

COOLING OF HOT WATER IN LONG PIPELINES

(during temperature-variation cycles)

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Our previous paper [1] dealt with heat losses in pipelines during hours of peak operation. The developed computation method to determine temperature drop and heat losses in hot water networks during cyclic temperature variation was checked by measurements. The computation method involved some approximations, neglects those justified empirically and by measurements. In this paper, the admissibility of the neglect will be theoretically analysed and justified.

1. Differential equation of the phenomenon

This phenomenon is expressed as:

$$c \cdot \rho \frac{\partial u}{\partial t} = \lambda \cdot \frac{\partial^2 u}{\partial x^2} - c \cdot \rho \cdot w \cdot \frac{\partial u}{\partial x} - k \cdot \frac{p}{q} (u - u_k) \quad (1)$$

where

u [°C]	water temperature
u_k [°C]	ambient temperature
t [h]	time
x [m]	longitudinal coordinate of the pipeline
c [kcal/kg, °C]	specific heat
ρ [kg/m ³]	density of water
λ [kcal/m, h, °C]	heat-conductivity coefficient of water
w [m/h]	flow rate
k [kcal/m ² , h, °C]	heat-transfer rate referred to the pipe outer surface
p [m]	outer circumference of the pipe
q [m ²]	inner cross-section area of the pipe

The first term on the right side of the differential equation (1) is the heat conduction in the fluid, the second one the temperature variation upon flow, and the third one the temperature drop due to heat transfer to the surroundings.

The water is fed into the pipeline at a predetermined time-dependent temperature (Fig. 1) with the boundary condition:

$$u(0, t) = g(t) \quad (2)$$

the function $g(t)$ being known.

From the comparison of the orders of magnitude of numerical coefficients in (1)

$$c \cdot \rho = 10^3 \text{ kcal/kg, m}^3$$

$$\lambda = 5 \cdot 10^{-1} \text{ kcal/m}^2, \text{ h, } ^\circ\text{C}$$

$$c \cdot \rho \cdot w = 3 \cdot 10^6 \text{ kcal/m}^2, \text{ h, } ^\circ\text{C}$$

$$k \frac{P}{q} = 2 \cdot 10^2 \text{ kcal/m}^2, \text{ h, } ^\circ\text{C}$$

The term for heat conductivity is seen to be small by at least 10^3 order compared with other terms, hence in our previous paper it was neglected as a first approximation.

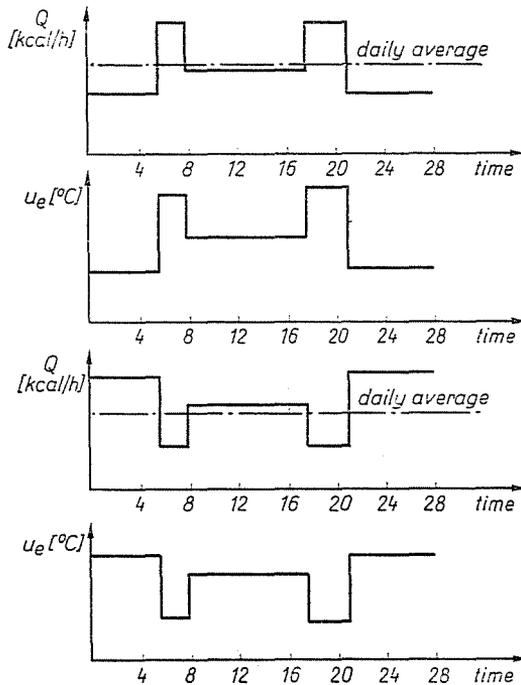


Fig. 1. The periodic temperature function as initial condition

In this paper a new solution will be described for the differential equation, taking the 2nd-order term into consideration.

In turn, the following minor simplifications supported by tests will be introduced.

a) In any given cross-section, the temperature distribution is a function of time alone; hence, seen in the space, this problem is reduced to a linear one.

b) Because of the process features, the initial distribution of temperature in the pipe will be omitted.

c) The ambient temperature is considered as constant.

2. Solution of the differential equation (1)

Introducing the transformation

$$u - u_k = \vartheta$$

and arranging yields for the overtemperature $\vartheta = \vartheta(x, t)$ the differential equation:

$$\frac{\partial \vartheta}{\partial t} - \frac{\lambda}{\rho \cdot c} \frac{\partial^2 \vartheta}{\partial x^2} + w \cdot \frac{\partial \vartheta}{\partial x} + \frac{k \cdot p}{\rho \cdot c \cdot q} \vartheta = 0 \quad (3)$$

With a view to the periodicity of function $g(t)$, the boundary condition (2) becomes:

$$u(0, t) = g(t) = \sum_{n=0}^{\infty} A_n \cos n \frac{2\pi}{T} t + B_n \sin n \frac{2\pi}{T} t \quad (4)$$

where, with the length of period T , we have:

$$A_n = \frac{2}{T} \int_0^T g(t) \cos n \frac{2\pi}{T} t \cdot dt$$

$$B_n = \frac{2}{T} \int_0^T g(t) \sin n \frac{2\pi}{T} t \cdot dt$$

$$n = 1, 2, \dots$$

and

$$A_0 = \frac{1}{T} \int_0^T g(t) \cdot dt$$

according to the sense.

