INVESTIGATION OF TANGENTIAL FORCES IN METAL CUTTING BY DIMENSIONAL ANALYSIS*

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1. Introduction

Determination of the tangential tool force for different types of metal cutting processes is one of the basic goals from practical as well as theoretical point of view. Forces wich arise during cutting are transmitted by the tool to the machine.

Knowledge of the tangential tool force permits to determine the deformation of work piece and thus the tolerance of machining. Knowing the tangential tool force one can also determine the right dimensions of the tool as well as the dimensions of certain parts of the machine.

Since the end of the last century there have been various attempts to theoretically determine tangential tool force, starting from the basic assumptions of the classical mechanics. These models gave formulas for the determination of tangential tool force in terms of properties of the material of work piece. Great many equations were derived from mechanical models, but in general, the values of the tangential tool force calculated by different equations differed significantly.

We refer to some of the most famous research works in this field: I. A. TIME (1870), TRESKA (1873), AFANASJEV (1883), GAUSNER (1892), ZVORIKIN (1893), BRIKS (1896), MERCHANT (1942), HUKS (1951), LOLADZE (1952), KRONENBERG (1957).

From the early twenties, a great many force measuring devices have been developed up to now, based on different (mechanical, hydraulical, pneumatical, inductive, with strain gauges, piezoelectric, etc.) principles. Results indicate that the models based on the classical mechanics are too simplified (e.g., they greatly simplify complex process of cutting) to obtain agreement between calculated and measured values for the tangential tool force.

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Therefore, most tangential tool force equations used today are based on graphoanalytical records. The formula given by Kronenberg is usually applied for tangential tool force as a function of chip cross section

$A = \delta s$:

$$F_1 = K_s A = \frac{C_k}{\varepsilon_k \sqrt{A}} \cdot A = C_k A^{\left(1 - \frac{1}{\varepsilon_k}\right)} \tag{1}$$

Extending this formula (1)

$$F_1 = C_{k_1} \delta^{x_1} x^{y_1} \tag{2}$$

where C_k and C_{k_1} depend both on the material of work piece, on the tool, and on the tool's rake angle γ: x_1 and y_1 and ε_k depend on the material of work piece alone.

In the thirties, attempts were made to determine tangential tool force according to the plastic properties of material. However, just as for mechanical models, the agreement between measured and calculated values of the tangential tool force was rather poor. This led to the conclusion that also this analysis ignored many important factors. Important works in this field are those by Rejtő (1926) [1], Kuznecov (1941) [2], Krivouhov (1944) [6].

Now the polytropic law, valid in the plastic region, will be applied to determine the compression stresses in a chip. This law is valid for the compression of cylindrical and prismatic specimens (with square cross section) and with different slenderness ratios of height h_0 to diameter of the unloaded specimen.

Equation of the polytropic compression is

$$F_0 h_0^m = Fh = C = \text{const.} \tag{3}$$

where F_0 is the force at the beginning of plastic compression referred to the initial height of the specimen h_0, F is the force referred to the actual height of the specimen $h_0 \gg h$, and m is the exponent of the polytropic change:

$$m = f\left(\frac{h_0}{h}\right).$$

According to Kuznecov, tangential tool force can be determined directly from the polytropic compression law. Solving Eq. (3) for F yields the force during plastic compression:

$$F = F_0 \left(\frac{h}{h_0}\right)^m. \tag{4}$$

Force at the beginning of plastic compression F_0 can be expressed as

$$F_0 = \sigma_0 A_0$$

where σ_0 is the yield stress and $A_0 = ab$ the initial cross section of the specimen. Substituting (4) we get

$$F = \sigma_0 A_0 \left(\frac{h}{h_0}\right)^m \tag{5}$$

Identifying the process of specimen compression in the plastic range with that of the chip (i.e. identifying with the cross section of the chip ab, where a is the thickness and b the width of the chip and ratio h_0/h with the chip's compression factor $\lambda = h_0(h)$) we get for the tangential tool force:

