COMPLEX SYSTEMS IN MEASURING INSTRUMENTS

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The modern technology and experimental techniques require numerous very sensitive and complicated instruments. The measuring chain (information flow) is often decomposed to a system of independent units, increased efforts are being made to utilize various physical processes. In the followings increased complexity and the combined application of elements with different physical characteristics are discussed.

The decisive part of a measuring intsrument is the detecting element. Its primary aim is sensing and undistorted transmission of the measurable quantity in form of a signal. With the exception of length mesuring instruments, there is hardly an instrument, where the physical character of the signal would be identical with that of the measurable quantity. Signal converters used as detecting elements are in general of a mixed structure. In their simulation, idealized system elements called transducers or dependent sources take a prominent part.

Transducers are suited to connect two subsystems of different physical character, although the term is also rather generally used to a device which absorbs energy from one system and supplies energy to a second system, even if both systems are electrical.

The application of the dependent (controlled) source to represent mixed systems can be illustrated on a wide variety of devices of great importance in engineering.

Generally the electromechanical transducer is characterized by a pair of equations relating the electrical and mechanical variables. The electromechanical systems can be treated by the methods of generalized mechanics. This is a useful area of application because it involves the interface of two separate sciences, i.e., mechanics and electromagnetism.

One widely used type of transducer is illustrated in Fig. 1. The coil mounted on an elastic element moves axially in the annular air gap of a permanent magnet, and is coupled both to electrical and mechanical elements which serve as inputs and outputs. The following investigation is made for the basic device, i.e., an electromagnetic driver for vibration testing (Fig. 2).

The Lagrange equations yields the description of motion for the system as follows:

$$L_c\ddot{q} + (R_c - R)\dot{q} + k_1\dot{x} - U(t) = 0 \tag{1}$$
$$M\ddot{x} + B\dot{x} + kx - k_1\dot{q} - f = 0$$

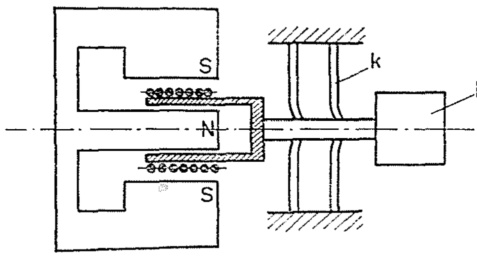

Fig. 1

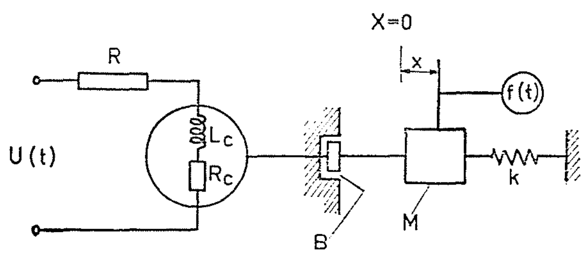

Fig. 2

where: L_c, R_c inducitivity and resistance of moving coil
$U(t)$, $f(t)$ voltage and force sources
q charge
x displacement
B coefficient of viscous damping
k spring constant
m moving mass
R resistance (out of coil)
k_1 magnetic constant.

The matrix equation, writing $\dot{q} = I$:

$$\begin{bmatrix} L_c s + R_c + R & k_1 \\ -k_1 & Ms + B + \dfrac{k}{s} \end{bmatrix} \begin{bmatrix} I \\ \dot{x} \end{bmatrix} = \begin{bmatrix} U \\ f \end{bmatrix} \tag{2}$$

Various transducer operations can be performed by choice of driving functions U and f and of system parameters. For example:

U	f	Type of device
Constant	impinging sound wave	microphone
Output form audio amplifier	$f = 0$ Coil connected to a diaphragm	loudspeaker
Servomechanism signal	$f = 0$ m is a hydraulic control valve	positioning element in an automatic feedback control system

. etc.
.
.

Illustrating one of these, the system equations will be computed for the device which is used to test the reliability of equipment assemblies under vibration.

