Z)^2 + b(j\Delta Z) + t_0'$$

respectively; whereof:

$$b = \frac{-(j^2 - n^2)t_0' + j^2 t_1' - n^2 t_j'}{(j-n)\, n\, j\, \Delta Z} .$$

Hence, the derivative with respect to the boundary surface as location may be written as:

$$\left.\frac{\partial t}{\partial z}\right|_{0+} \approx \frac{-(j^2 - n^2)t_0' + j^2 t_1' - n^2 t_j'}{(j-n)\, n\, j\, \Delta Z} . \tag{I. 8}$$

For the case $j = 2n$, the correlation (I. 8) passes over to Eq. (I. 5), the front being at the main grid point, and the formula for even division has to be applied.

γ) For the case $j > 2n$, the derivative with respect to the boundary surface may be computed by means of even division from known values at the locations $j = 0$, n and $2n$.

δ) For the case $(k-2)\, n \leq j < (k-1)\, n$, the difference formula can be determined according to *Fig. 3*

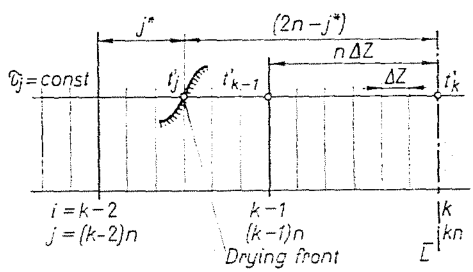

Fig. 3. Penetrating front, in the position precedent to the median (symmetry) plane

Introducing the following term according to *Fig. 3*:

$$j^* = j - (k-2)\, n$$

considerations similar to the foregoing lead to: (also on the basis of *Fig. 3*)

$$\left.\frac{\partial t}{\partial z}\right|_{L-} = b = \frac{-[(2n - j^*)^2 - n^2]t_k' + (2n - j^*)^2 t_{k-1}' - n^2 t_j'}{n(2n - j^*)(j^* - n)\Delta Z} . \tag{I.9}$$

If $j = (k-2)\, n$, then $t_j' = t_{(k-2)\cdot n} = t_{k-2}$ and the main grid point $j^* = 0$ and the correlation (I.9) turns into Eq. (I.6).

ε) Determination of the first derivatives with respect to location, for the case of the penetrating front:

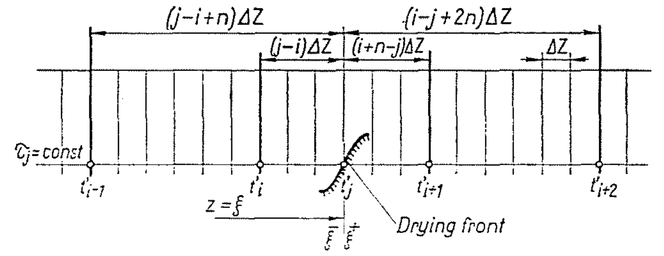

Fig. 4. Moving (penetrating) front, in a general position

The penetrating front in general position is illustrated in *Fig. 4.*

If

$$n < j < 2n, \qquad \text{then } i = n$$

$$2n < j < 3n, \qquad \text{then } i = 2n$$

and

$$(k-2)\,n < j < (k-1)\,n, \qquad \text{then } i = (k-2)\,\mathrm{n}.$$

Accepting the approximation of second degree:

$$\left.\frac{\partial t}{\partial z}\right|_{\xi-} = \frac{(j-i)^2 t'_{i-1} - (j-i+n)^2 t'_i + n[2(j-i)+n]\,t'_j}{n(j-i+n)\,(j-i)\,\Delta Z} \tag{I.10}$$

and

$$\left.\frac{\partial t}{\partial z}\right|_{\xi+} = \frac{-n[2(\mathrm{i}-j)+3n]\,t'_j + (i+2n-j)^2 t'_{i+1} - (i+\mathrm{n}-\mathrm{j})^2 t'_{i+2}}{n(i+n-j)\,(i+2n-j)\,\Delta Z}\,. \tag{I.11}$$

If $j = 2n, 3n, \ldots (k-2)n$, namely if the front is situated at internal main grid points, then the derivatives may be computed by means of even division formulas.

ζ) Approximation of second derivatives with respect to location:
A general position is shown in *Fig.* 5, resulting for region I in the following correlation:

$$\frac{\partial^2 t}{\partial z^2} = 2\,\frac{(j-1-i)t_{i-1} - (n+j-i-1)t_i + n\,t_{j-1}}{n(j-1-i)\,(n+j-1-i)\,\Delta Z^2}\,. \tag{I.12}$$

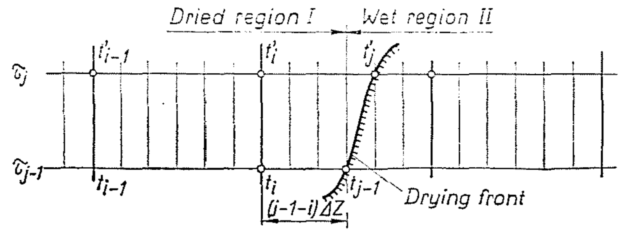

Fig. 5. Grid for approximation of second derivatives, with respect to location

Notes:

1. For the case $0 < j < n$, no internal point exists in region I, hence correlation (I. 12) cannot be applied;

2. for the case $j = n + 1$, $i = n$ (even division) the front meets the main grid point, and similarly, at time levels

$$j = 2n + 1,\ 3n + 1,\ \ldots (k - 1)n + 1;$$

3. for the case $n + 2 \leq j \leq 2n$, $i = n$, according to Eq. (I. 12), and for

$$2n + 2 \leq j \leq 3n, \qquad i = 2n, \text{ according}$$

to Eq. (I. 12) and furthermore for

$$(k - 1)n + 2 \leq j \leq k\,n, \qquad i = (k - 1)n$$

Eq. (I. 12) can be applied too.

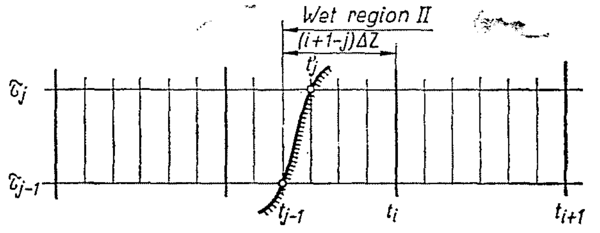

Fig. 6. Notations applied for the wet region *II*

A general position within the wet region *II* is shown in *Fig. 6*,

$$\frac{\partial^2 t}{\partial z^2} \approx 2\,\frac{n t_{j-1} - (i+1-j+n) t_i + (i+1-j)\, t_{i+1}}{n(n + i + 1 - j)\,(i + 1 - j)\,\Delta Z^2}\ . \tag{I.13}$$

Notes:

For the cases:

$$\begin{array}{ll} 0 < j \leq n, & i = n \\ n + 2 \leq j \leq 2n, & i = 2n \\ (k - 2)n + 2 \leq j \leq (k - 1)n, & i = (k - 1)n \end{array}$$

the correlation (I. 13) may be applied.

For $j = n + 1,\ 2n + 1, \ldots\ (k-2)n + 1$, the even division formula may be applied.

If $j > (k-1)\,n$, there is no internal point within region II.

II. Boundary conditions expressed by differential equations

The boundary condition (4) in *Part I* is written as

$$\alpha_h(t_G - t_F) = -\lambda_{hI} \left.\frac{\partial t}{\partial z}\right|_{0+} .$$

1. For the case $0 < j \leq n$ and with the application of Eq. (I. 7):

$$\alpha_h(t_G - t_0') = -\lambda_{hI} \frac{t_j' - t_0'}{j\Delta Z} .$$

Herefrom

$$t_0' = \frac{j\Delta Z\, \alpha_h t_G + \lambda_{hI}\, t_j'}{j\Delta Z\, \alpha_h + \lambda_{hI}} \;; \qquad \text{(II.1)}$$

2. for the case $n < j < 2n$ and with the application of Eq. (I. 8),

$$t_0' = \frac{nj(j-n)\, \Delta Z\, \alpha_h\, t_G + \lambda_{hI}\, j^2\, t_1' - \lambda_{hI}\, n^2\, t_j'}{\lambda_{hI}\,(j^2 - n^2) + \alpha_h\, n\, j(j-n)\Delta Z} \;; \qquad \text{(II.2)}$$

3. for the case $2n \leq j \leq k\, n$ and with the application of Eq. (I. 5),

$$t_0' = \frac{2\, n\, \Delta Z\, \alpha_h\, t_G + 4\, \lambda_{hI}\, t_1' - \lambda_{hI}\, t_2'}{3\, \lambda_{hI} + 2\, n\, \Delta Z\, \alpha_h} \;. \qquad \text{(II.3)}$$

Introducing terms:

$$d = \frac{1}{\beta} \quad \text{and} \quad e = \frac{RT}{D_e\, M_w}$$

permits to write the boundary condition (5) in *Part I* with respect to the front, as

$$-\lambda_{hI} \left.\frac{\partial t^{I}}{\partial z}\right|_{\xi-} = -\lambda_{hII} \left.\frac{\partial t^{II}}{\partial z}\right|_{\xi+} + \frac{s}{d + \xi e}\,(t_\xi - t_{hp})\, r .$$

