ELASTIC STRESSES IN TORICONICAL PRESSURE VESSEL HEADS

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Introduction

Beside elliptical and torispherical pressure-vessel heads, the most important head type of thin-walled pressure vessels used in the chemical industry is the toriconical one. It is most extensively applied on evaporators and crystallizators, on dosing tanks. It is curious to find little relevant data in the literature while torispherical heads are of much concern.

The classic work by Watts and Lang concerned with the junction of conical and cylindrical shells, has been further developed by Taylor and Wenk [1]. Collected papers [1] report on test results also for toriconical shells.

Shield and Drucker [2] give a theoretical formula for limit pressure causing large plastic deformations. This formula has been confirmed by Save's tests [3]. These reports fail, however, to give a full picture of the elastic stress state of the structure, however fundamental for the design.

Our analyses aimed at determining the effect of the variation of head-size proportions on the elastic stresses. Analyses were made by the method of finite differences, based on the linear theory of thin shells, on a computer ODRA 1204 programmed in ALGOL-60. The method is identical with that in [4].

The toriconical vessel head is of the geometry shown in Fig. 1. Head recommended by the Hungarian standard may have a half cone-apex angle $\alpha = 30$, 45 or $60°$, and a knuckle radius $r/D = 0.1$. The standard permits, however, heads of other size proportions, too.

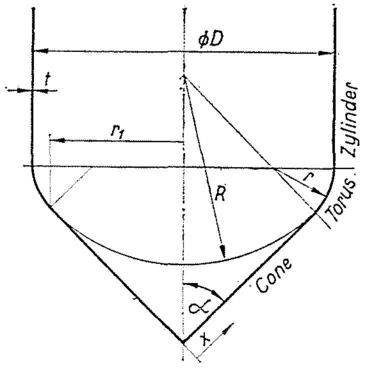

Fig. 1. Geometry of the toriconical vessel head

Stresses

Stress curves in a head of dimensions $\alpha = 60°$; $r/D = 0.1$; $D/2t = 100$ are shown in Fig. 2. The figure shows relative stress values, to define as:

$$I = \frac{\sigma}{p \cdot D/2t} . \tag{1}$$

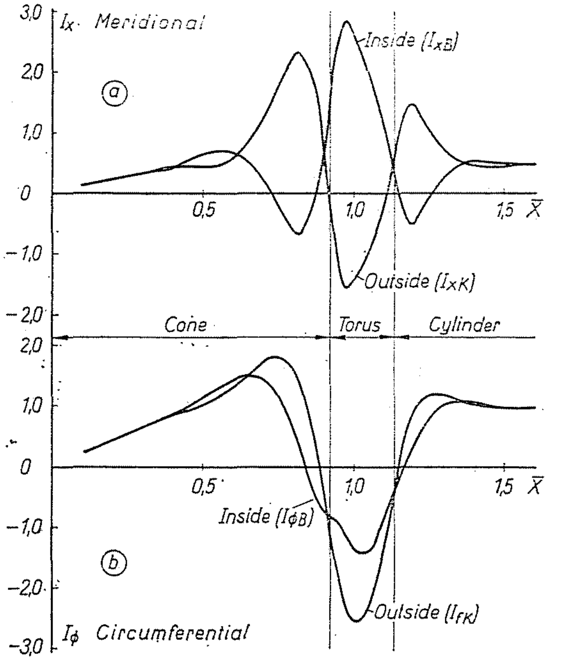

Fig. 2. Stress-index curves for $\alpha = 60°$, $r/D = 0.1$, $D/2t = 100$

Both meridional and circumferential stresses have been represented separately. Relative arch lengths from the cone apex along the meridian curve have been plotted on the horizontal axis:

$$\bar{x} = \frac{x}{D/2} . \tag{2}$$

The major part of the cone is seen to be in membrane-stress state. Towards the knuckle, a high bending stress of meridional direction arises, causing a stress maximum on the outer shell surface (Fig. 2a). In the knuckle it has a minimum on the outer surface, and a maximum in the inner surface, this latter being the highest stress throughout the head. Proceeding further, the membrane-stress state is restored in the cylinder. The circumferential bending stresses are lower (Fig. 2b), the secondary membrane hoop stress of compression is significant.