In this case the forcing function f is zero and $U(t)$ is sinusoidal. Since the steady-state vibrations are of interest the complex notation can be used.

$$\text{Let: } U(t) = U_0 \cos \omega t = Re[U_0 e^{i\omega t}] ,$$

where ω is the angular frequency. Because of the assumed linearity of the system, also I and $\dot{x}$ are sinusoidal:

$$\dot{q} = I_0 \cdot e^{i\omega t} ; \quad \dot{x} = \dot{x}_0 e^{i\omega t} ,$$

The equations become

$$iL_c\omega I_0 + (R_c + R)I_0 + ik_1\omega X_0 = U_0$$

$$-k_1 I_0 - M\omega^2 \dot{X}_0 + iB\omega\dot{X}_0 \times k\dot{X}_0 = 0$$

in matrix form:

$$\left[\begin{array}{c|c} R_c + R + iL_c\omega & ik_1\omega \\ \hline -k_1 & k - M\omega^2 + iB\omega \end{array}\right] \begin{bmatrix} I_0 \\ X_0 \end{bmatrix} = \begin{bmatrix} U_0 \\ 0 \end{bmatrix} \tag{3}$$

The matrix solution of Eq. (3) is

$$\begin{bmatrix} I_0 \\ X_0 \end{bmatrix} = \mathrm{A}^{-1} \begin{bmatrix} U_0 \\ 0 \end{bmatrix} = \frac{1}{\det \mathrm{A}} \begin{bmatrix} \Delta_{11} & U_0 \\ \Delta_{21} & U_0 \end{bmatrix} ; \tag{4}$$

A is the system matrix and Δ_{11}, Δ_{21} are cofactors of $\mathbf{A}^{-1}$;

$$i.e.\colon \Delta_{11} = k - M\omega^2 + iB\omega$$
$$\Delta_{21} = k_1$$
$$\begin{aligned}\det[\mathbf{A}] &= (R_c + R + iL_c\omega)(k - M\omega^2 + iB\omega) + ik_1^2\omega = \\ &= (R_c + R)(k - M\omega^2) - L_c B\omega^2 + \\ &+ i\omega\left[(k - M\omega^2)L_c + (R_c + R)B + k_1^2\right].\end{aligned}$$

A single resonant frequency is possible, since the imaginary component of det [**A**] vanishes for a single real positive value of ω.

Requirements of up-to-date instrument technology should be satisfied by more complicated systems than those shown here. We can mention as an example the instruments detecting chemical colour reactions with liquid flow input, the information flowing on an optical subsystem, through a photoelectric converter to an analogue electric system, where it is passing, after suitable amplification and formation, converted into a digital signal, further through the electronic subsystem to the digital display units, or to the mechanical printer.

Systems discussed so far are characterized by the fact that their subsystems can be well separated according to their physical character. Signal flow is passing through suitably formed converters, like ports, towards display, recording, or other utilization. More complicated is the configuration where the above subsystems interact by means of one or more elements extend over the field of the other sub-system. Multi-purpose elements represent typical possibilities of such interaction, e.g. a tension wire instrument, where the torsion spring is functioning in the mechanical system as guiding element and spring, while in the electric one as conductor (and resistance).

Fig. 3. shows an example of the model of an apparatus consisting of interacting subsystems within the mechanical-electrical system. The problem is symbolically solved by generalized network theory for system engineering. The idealized model of the system is shown in Fig. 3/a. The common element of the two systems is the body Z, which is rigid in the mechanical sense, but changing in length to the effect of a current (e.g. thermal expansion or other effects). The characteristics are shown in Fig. 4. Performance in the electric system (Ohm's rule) is shown in Fig. 4/a, the mechanical characteristic (rigid body) in Fig. 4/b, while Fig. 4/c shows the characteristics of changes to the combined effect of the two systems, described e.g. by formula

$$l(t) = a\left[i(t)\right]^2 - l_0 \tag{5}$$

where a is a constant, l_0 the original length. System equations can be written as cut-set and loop matrices determined on the basis of the structure graph shown in Fig. 3/b. The cut-set matrix is