1. For the case $0 < j < n$ and with the application of Eqs (I. 7) and (I. 11):

$$-\lambda_{hI} \frac{t_j' - t_0'}{j\Delta Z} = -\lambda_{hII} \frac{-n\,[2\,(i-j) + 3n]\,t_j' + (i+2n-j)^2\, t_{i+1}' - (i+n-j)^2\, t_{i+2}'}{n(i+n-j)\,(i+2n-j)\,\Delta Z} + $$

$$+ \frac{s}{d + j\Delta Z\, e}\,(t_j' - t_{hp})\, r$$

and if $j < n$ then $i = 0$; and substituting Eq. (II. 1) for t_0' yields

$$t_j' = \frac{\dfrac{\lambda_{hI} C_0 j \Delta Z \alpha_h t_G}{j \Delta Z \alpha_h + \lambda_{hI}} + \dfrac{s C_0 j \Delta Z r}{d + j \Delta Z e} t_{hp} + \lambda_{hII} j (2n-j)^2 t_1' - \lambda_{hII} j (n-j)^2 t_2'}{\lambda_{hI} C_0 - \dfrac{\lambda_{hI}^2 C_0}{j \Delta Z \alpha_h + \lambda_{hI}} + \lambda_{hII} j n (3n-2j) + \dfrac{s C_0 r j \Delta Z}{d + j \Delta Z e}} \tag{II.4}$$

where

$$C_0 = n(n-j)(2n-j);$$

2. for the case: $j = n$ (when the front is located at the first main grid point), considering Eq. (I. 5), the boundary condition can be written as

$$-\lambda_{hI} \frac{t_1' - t_0'}{n \Delta Z} = -\lambda_{hII} \frac{-3t_1' + 4t_2' - t_3'}{2n \Delta Z} + \frac{s}{d + n \Delta Z e} (t_1' - t_{hp}) r$$

and the substitution of t_0' by Eq. (II. 1) gives

$$t_1' = \frac{\dfrac{2 \lambda_{hI} n \Delta Z \alpha_h t_G}{n \Delta Z \alpha_h + \lambda_{hI}} + \dfrac{2 s n \Delta Z r t_{hp}}{d + n \Delta Z e} + 4 \lambda_{hII} t_2' - \lambda_{hII} t_3'}{2 \lambda_{hI} - \dfrac{2 \lambda_{hI}^2}{n \Delta Z \alpha_h + \lambda_{hI}} + 3 \lambda_{hII} + \dfrac{2 s n \Delta Z r}{d + n \Delta Z e}}; \tag{II.5}$$

3. for the case $n < j < 2n$ with the substitution of Eqs (I. 10) and (I. 11) and considering that

$$i = n \text{ and } t_n' = t_1'$$

the boundary condition will be:

$$\begin{aligned} -\lambda_{hI} \frac{(j-n)^2 t_0' - j^2 t_1' + n(2j-n) t_j'}{n(j-n) j \Delta Z} &= \\ = -\lambda_{hII} \frac{-n(5n-2j) t_j' + (3n-j)^2 t_2' - (2n-j)^2 t_3'}{n(2n-j)(3n-j) \Delta Z} &+ \\ + \frac{s}{d + j \Delta Z e} (t_j' - t_{hp}) r . & \end{aligned}$$

Introducing:

$$C_1 = j(j-n)$$
$$C_2 = (2n-j)(3n-j)$$

and substituting Eq. (II. 2) for t_0', the correlation can be solved and yields

$$t_j' = \frac{1}{u_1}\left\{ \frac{sC_1 C_2 n\Delta Z\, rt_{hp}}{d + j\Delta Ze} - \frac{\lambda_{hI} C_1 C_2 n(j-n)^2 \Delta Z \alpha_h t_G}{\lambda_{hI}(j^2 - n^2) + \alpha_h n C_1 \Delta Z} + \right.$$
$$+ \left[\lambda_{hI} j^2 C_2 - \frac{\lambda_{hI}^2 C_2 (j-n)^2 j^2}{\lambda_{hI}(j^2 - n^2) + \alpha_h n C_1 \Delta Z}\right] t_1 + \tag{II.6}$$
$$\left. + \lambda_{hII} C_1 (3n - j)^2 t_2' - \lambda_{hII} C_1 (2n - j)^2 t_3' \right\}$$

where

$$u_1 = \lambda_{hI} C_2 n(2j - n) - \frac{\lambda_{hI}^2 C_2 n^2 (j-n)^2}{\lambda_{hI}(j^2 - n^2) + \alpha_h n C_1 \Delta Z} +$$
$$+ \lambda_{hII} C_1 n(5n - 2j) + \frac{sC_1 C_2 n \Delta Z\, r}{d + j\Delta Ze};$$

4. for the case $j = 2n$, on the basis of Eqs (I. 5) and (I. 6), the boundary condition becomes:

$$-\lambda_{hI} \frac{t_0' - 4t_1' + 3t_2'}{2n\Delta Z} = -\lambda_{hII} \frac{-t_4' + 4t_3' - 3t_2'}{2n\Delta Z} +$$
$$+ \frac{s}{d + 2n\Delta Ze}(t_2' - t_{hp})\, r$$

replacing t_0' by Eq. (II. 3) and rearranging:

$$t_2' = \frac{1}{u_2}\left\{ \frac{2\, sn\Delta Zr\, t_{hp}}{d + 2n\Delta Ze} - \frac{2\lambda_{hI} n\Delta Z \alpha_h t_G}{3\lambda_{hI} + 2n\Delta Z\alpha_h} + \right.$$
$$\left. + \left[4\lambda_{hI} - \frac{4\lambda_{hI}^2}{3\lambda_{hI} + 2n\Delta Z\alpha_h}\right] t_1' + 4\lambda_{hII} t_3' - \lambda_{hII} t_2' \right\}; \tag{II.7}$$

where

$$u_2 = 3\lambda_{hI} - \frac{\lambda_{hI}^2}{3\lambda_{hI} + 2n\Delta Z\alpha_h} + 3\lambda_{hII} + \frac{2s\, n\, \Delta Z\, r}{d + 2n\Delta Ze};$$