$$F_1 = \sigma_0 \, ab \, \lambda^m \, .$$

Using the polytropic law for compression in case of turning, Krivouhov gets a more complicated formula for the tangential tool force

$$F_1 = \frac{\sigma_0 s \lambda^m R}{(\lambda - 1)(m - 1)} \left(1 - \frac{1}{\left[\frac{\delta(\lambda - 1)}{R} + 1\right]^{m-1}}\right) \tag{6}$$

where $R = D/2$ is the outer radius of work piece and δ is the depth of cut. For $R = \infty$, i.e. in the case of machining a flat plane, the last expression becomes undetermined and equals $0.\infty$. This equation is a special case of L'Hospital's rule.

If higher accuracy is needed, dynamometry methods are more efficient.

In this work we use dimensional analysis as a function of the primary factors, while the other effects are involved in an empirical coefficient. So far, dimensional analysis has been applied in two cases, to determine the temperature of tool and the relationship between tool life and cutting speed.

2. Application of dimensional analysis to tangential tool force

Experiments showed tangential tool forces to depend on the stress of plastic compression in chip σ, the chip cross section $a = \delta s$, depth of cut δ and feed s, hence,

$$F_1 = f(\delta, \sigma, s) \, . \tag{7}$$

Expanding this function to series of the type

$$F_1 = \sum A_i \, \sigma^x \, \delta^y \, s^z \tag{8}$$

the following condition must be satisfied

$$\mathrm{Dim}(F_1) = \mathrm{Dim} \, | \sigma^x \, \delta^y \, s^z | \, . \tag{9}$$

The basic quantities needed for the application of the dimensional analysis are known to be:

$$
\begin{aligned}
&& (L] &= \text{length}\\
&& [M] &= \text{mass}\\
&& [S] &= \text{time}\\
[F] &= [LMS^{-2}] && = \text{force}\\
[\sigma] &= L^{-1}MS^{-2}] && = \text{stress}\\
[\delta] &= [L] && = \text{depth of cut}\\
[s] &= [L] && = \text{feed.}
\end{aligned}
$$

Substituting these values in Eq. (9):

$$[LMS^{-2}] = [L^{-1}MS^{-2}]^x\,[L]^y\,[L]^z \tag{10}$$

writing

$$\left.\begin{aligned} 1 &= -x + y + z\\ 1 &= x\\ -2 &= -2x \end{aligned}\right\} \tag{11}$$

yields

$$\left.\begin{aligned} x &= 1\\ z &= 2 - y \end{aligned}\right\}. \tag{12}$$

Substituting (12) into (8) we get

$$F_1 = \sum A_i\,\sigma^y\,s^{(2-y)} \tag{13}$$

or

$$F_1 = \sum A_i\,\sigma \left(\frac{\delta}{s}\right)^y s^2. \tag{14}$$

Introducing the coefficient $g = \delta/s$:

$$F = \sum a_i\,\sigma s^2 g^y\,. \tag{15}$$

From (15):

$$\sum A_i\,g^y = f(g)$$

be

$$f(g) = B$$

the expression for tangential tool force will take the form

$$F_1 = B\sigma\,s^2; \quad B = f\,(g). \tag{16}$$

By analyzing the last expression, we conclude that the value of the tangential tool force depends on the plastic properties of the cut material, i.e. on the compression stress in the cross section of the chip.

To determine tangential tool force from (16) we have to know the compression stress σ in the chip and the empirical relationship

$$B = f\ (g)$$

(in graphical or analytical form), which corresponds to the actual working conditions, and which can be obtained experimentally.

The value of the compression stress in the chip, as already mentioned, could be obtained from the polytropic law, to be used for two alternatives of deriving the tangential tool force for the work on lathe.

2.1. *First alternative of the expression for tangential tool force*

According to the physical law of constant volume before and after compression in the plastic range:

$$V_0 = A_0 h_0 = V = Ah \tag{17}$$

or

$$\frac{A}{A_0} = \frac{h_0}{h} = \lambda . \tag{18}$$

The factor of compression λ is the ratio of the cross sections or of the height values of the specimen after and before compression.