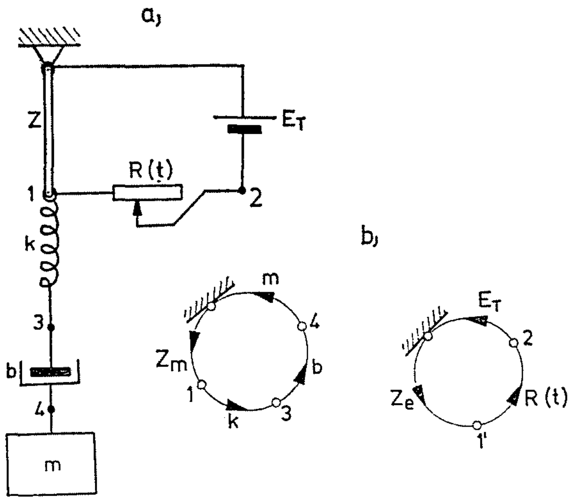

Fig. 3

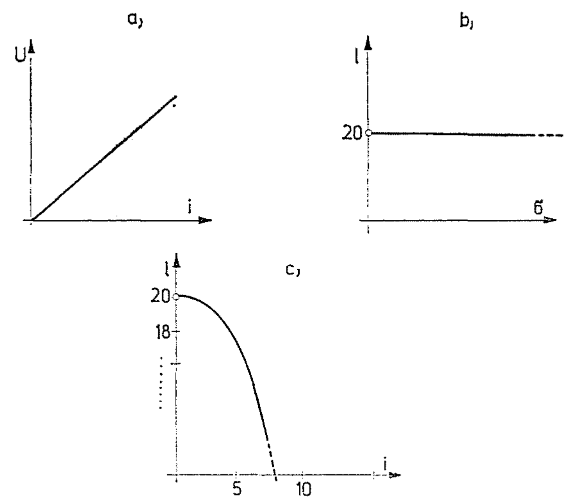

Fig. 4

$$
\begin{array}{ccc}
 & & \begin{array}{ccccccc} Z_e & E_t & R_{(t)} & Z_m & m & b & k \end{array} \quad \leftarrow \text{Elements} \\
\mathbf{S} = & \begin{array}{c} 1 \\ 2 \\ \\ 1 \\ 3 \\ 4 \end{array} & \left[\begin{array}{ccc|cccc}
-1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 1 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & -1
\end{array}\right]
\begin{array}{l} \left\{ \begin{array}{l} \text{Electrical} \\ \text{subsystem} \end{array} \right. \\ \\ \left\{ \begin{array}{l} \text{Mechanical} \\ \text{subsystem} \end{array} \right. \end{array} \\
\uparrow & & \\
\text{Nodes} & &
\end{array}
\qquad (6)
$$

The loop matrix is

$$\mathbf{B} = \left[\begin{array}{ccc|cccc} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right] \tag{7}$$

Denoting the vector of through variables by Y, that of across variables by X, and the matrix formed of these by $\mathbf{V}$, further if $\mathbf{C}$ is the so-called coefficient matrix describing the characteristics of the individual elements, and K is the vector of constants in the equations, then the system of matrix equations characterizing the system is:

$$\begin{array}{lll} \mathbf{S} \cdot Y = 0 & \text{(node rule)} \\ \mathbf{B} \cdot X = 0 & \text{(loop rule)} \\ \mathbf{C} \cdot V = K & \text{(elementary equations),} \end{array} \tag{8}$$

These can be rewritten in the form:

$$\left[\begin{array}{c|c} \mathbf{S} & 0 \\ \hline 0 & \mathbf{B} \\ \hline \mathbf{R}_1 & \mathbf{R}_2 \end{array}\right] \cdot \begin{bmatrix} Y(\mathrm{t}) \\ X(\mathrm{t}) \end{bmatrix} = \begin{bmatrix} 0 \\ K \end{bmatrix}, \tag{9}$$

where submatrices $\mathbf{R}_1$ and $\mathbf{R}_2$ of the hypermatrix in the equation originate from matrix $\mathbf{C}$.