5. for the case $2n < j < (k-2)n$ and with the application of Eqs (I. 10) and (I. 11) and with the following considerations, namely

$$\begin{array}{lll} 2n < j < 3n, & \text{then} & i = 2n \\ 3n < j < 4n, & \text{then} & i = 3n \\ \vdots & & \vdots \\ (k-3)n < j < (k-2)n & & i = (k-3)n \end{array}$$

the boundary condition becomes:

$$-\lambda_{hI}\frac{(j-i)^2 t'_{i-1}-(j-i+n)^2 t'_i+n[2(j-i)+n]t'_j}{n(j-i+n)(j-i)\Delta Z}=$$

$$=-\lambda_{hII}\frac{-n[2(i-j)+3n]\,t'_j+(i+2n-j)^2 t'_{i+1}-(i+n-j)^2 t'_{i+2}}{n(i+n-j)(i+2n-j)\Delta Z}+$$

$$+\frac{s}{d+j\Delta Ze}(t'_j-t_{hp})\,r\,.$$

Introducing

$$C_3=(j-i+n)(j-i)$$

$$C_4=(i+n-j)(i+2n-j)$$

and solving with respect to t'_j yields:

$$t'_j=\frac{1}{u_3}\left\{\frac{sC_3C_4n\Delta Zr}{d+j\Delta Ze}t_{hp}-\lambda_{hI}C_4(j-i)^2t'_{i-1}+\lambda_{hI}C_4(j-i+n)^2t'_i+\right.$$
$$\left.+\lambda_{hII}C_3(i+2n-j)^2t'_{i+1}-\lambda_{hII}C_3(i+n-j)^2t'_{i+2}\right\};\qquad\text{(II.8)}$$

where

$$u_3=\lambda_{hI}C_4n[2(j-i)+n]+\lambda_{hII}C_3n[2(i-j)+3n]+\frac{sC_3C_4n\Delta Zr}{d+j\cdot\Delta Ze};$$

6. for the case $(k-2n)<j<(k-1)n$, $i=(k-2)n$, thus $t'_{i+2}=t'_k$. The unknown term t'_j may be computed on the basis of Eq. (II. 8), but first t'_k must be determined on the basis of Eq. (II. 16);

7. for the case $(k-1)n<j<k\,n$ the characteristics known are seen in *Fig. 7.*

Accepting the approximation of second degree for the temperature distribution yields

$$\left.\frac{\partial t^{II}}{\partial z}\right|_{\xi+}\approx 2\frac{t'_k-t'_j}{(kn-j)\Delta Z}$$

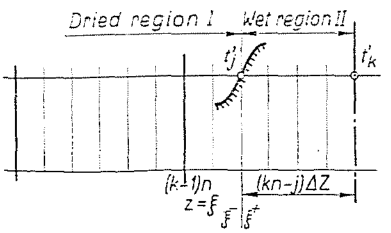

Fig. 7. Penetrating front, close to the median (symmetry) plane

and since $i = (k-1)\,n$, further with the application of Eq. (I. 10), the boundary condition can be written as

$$-\lambda_{hI}\frac{(j-i)^2 t'_{i-1} - (j-i+n)^2 t'_i + n[2(j-i)+n]\,t'_j}{n(j-i+n)(j-i)\Delta Z} =$$
$$= -\lambda_{hII}\frac{2(t'_k - t'_i)}{(kn-j)\Delta Z} + \frac{s}{d+j\Delta Ze}(t'_j - t_{hp})\,r$$

and t'_k may be computed on the basis of the second boundary condition, according to Eq. (II. 17).
Herefrom:

$$t'_j = \frac{1}{u_4}\left\{\frac{sC_3\,n\Delta Z(kn-j)r}{d+j\Delta Ze}t_{hp} - \lambda_{hI}(kn-j)(j-i)^2 t'_{i-1} + \right.$$
$$\left. + \lambda_{hI}(kn-j)(j-i+n)^2 t'_i + 2\,\lambda_{hII}\,nC_3\,t'_k\right\}; \qquad \text{(II.9)}$$

where

$$u_4 = \lambda_{hI}(kn-j)\,n\,[2(j-i)+n] + 2\lambda_{hII}\,nC_3 + \frac{sC_3\,n(kn-j)\,\Delta Zr}{d+j\Delta Ze};$$

8. for the case $j = 3n,\ 4n,\ \ldots\ (k-2)\,n$, the drying front will coincide with a main grid point, therefore the even division difference formulas may be applied.