At the maximum meridional stress, in the inner knuckle surface, the resultant circumferential stress is seen to be a compressive one. According to the Mohr theorem of failure, the maximum equivalent stress index is:

$$I_R = [I_{xB} - I_{\varphi B}]_{\max} . \tag{3}$$

It is advisable to introduce the maximum equivalent membrane-stress index too:

$$I_M = \max\{I_{Mt}, \quad I_{Mc}\}, \tag{4}$$

where:

$$I_{Mt} = \max, \text{ torus } \{[I_{xM}]_{\max}, [I_{xM} - I_{\varphi M}]_{\max}\} \tag{5}$$

is the index of the maximum equivalent membrane stress in the torus according to Mohr, and

$$I_{Mc} = [I_{\varphi M}]_{\max,\text{cone}} \tag{6}$$

is the index of maximum membrane stress in the cone. For thin shells, where $D/2t > 50$, $I_{Mt} > I_{Mc}$. Values of I_R and I_M may serve as basis of head design.

Maximum values of indices of each stress component in the knuckle, and values of I_{Mc} are compiled in Table 1 for the recommended dimensions. I_{xM}, I_{xH}, $I_{\varphi M}$, $I_{\varphi H}$ are basic values, and the other ones can be obtained by their superposition. Extreme fibre-stress indices in the shell corresponding to Fig. 2 are:

$$\begin{aligned} I_{xB} &= I_{xM} + I_{xH} , \\ I_{xK} &= I_{xM} - I_{xH} , \\ I_{\varphi B} &= I_{\varphi M} + I_{\varphi H} , \\ I_{\varphi K} &= I_{\varphi M} - I_{\varphi H} . \end{aligned} \tag{7a-d}$$

Table 1

α	$D/2t$	I_{xM}	I_{xH}	$I_{\varphi M}$	$I_{\varphi H}$	I_R	I_{Mt}	I_{Mc}
30°	25	0.513	1.087	0.186	0.325	1.600	0.513	0.948
	50	0.514	1.325	−0.127	0.405	1.865	0.641	1.020
	100	0.514	1.537	−0.535	0.461	2.125	1.049	1.083
	150	0.514	1.568	0.807	0.470	2.418	1.321	1.115
	200	0.514	1.539	−1.007	0.462	2.598	1.521	1.128
45°	25	0.533	1.486	−0.207	0.442	2.020	0.740	1.023
	50	0.524	1.708	−0.648	0.498	2.399	1.190	1.145
	100	0.547	1.731	−1.176	0.508	2.946	1.723	1.240
	150	0.564	1.597	1.373	0.457	3.166	1.937	1.283
	200	0.567	1.495	−1.563	0.429	3.254	2.130	1.312
60°	25	0.585	1.985	−0.632	0.559	2.643	1.225	1.204
	50	0.621	2.170	−1.175	0.589	3.418	1.796	1.406
	100	0.640	2.153	−1.826	0.600	4.018	2.547	1.574
	150	0.690	2.127	−1.967	0.570	4.273	2.840	1.651
	200	0.698	2.120	−2.162	0.581	4.400	3.099	1.670

Table 1 shows stress indices to increase with reducing shell thickness (increasing $D/2t$ ratios), except for bending-stress indices I_{xH} and $I_{\varphi H}$ which decrease again over $D/2t = 100$. Namely, in very thin shells, the stress-increasing effects of cone-to-torus and torus-to-cylinder junctions do not add up any more, because of damping.

The increasingly negative value of the circumferentical secondary membrane stress index $I_{\varphi M}$ is of interest.

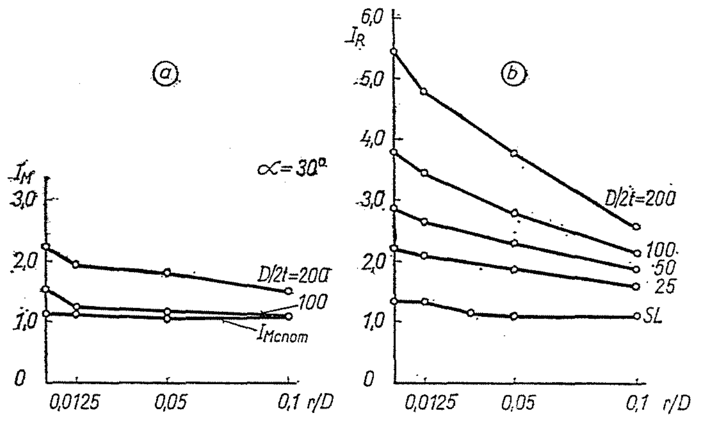

Fig. 3. Curves of I_R and I_M for $\alpha = 30°$

Figs 3 to 5 show the effect of decreasing knuckle radius on values of I_R and I_M. Calculations refer to $r/D = 0.0125, 0.05$ and 0.1. Calculated points have been connected by straight lines.