From (3) we have

$$\frac{F}{F_0} = \left(\frac{h_0}{h}\right)^m = \lambda^m \tag{19}$$

so that multiplying the left side of equation by

$$\frac{A_0}{A_0} \cdot \frac{A}{A}:$$

$$\frac{F}{F_0}\left(\frac{A_0}{A_0} \cdot \frac{A}{A}\right) = \frac{\sigma}{\sigma_0} \cdot \frac{A}{A_0} = \frac{\sigma}{\sigma_0}\lambda . \tag{20}$$

Equalizing the right sides of Eqs (19) and (20) yields for the stress of compression in the plastic range:

$$\sigma = \sigma_0 \lambda^{m-1} \tag{21}$$

Identifying the stress of plastic compression with the stress in the chip and substituting (21) in (16):

$$F_1 = B\sigma_0 \lambda^{m-1} s^2 \tag{22}$$

we get

$$B = \frac{F_1}{\sigma_0 \lambda^{m-1} s^2} = f(g),$$

an empirical relationship. If this relationship $B = f(g)$ is known for the material of the work piece, then

$$\sigma_{02} = \sigma_0 = k_0 .$$

For given conditions of cutting we may perform a short experiment and determine λ from the expression

$$\lambda = \frac{G_s}{\delta_s \gamma l_s} \tag{23}$$

where G_s is the weight, l_s the length and γ the specific weight of the chip.

Assume $m = 1.25$ is the exponent of the polytropic change (the value recommended by Kuznecov for steel), then all the necessary elements for the calculation of tangential tool force for cutting steel materials are known. It is important to note that $B = f(g)$ depends on the working conditions, mainly on the tool geometry as well as on the lubrication and cooling of the system.

2.2 *Second alternative for deriving the expression of tangential tool force*

The equation of the polytropic change (3)

$$Fh^m = C$$

divided by the cross section area corresponding to the height h of the specimen gives:

$$\frac{F}{A} h^m = \sigma h^m = \frac{C}{A} . \tag{24}$$

Since the volume of the specimen before compression is (17)

$$V = Ah$$

we have

$$A = \frac{V}{h} . \tag{25}$$

Substituting (25) into (24) and solving for σ, we get

$$\sigma = C_1 h^{1-m} \tag{26}$$

where

$$C_1 = \frac{C}{V} = \text{const} .$$

But, for side machining on the lathe, the length of chip element, for a given depth of cut δ and given approach angle of a tool $\varkappa$ is proportional to the feed s, i.e.

$$h = C_2 s; \qquad C_2 = \text{const.} \tag{27}$$

Therefore, resubstituting (27) into (26)

$$\sigma = B_1 s^{1-m} \tag{28}$$

where $B_1 = C_1 C_2^{1-m} = \text{const.}$
Substituting (28) into (16)

$$F_1 = B_2 s^{3-m} \tag{29}$$

where $BB_1 = B_2 = f(g)$. Since $B = f(g)$ and $B_1 = \text{const.}$, from (29) it follows that

$$B_2 = \frac{F_1}{s^{3-m}} = f(g)\,. \tag{30}$$

Assume B_2 to be proportional to the slenderness ratio of a chip g:

$$B_2 = B_0 g \tag{31}$$

where $B_0 = \text{const.}$, yields tangential tool force

$$F_1 = B_0 g s^{3-m}\,. \tag{32}$$

Substituting the slenderness ratio into (32):

$$F_1 = B_0 \delta\, s^{2-m}\,. \tag{33}$$

Be (as in alternative 1) the exponent of the polytropic change $m = 1.25$, which corresponds to the steel, we shall finally get for the tangential tool force for machining on lathe

$$F_1 = B\delta s^{0,75}\,. \tag{34}$$

Comparing Eqs (33) and (2), this latter obtained by graphoanalytical method (from measured data) where the exponents $x = 1.00$ and $y = 0.75$ represent the average values for steel, they are seen to represent identical formulas, even identical numerical values. Constant B_0 is equivalent to the specific tangential tool force C_{k1}.