Then, if $j = 3n \rightarrow i = 3n$
$j = 4n \rightarrow i = 4n$
etc. $\vdots \qquad \vdots$
$j = (k-2)n \rightarrow i = (k-2)n$

and the boundary condition can be written as:

$$-\lambda_{hI}\frac{t'_{i-2} - 4t'_{i-1} + 3t'_i}{2n\Delta Z} = -\lambda_{hII}\frac{-3t'_i + 4t'_{i+1} - t'_{i+2}}{2n\Delta Z} + \frac{s}{d+j\Delta Ze}(t'_i - t_{hp})\,r\,,$$

wherefrom

$$t'_i = \frac{\dfrac{2sn\Delta Zr}{d+j\Delta Ze}t_{hp} - \lambda_{hI}\,t'_{i-2} + \lambda_{hI}4t'_{i-1} + 4\lambda_{hII}\,t'_{i+2} - \lambda_{hII}\,t'_{i+2}}{3\lambda_{hI} + 3\lambda_{hII} + \dfrac{2sn\Delta Zr}{d+j\Delta Ze}}; \qquad \text{(II.10)}$$

9. for the case $j = (k-1)n$, the drying front will meet the last internal main grid point.

Then, on the basis of *Fig.* 7 and since

$$i = (k-1)n \text{ and } t'_i = t'_{k-1};$$

the boundary condition can be written as:

$$-\lambda_{hI}\frac{t'_{k-3}-4t'_{k-2}+t'_{k-1}}{2n\Delta Z} = -\lambda_{hII}\,2\,\frac{t'_k - t'_{k-1}}{n\Delta Z} + \frac{s}{d+j\Delta Ze}(t'_{k-1}-t_{hp})\,r$$

wherefrom

$$t'_{k-1} = \frac{\dfrac{2sn\Delta Zr}{d+j\Delta Ze}t_{hp} - \lambda_{hI}\,t'_{k-3} + \lambda_{hI}\,4\,t'_{k-2} + 4\lambda_{hII}\,t'_k}{3\lambda_{hI} + 4\lambda_{hII} + \dfrac{2sn\Delta Zr}{d+j\Delta Ze}}\,. \tag{II.11}$$

Formulation of the boundary condition ((7) in *Part I*) at the median (symmetry) plane:

$$\left.\frac{\partial t^{II}}{\partial z}\right|_{L-} = 0\,.$$

1. For the case $0 \leq j \leq (k-2)n$, two main grid points are provided to the left of the symmetry axis and thus the even division difference formulas may be applied. According to Eq. (I. 6),

$$\left.\frac{\partial t^{II}}{\partial z}\right|_{L-} \approx \frac{3t'_k - 4t'_{k-1}+t'_{k-2}}{2\cdot(n\Delta Z)} = 0\,.$$

Herefrom:

$$t'_k = \frac{4t'_{k-1} - t'_{k-2}}{3}\,; \tag{II.12}$$

2. for the case $(k-2)n<j<(k-1)n$, with respect to Eq. (I. 9),

$$t'_k = \frac{(2n-j^*)^2\,t'_{k-1} - n^2\,t'_j}{(2n-j^*)^2 - n^2} \tag{II.13}$$

where

$$j^* = j-(k-2)n;$$

3. for the case $j=(k-1)n$

$$\left.\frac{\partial t^{II}}{\partial z}\right|_{L-} = \frac{t'_k - t'_{k-1}}{n\Delta Z} = 0\,,$$

wherefrom

$$t'_k = t'_{k-1}; \tag{II.14}$$

4. for the case $(k-1)n < j \leq k\, n$

$$\left.\frac{\partial t^{II}}{\partial z}\right|_{L-} = \frac{t'_k - t'_j}{(kn-j)\Delta Z} = 0\,,$$

wherefrom

$$t'_k = t'_j \tag{II.15}$$

The boundary conditions at the median (symmetry) plane may be formulated on the basis of symmetry too, according to *Fig.* 8.

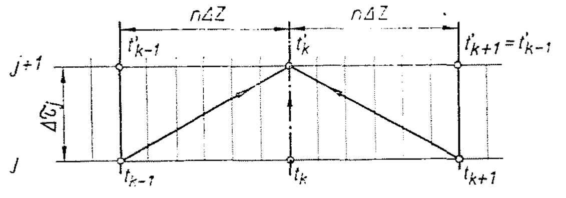

Fig. 8. Calculation of the temperature of the median (symmetry) plane

5. For the case $0 < j \leq (k-1)n$, the differential equation ((6) in *Part I*) transcribed to difference equation, yields

$$\frac{t'_k - t_k}{\Delta\tau_j} = a_{hII}\frac{2}{n^2\Delta Z^2}\,(t_{k-1} - t_k)\,.$$

Herefrom

$$t'_k = t_k + \frac{2a_{hII}\,\Delta\tau_j}{(n\Delta Z)^2}\,(t_{k-1} - t_k)\,; \tag{II.16}$$

6. for the case $(k-1)n < j \leq k\, n$, on the basis of *Fig.* 9

$$t'_k = t_k + \frac{2a_{hII}\,\Delta\tau_j}{(kn-j)^2\Delta Z^2}\,(t_j - t_k)\,. \tag{II.17}$$

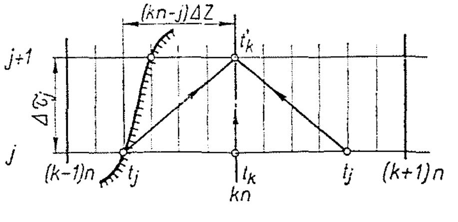

Fig. 9. Evaporating "plane", close to the median (symmetry) plane

III. Determination of the penetration velocity of the front

Let the penetration velocity be

$$\frac{dz}{d\tau} = v$$

of a value changing from point to point.