On figures marked *b*, also the limits *SL* corresponding to Hungarian-standard shape factors have been indicated, understood as

$$SL = \max\left\{\frac{y}{2}, \quad I_{Mc\,\mathrm{nom}}\right\}, \tag{8}$$

where y is defined by

$$t \geqq \frac{pD}{4(\sigma_m \cdot v)} \cdot y, \tag{9a}$$

and

$$I_{Mc\,\mathrm{nom}} = \frac{r_1}{D/2} \cdot \frac{1}{\cos\alpha} \tag{9b}$$

is the maximum nominal membrane stress index in the cone.

Figs 3a, 4a, 5a show values of $I_{Mc\,nom}$, for every r/D. For $D/2t<50$ values of I_M are lower than of $I_{Mc\,nom}$, hence they have been omitted.

The maximum resultant stress intensity I_R is seen to exceed the standard limit SL strongly.

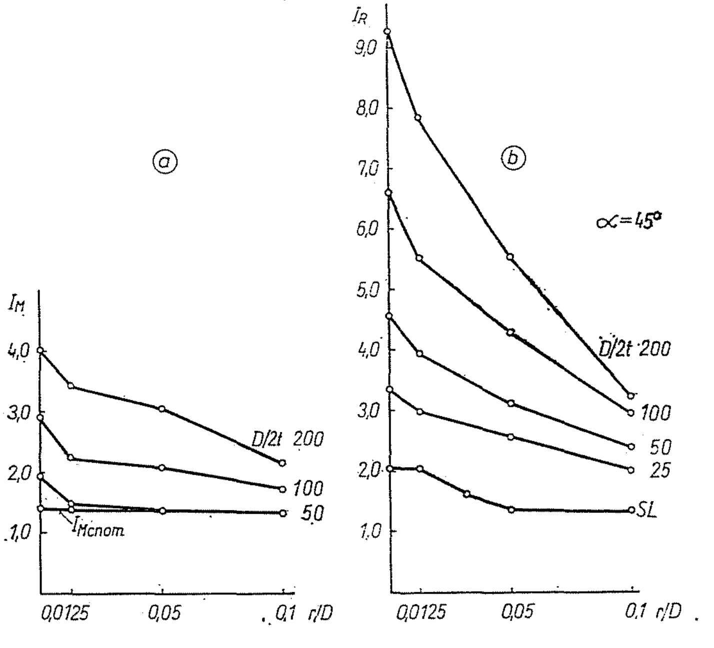

Fig. 4. Curves of I_R and I_M for $\alpha = 45°$

In case of thinner shells ($D/2t > 100$) the standard limit SL is seen to be lower than the equivalent membrane-stress index I_M. Heads of standard design may undergo significant plastic deformations. Of course, equivalent stress indices I_R and I_M are characteristic only of the initial plastic deformation. They are not suitable to determine the ultimate head pressure.