In the tangential tool force equation, compression stress σ may be substituted for some other quantities with the same dimension and still preserve dimensional identity. Using shear stress τ_s in the plane of shear instead of σ in the equation, we get

$$F_1 = B\,\tau_s\text{-}s^2; \quad B = f(g);\, g = \frac{\delta}{s}\,. \tag{35}$$

Mechanical experiment involved the following shear stress/strain relationship:

$$\tau = \tau_1\,\varepsilon^m\,.$$

Extrapolation of the above relationship to the range of high deformations which corresponds to cutting ($\varepsilon_s = 2.5$) yields stresses close to the stress in the plane of shear τ_s:

$$\tau_s = \tau_1\,2.5^m = \tau_{2.5} \tag{36}$$

where $\tau_1 = \tau_{\varepsilon=1}$ and m is the slope in log-log scale.

The shear stress in the plane of shear can be approximated as:

$$\tau_{2.5} = \frac{0.6\,\sigma_M}{1-1.7\psi} \tag{37}$$

instead of the relationship $\tau = f(\varepsilon)$, where σ_M is the tensile strength of the material and ψ its contraction.

Substituting (36) into (35) we get for the tangential fool force

$$F_1 = B\,\tau_1\,2.5^m\,s^2;\; B = f(g) \tag{38}$$

so that

$$B = \frac{F_1}{\tau_1\,2.5\,s^2} = f(g) \tag{39}$$

which may be determined experimentally.

From the approximate value of the shear stress in the plane of shear, the tangential tool force is, substituting (37) into (35):

$$F_1 = B\,\frac{0.6\,\sigma_M}{1-1.7\,\psi}\,s^2 \tag{40}$$

where

$$B = \frac{1-1.7\,\psi_M}{0.6\,\sigma}\cdot\frac{F_1}{s^2} = f(g) \tag{41}$$

with the same dependence as in the previous case . Consequently, tangential tool force can be determined from the mechanical characteristics of the material and $B = f\,(g)$.

3. Conclusion

From the expression of the first alternative the determination of the tangential tool force on the lathe is seen to depend on the knowledge of material yield point $\sigma_0 = \sigma_{0.2}$ to be determined by mechanical tests. Chip compression factor λ as a measure of the plastic deformation may be determined by a short cutting experiment. Finally, dependence of the coefficient B on the slenderness of chip must also be determined experimentally. We note that λ and $B = f(g)$ depend on the actual working conditions, tool geometry, etc.

Comparing the first alternative equation for tangential tool force

$$F_1 = B\,\sigma_0\,\lambda^{m-1}\;s^2;\; B = f(g) \tag{22}$$

to that by KUZNECOV

$$F_1 = \sigma_0\,ab\,\lambda^m$$

they are seen to differ considerably, although they both use the polytropic law for compression in the plastic range. It is clear that the first expression obtained by dimensional analysis is more complete, because it includes function $B = f(g)$, taking into account actual working conditions.

The second alternative equation for tangential tool force that led to the well-known extended formula shows that the use of the polytropic law of compression is correct for cutting problems. Actually, starting assumptions in both are the same.

Because expanded formula for the determination of the tangential tool force is obtained from the graphoanalytic method (based on tangential tool force records), the second alternative derivation seems to prove that assumptions in the first one were correct, making an experimental proof of the first alternative needless.

Last but not least, the general tangential tool force equation

$$F_1 = B\sigma s^2; \qquad B = f(g)$$

obtained by dimensional analysis gives wide possibilities for process analysis. For example, by substituting the compressive stress σ for the stress in the plane of shear τ_s one may get convenient formulas for the determination of tangential tool force on the lathe.

Summary

Dimensional analysis is convenient for determining the tangential tool force, taking into consideration the primary effects on tool force (stresses in the chip, depth of cut and feed). Some possible uses of the obtained equations are presented. For example, replacing the stress in equations obtained by dimensional analysis by stresses measured in other way (substituting the shear stress in the plane of shear), relationships suitable for further research will result.

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