The drying velocity is written as:

$$\dot{j}_w = g\frac{dz}{d\tau} = gv = \frac{m}{d+Ze}(t_z - t_{hp}),$$

where

$$d = \frac{1}{\beta} = \frac{1}{K_G M_W}$$

and

$$e = \frac{RT}{D_e M_W}.$$

At the initial instant of time, the drying velocity may be written as

$$gv_0 = \frac{m}{d}(t_0 - t_{hp}) \qquad \text{(III.1)}$$

if

$$\tau = 0, \ j = 0.$$

At the instant of time $j = 1$

$$gv_1 = \frac{m}{d+\Delta Ze}(t_j - t_{hp})$$

and at $j = j$

$$gv_j = \frac{m}{d+j\Delta Ze}(t_j - t_{hp}). \qquad \text{(III.2)}$$

In the numerical computation the penetration velocity may be regarded as constant during the period $\Delta\tau$ and equal to its value at the beginning of the time step, since the computation method has been chosen so that the time step $\Delta\tau$ is always determined by that portion ΔZ of the drying front which penetrates into the material at the end of the time step, thus reaching an auxiliary grid point.

On the basis of the foregoing, we obtain:

$$\Delta\tau_j = \Delta Z \frac{1}{v_{j-1}} \tag{III.3}$$

namely

$$j = 0 \quad \Delta\tau = 0 \,,$$

and

$$j = 1 \quad \Delta\tau_1 = \frac{\Delta Z}{v_0} \,.$$

IV. Computation of the internal points of the dried region

The heat-balance of the dried region yielded the differential equation (3) in *Part I*. Transforming it to a difference equation, the internal points may be determined numerically.

1. For the case $0 < j \leq n$, there in no internal point (main grid point) for the material I yet;

2. for the case $j = n + 1$, only two points occur within the dried material, thus the approximation of the second derivative with respect to location cannot be carried out in the usual way.

The temperature change at z:

$$dt_z = \left(\frac{\partial t}{\partial \tau}\right)_z d\tau + \left(\frac{\partial t}{\partial z}\right)_z dz \,.$$

Herefrom

$$\frac{dt_z}{dz} = \left(\frac{\partial t}{\partial \tau}\right)_z \frac{1}{\dfrac{dz}{d\tau}} + \left(\frac{\partial t}{\partial z}\right)_z .$$

Location z may be constidered as the common point of both the dried region I and the wet region II, therefore with the application of Eqs (3) and (6) in *Part I* we obtain:

$$\left(a_{hI} \frac{\partial^2 t^I}{\partial z^2} + b \frac{dz}{d\tau} \frac{\partial t^I}{\partial z}\right) \frac{1}{\dfrac{dz}{d\tau}} + \frac{\partial t^I}{\partial z} = a_{hII} \frac{\partial^2 t^{II}}{\partial z^2} \frac{1}{\dfrac{dz}{d\tau}} + \frac{\partial t^{II}}{\partial z} \,.$$

Herefrom

$$\frac{\partial^2 t^I}{\partial z^2} = \frac{a_{hII}}{a_{hI}} \frac{\partial^2 t^{II}}{\partial z^2} + \frac{dz}{d\tau}\left[\frac{\partial t^{II}}{\partial z} - (1+b)\frac{\partial t^I}{\partial z}\right]\frac{1}{a_{hI}} \tag{IV.1}$$

and the substitution of Eq. (IV. 1) into Eq. (3) in *Part I* gives:

$$\frac{\partial t^{I}}{\partial \tau} = a_{hII}\frac{\partial^2 t^{II}}{\partial z^2} + \frac{dz}{d\tau}\left[\frac{\partial^2 t^{II}}{\partial z} - (1+b)\frac{\partial t^{I}}{\partial z}\right] + b\frac{dz}{d\tau}\,\frac{\partial t^{I}}{\partial z}. \qquad \text{(IV.2)}$$

Transcribing Eq. (IV. 2) to a difference equation, at the instant of time $j = n + 1$ and at the location $i = n$ — at the first main grid point — the temperature variation can be expressed by

$$\frac{t_i'-t_i}{\Delta\tau_j} = a_{hII}\,\frac{t_i - 2t_{i+1} + t_{i+2}}{(n\Delta Z)^2} + v_{j-1}\left(\frac{t_{i+1} - t_i}{n\Delta Z} - \frac{t_i - t_{i-1}}{n\Delta Z}\right). \qquad \text{(IV.3)}$$