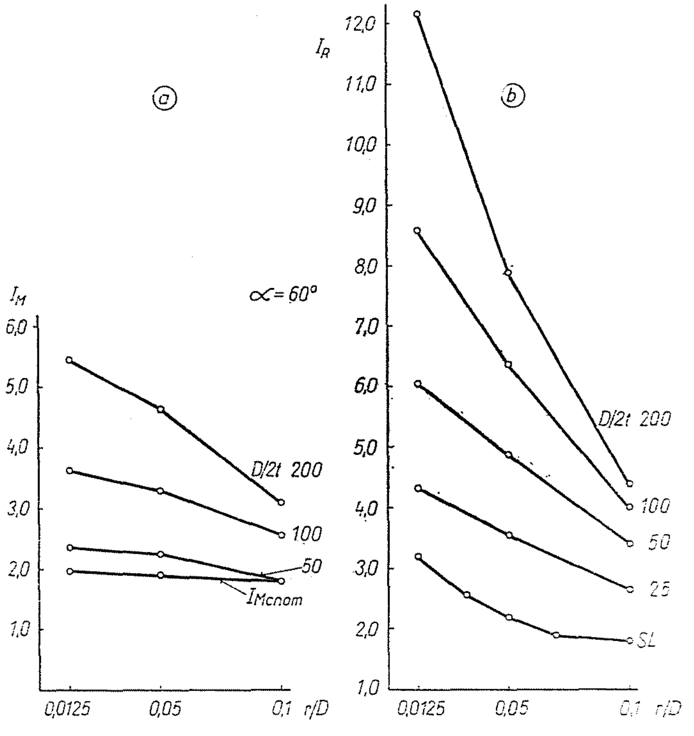

Fig. 5. Curves of I_R and I_M for $\alpha = 60°$

Equivalent torispherical head

From the Geckeler approximation, toriconical and torispherical head-stress curves are expected to be similar. This is also confirmed by limit pressure tests [3]. According to our numerical analyses, this similarity is a real one. Results are tabulated in Table 2 for $r/D = 0.1$, $\alpha = 30, 45$ and $60°$ as well as $D/2t = 25$ and 100.

Table 2

α	$D/2t$	Toriconical		Torispherical	
		I_R	I_M	I_R	I_M
30°	25	1.600	0.513	1.551	0.518
	100	2.125	1.049	2.159	1.181
45°	25	2.020	0.740	1.982	0.977
	100	2.946	1.723	2.963	1.850
60°	25	2.643	1.225	2.847	1.439
	100	4.018	2.547	4.066	2.650

Sharp junction

Sharp junction is interesting to be examined both from practical and theoretical aspects. The standard permits to apply such heads for $\alpha < 30°$, besides, it is interesting to see the results of a numerical method based on the theory of thin shells applied for very high curvatures.

Figs 3 and 4 show the numerical method to give a fair approximation even for $r/t = r/D \cdot 2 \cdot D/2t = 0.0125 \cdot 2 \cdot 25 = 0.625$ to the elementary solution neglecting the singularity of the sharp junction, described in detail in [5].

It should be stressed that this congruency is only valid within the validity of the theory of thin shells, this latter being not valid in sharp corners.

Notations

p	kp/cm^2	pressure
r	cm	knuckle radius
r_1	cm	greatest cone radius
t	cm	wall thickness
x	cm	arc lenth from the cone apex
$\bar{x}$		relative arc length
y		standard shape factor
D	cm	medium diameter of the cylindrical part
I		stress index
I_R		maximum equivalent stress index
I_M		maximum equivalent membrane-stress index
$I_{Mc\,nom}$		maximum nominal cone-membrane-stress index
R	cm	crown radius of the equivalent torispherical head
SL		standard limit
$\alpha°$		semi-cone-angle
σ	kp/cm^2	stress
$\sigma_m \cdot v$		product of allowable stress and weld-efficiency factor

Subscripts

x	meridional
φ	circumferential
M	membrane
H	bending
B	inner
K	outer
c	cone
t	torus

Summary

The finite-difference method based on the linear theory of thin shells has been applied to examine the effect of varying the geometry of toriconical pressure-vessel heads on the developing elastic stresses. Stress-index values are presented for practically occurring dimensions. Calculations confirmed the similarity between stress state in toriconical and torispherical heads, as well as the applicability of the method of finite differences on shells of relatively sharp curvature.

References

1. Pressure Vessel and Piping Design, coll. papers 1927—1959. Ed. ASME, New York, 1960.
2. SHIELD, R. T.—DRUCKER, D. C.: Design of thin-walled torispherical and toriconical pressure-vessel heads, Journal of Applied Mechanics, June 1961.
3. SAVE, M. A.—MASSONNET, C. E.: Plastic Analysis and Design of Plates, Shells and Discs, North-Holland Publ. Co., Amsterdam, 1972.
4. REUSS, P.: Elastic stresses in torispherical pressure-vessel heads, Periodica Polytechnica, *18*, 253, (1974)
5. REUSS, P.: Influence of skirt support on the stresses in large vessels with a conical bottom, Periodica Polytechnica, *18*, 167, (1974)

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