The system equation can be decomposed to a system of linear differential equations with constant coefficients which are easy to handle by a computer, and yield, depending on initial conditions and on the parameters of the individual elements, the time functions characterizing the dynamics of the system.

Systems of mixed type can also be well handled by the state space method. To determine the actual system state between vector $X(t)$ containing the state parameters of the system, and input vector $u(t)$, the following state vector-differential equation can be applied:

$$X(t) = \mathbf{A}x(t) + \mathbf{B}u(t)\,; \tag{10}$$

where $\mathbf{A}$ is the so-called system matrix,
$\mathbf{B}$ the so-called input matrix.

If also the input to output relation is wanted another equation is required:

$$v(\mathrm{t}) = \mathbf{C}x(t) + \mathbf{D}u(t)\,; \tag{11}$$

where $v(t)$ is the output vector,
$\mathbf{C}$ the output matrix,
$\mathbf{D}$ the so-called auxiliary matrix.

From the aspect of our investigations, the structure of the system matrix deserves attention. In this matrix, namely, parameters of the individual elements are arranged in an interesting way. In the case of mixed type systems matrix **A** can be partitioned so that parameters of the individual sub-systems are clearly separated in sub-matrices each, while the remaining ones may be considered as connecting matrices. As an example let us examine the system

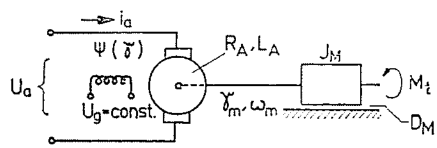

Fig. 5

matrix of the constant excitation direct current motor with the model shown in Fig. 5 (as a mixed electrical-mechanical system):

$$\begin{array}{cc} \text{Mechanical} & \text{Connecting} \end{array}$$
$$\left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{D_M}{J_M} & 0 & \dfrac{K_M}{J_M} \\ \hline 0 & 0 & 0 & 1 \\ 0 & -\dfrac{K_V}{L_A} & 0 & -\dfrac{R_A}{L_A} \end{array}\right] \tag{12}$$
$$\begin{array}{cc} \text{Connecting} & \text{Electrical} \end{array}$$

where D_M is the damping coefficient of the motor,
L_M inertia to the motor rotor (reducing also the connected elements, such as coupling, gears, etc., to a single element),
R_A, L_A resistance and inductance of the armature, resp.,
K_V voltage vs. angular velocity characteristics of the motor,
K_M current vs. torque characteristics of the motor.

The state method, beside other favourable characteristics, gives good information on the parameters of the mixed system, and the effects having an influence on the individual "pure" sub-systems and on coupling can be read off the system matrix. The individual systems may "become" of the mixed type also in a way different from that described above. As an illustrative example, let us examine an instrument of the so-called homogeneous system, let it have a purely electrical (electronic) structure (such as a digital voltmeter despite of the opto-electronic display, a tube voltmeter, etc.). Examination of the signal flow of this instrument permits to plot, the system graph and favourable or unfavourable aspects of the structure can be evaluated on the basis of previously fixed objective functions. When continuing, in accordance with other examination purposes (e.g. determination of thermal state), another model should be composed containing heat sources, heat capacities, etc., and this can also be examined "in itself" by the methods of system engineering. Again another model is produced if the instrument is examined e.g. from the aspect of reliability. These different models of the same measuring apparatus may have various connecting elements, and we are back at the mixed system model, at the problem that interactions have to be determined to a sufficient depth by methods with underlying principles described above.

Summary

Influence of the complexity of the measuring instruments to their analysis and synthesis has been discussed. The combined application of elements with different physical characteristics results in mixed system. The conventional input/output and the states-space method are applied for the analysis of mixed-type measuring systems.

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