Substituting Eq. (IV. 3) and rearranging gives

$$\begin{aligned} t_i' = {} & \frac{1}{n}t_{i-1} + \left[\frac{a_{hII}\,\Delta\tau_j}{(n\Delta Z)^2} + 1 - \frac{2}{n}\right]t_i + \\ & + \left[\frac{1}{n} - \frac{2a_{hII}\,\Delta\tau_j}{(n\Delta Z)^2}\right]t_{i+1} + \frac{a_{hII}\,\Delta\tau_j}{(n\Delta Z)^2}\,t_{i+2}. \end{aligned} \qquad \text{(IV.4)}$$

Similarly, if $j = (k-1)n + 1$, then at $i = (k-1)n$ and because of the symmetry condition $t_i = t_{i+2}$, we obtain

$$t_i' = \frac{1}{n}t_{i-1} + \left[\frac{2\,a_{hII}\,\Delta\tau_j}{(n\Delta Z)^2} + 1 - \frac{2}{n}\right]t_i + \left[\frac{1}{n} - \frac{2a_{hII}\,\Delta\tau_j}{(n\Delta Z)^2}\right]t_{i+1}. \qquad \text{(IV.5)}$$

3. Points, determinable by even division, with respect to location, applying Eqs (I. 1), (I. 2) and (I. 3)

— for the time steps $2n + 1 \leq j \leq 3n$, at $i = n$;

— for the time steps $3n + 1 \leq j \leq 4n$, at $i = n, 2n$, etc.

⋮

— for the time steps $(k-1)n + 1 \leq j \leq k\,n$,

at $i = n, 2n, 3n, \ldots\ (k-2)n$; respectively, Eq. (3) in *Part I* can be rewritten as

$$\frac{t_i' - t_i}{\Delta\tau_j} = a_{hI}\,\frac{t_{i+1} - 2t_i + t_{i-1}}{(n\Delta Z)^2} + bv_{j-1}\,\frac{t_{i+1} - t_i}{n\Delta Z}.$$

Substituting Eq. (III. 3) and rearranging, yields:

$$t_i' = \frac{a_{hI}\,\Delta\tau_j}{(n\Delta Z)^2}\,t_{i-1} + \left[1 - \frac{b}{n} - \frac{2a_{hI}\,\Delta\tau_j}{(n\Delta Z)^2}\right]t_i + \left[\frac{a_{hI}\Delta\tau_j}{(n\Delta Z)^2} + \frac{b}{n}\right]t_{i+1}. \qquad \text{(IV.6)}$$

4. Points, determinable by uneven division, applying Eqs (I. 1), (I. 2) and (I. 12),

— for time steps $n+2 \leq j \leq 2n$, at $i = n$

— for time steps $2n+2 \leq j \leq 3n$, at $i = 2n$

$\vdots \qquad \vdots$

— for time steps $(k-1)n+2 \leq j \leq kn$, at $i=(k-1)n$, respectively, Eq. (3) in *Part I* becomes:

$$\frac{t_i' - t_i}{\Delta\tau_j} = 2a_{hI}\frac{(j-1-i)\,t_{i-1} - (n+j-i-1)\,t_i + nt_{j-1}}{n\,(j-1-i)\,(n+j-1-i)\,\Delta Z^2} + \\ + bv_{j-1}\frac{t_{j-1}-t_i}{(j-1-i)\,\Delta Z}\,.$$

Substituting the term v_{j-1} by Eq. (III. 3) and rearranging gives:

$$t_i' = \frac{2\,a_{hI}\,\Delta\tau_j}{n(n+j-1-i)\Delta Z^2}\,t_{i-1} + \left[1 - \frac{b}{(j-1-i)} - \frac{2a_{hI}\,\Delta\tau_j}{n(j-1-i)\Delta Z^2}\right]t_i + \\ + \left[\frac{2a_{hI}\,\Delta\tau_j}{(j-1-i)\,(n+j-1-i)\,\Delta Z^2} + \frac{b}{(j-1-i)}\right]t_{j-1}. \qquad \text{(IV.7)}$$

5. For time step $j = 2n+1$, at $i = 2n$

Eq. (IV. 4) can be applied; 2 grid points being already provided at the former time step, the even division formula could be applied too.

The equations to be applied in different cases are as follows:

— For the time step $0 < j \leq n$ no internal main grid point occurs;

— for the time step $j = n+1$, at $i = n$
  t_n' can be computed according to Eq. (IV. 4);

— for the time step $n+2 \leq j \leq 2n$, at $i = n$
  t_i' can be computed according to Eq. (IV. 7);

— for the time step $j = 2n+1$, at $i = 2n$
  t_i' can be computed according to Eq. (IV. 4);

— for the time step $2n+2 \leq j \leq 3n$, at $i = 2n$
  t_i' can be computed according to Eq. (IV. 7);

$\vdots$

— for the time step $j = (k-1)n+1$, at $i = (k-1)n$
  t_i' can be computed according to Eq. (IV. 5);

— for the time step $(k-1)n+2 \leq j \leq kn$, at $i = (k-1)n$
  t_i' can be computed according to Eq. (IV. 7);

— for the time step $2n+1 \leq j \leq 3n$, at $i = n$

t_i' can be computed according to Eq. (IV. 6);

— for the time step $3n + 1 \leq j \leq 4n$, at $i = n, 2n$
t_i' can be computed according to Eq. (IV. 6);

$\vdots$

— for the time step $(k-1)n + 1 \leq j \leq kn$,
at $i = n, 2n, 3n \ldots (k-2)n$
t_i' can be computed according to Eq. (IV. 6).

V. Calculation of the internal points of the wet region

The temperature variation of the material with respect to time can be determined by means of the enthalpy balance of the wet region, Eq. (6) in *Part I*. If differential equation (6), *Part I*, becomes transformed into a difference formula, a distinction must be made between the applicability of even and uneven division formulas.

1. The internal points, computable by even division with respect to location, are as follows:

— for time levels $0 < j \leq n + 1$,
at $i = 2n, 3n \ldots (k-1)n$

— for time levels $n + 2 \leq j \leq 2n + 1$,
at $i = 3n, 4n, \ldots (k-1)n$

$\vdots$

— for the time levels $(k-3)n + 2 \leq j \leq (k-2)n + 1$,
at $i = (k-1)n$

$$\frac{t_i' - t_i}{\Delta\tau_j} = a_{hII} \frac{t_{i+1} - 2\,t_i + t_{i-1}}{(n\Delta Z)^2}\,.$$

Herefrom

$$t_i' = \frac{a_{hII}\,\Delta\tau_j}{(n\Delta Z)^2}\,t_{i-1} + \left[1 - \frac{2a_{hII}\,\Delta\tau_j}{(n\Delta Z)^2}\right]t_i + \frac{a_{hII}\,\Delta\tau_j}{(n\Delta Z)^2}\,t_{i+1} \tag{V.1}$$

may be obtained.

2. The internal points — first points after the front — computable by uneven division with respect to location, are as follows:

— for the time levels $0 < j \leq n + 1$, at $i = n$

— for the time levels $n + 2 \leq j \leq 2n + 1$, at $i = 2n$

$\vdots$

— for the time levels $(k-2)n + 2 \leq j \leq (k-1)n$,
at $i = (k-1)n$

while after time step $j > (k-1)n$, no internal main grid point will occur in the material *II*.

With the application of Eq. (I. 13) the following correlation can be obtained:

$$\frac{t_i' - t_i}{\Delta\tau_j} = 2a_{hII} \frac{nt_{j-1} - (i+1-j+n)\,t_i + (i+1-j)\,t_{i+1}}{n\,(n+i+1-j)\;(i+1-j)\;\Delta Z^2},$$

wherefrom

$$t_i' = \frac{2a_{hII}\,\Delta\tau_j}{(n+i+1-j)\,(i+1-j)\,\Delta Z^2}\,t_{j-1} + t_i\left[1 - \frac{2a_{hII}\,\Delta\tau_j}{n(i+1-j)\Delta\tau^2}\right] + \\ + \frac{2a_{hII}\,\Delta\tau_j}{n\,(n+i+1-j)\,\Delta Z^2}\,t_{i+1}\,. \tag{V.2}$$

Notations

a	[m²/h]	temperature conductivity
c	[kcal/kg °C]	specific heat
D	[m²/h]	diffusion coefficient
F	[m²]	surface
i	[pcs]	number of location steps
j	[pcs]	number of time steps
j_W	[kg/m² h]	drying velocity
k	[pcs]	number of location steps on the main grid
L	[m]	thickness
m	[kg]	mass
n	[pcs]	distribution number of the auxiliary grid
P, p	[atm]	pressure
q	[kcal/m² h]	heat flux density
r	[kcal/kg]	evaporation heat
R	[atm m³/kmol °K]	universal gas constant
s	[atm/°C]	slope of the tension curve
t	[°C]	temperature
v	[m/h]	penetration velocity of the front
V	[m³]	volume
W	[kg]	mass of moisture
z, Z	[m]	distance

Greek letters

α	[kcal/m² h °C]	heat transfer coefficient, specific
β	[kg/m² h atm]	mass transfer coefficient, specific
ϱ	[kg/m³]	density
λ	[kcal/m h °C]	thermal conductivity
ζ	[m]	thickness
ψ	[m³/m³]	porosity
τ	[h]	time

Subscripts

e	equilibrium state
$é$	interface (intercontacting surface)
F	boundary surface of air and drying material
G	(essential) mass of air

h	temperature
hp	dew-point value
p	constant pressure
ms	material, solid
W, WL	water
WG	water vapor
o	(original) initial value
z	at the location z
I	dried region (layer)
II	wet region (layer)

Summary

The drying of wet (centrifuga-wet) granular materials is an often occurring task in industry. The bed established by such particles is macroporous. In the case of drying macroporous beds, the evaporating "plane" tends to penetrate into the bed, thus forming a double layer (i.e. where one of the layers is already dried up while the other one is wet). In the course of the drying process the thickness of the layers is changing. In *Part I* an approximate analytical correlation has been developed for the determination of the drying time of macroporous beds. — The exact computer-aided calculation method is presented in *Part II*. Computer outputs and approximation results are compared by means of a numerical example.

Reference

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