The complex form of the Fourier's series (4):

$$g(t) = \sum_{n=-\infty}^{\infty} c_n \cdot e^{in \frac{2\pi}{T} t} \quad (5)$$

where

$$\begin{aligned} C_0 &= A_0 \\ C_n &= \frac{A_n - iB_n}{2} \\ C_{-n} &= \frac{A_n + iB_n}{2} \end{aligned}$$

according to the sense.

Writing boundary condition (5) for the transformed differential equation (3):

$$\vartheta(0, t) = u(0, t) - u_k = g(t) - u_k = f(t) = \sum_{n=-\infty}^{\infty} C_n^* \cdot e^{in \frac{2\pi}{T} t} \quad (6)$$

where

$$C_0^* = C_0 - u_k \quad C_n^* = C_n$$

Let us introduce for Eq. (3) the transformation:

$$\vartheta = e^{\gamma x + \delta t} v(x, t)$$

where constants γ and δ are defined so as to zero the coefficients of $\frac{\partial v}{\partial x}$ and of function $v(x, t)$ in the transformed differential equation, resulting for γ and δ in:

$$\begin{aligned} \gamma &= \frac{w \cdot \varrho \cdot c}{2\lambda} > 0 \\ \delta &= -\frac{k \cdot p}{\varrho \cdot c \cdot q} - \frac{w^2 \cdot \varrho \cdot c}{4\gamma} < 0 \end{aligned}$$

The differential equation (3) with function $v(x, t)$ becomes:

$$\frac{\partial v}{\partial t} - \frac{\lambda}{\varrho \cdot c} \frac{\partial^2 v}{\partial x^2} = 0 \quad (7)$$

Thus, the solution $\vartheta(x, t)$ can be written as:

$$\vartheta = e^{\frac{w \cdot \varrho \cdot c}{2\lambda} x - \left(\frac{k \cdot p}{\varrho \cdot c \cdot q} + \frac{w^2 \cdot \varrho \cdot c}{4\lambda} \right) t} \cdot v(x, t) \quad (8)$$

hence:

$$v(x, t) = e^{-\frac{w \cdot \rho \cdot c}{2\lambda} x + \left(\frac{k \cdot p}{\rho \cdot c \cdot q} + \frac{w^2 \cdot \rho \cdot c}{4\lambda}\right) t} \cdot \vartheta(x, t) \quad (9)$$

and the boundary condition:

$$v(0, t) = e^{\left(\frac{k \cdot p}{\rho \cdot c \cdot q} + \frac{w^2 \cdot \rho \cdot c}{4\lambda}\right) \cdot t} \cdot \sum_{n=-\infty}^{\infty} C_n^* e^{in \frac{2\pi}{T} t} \quad (10)$$

Let us write the solution $v(x, t)$ as:

$$v = \sum_{n=-\infty}^{\infty} v_n(x, t) \quad (11)$$

Now, individual terms at the boundary become, according to Eq. (9):

$$v_n(0, t) = C_n^* \cdot e^{\left(\frac{k \cdot p}{\rho \cdot c \cdot q} + \frac{w^2 \cdot \rho \cdot c}{4\lambda} + in \frac{2\pi}{T}\right) t} \quad (12)$$

Now, terms in function series (10) are sought for in the form:

$$v_n(x, t) = C_n^* e^{a_n x + b_n t} \quad (13)$$

where constants a_n and b_n are determined from differential equation (7) and boundary condition (10):

$$a_n = -\sqrt{\frac{k \cdot p}{\lambda \cdot q} + \left(\frac{w \cdot \rho \cdot c}{2\lambda}\right)^2} + in \frac{2\pi}{T} \frac{\rho \cdot c}{\lambda}$$

$$b_n = \frac{k \cdot p}{\rho \cdot c \cdot q} + \frac{w^2 \cdot \rho \cdot c}{4\lambda} + in \frac{2\pi}{T}$$

to

$$\vartheta_n(x, t) = C_n^* \cdot e^{\left[\frac{w \cdot \rho \cdot c}{2\lambda} x - \sqrt{\frac{k \cdot p}{\lambda \cdot q} + \left(\frac{w \cdot \rho \cdot c}{2\lambda}\right)^2} + in \frac{2\pi}{T} \frac{\rho \cdot c}{\lambda}\right] x + in \frac{2\pi}{T} t}$$

Solution $u(x, t)$:

$$u = u_k + \sum_{n=-\infty}^{\infty} C_n^* \cdot e^{(r - \sqrt{s + r^2 + id_n}) x + in \frac{2\pi}{T} t} \quad (14)$$

where

$$r = \frac{w \cdot \rho \cdot c}{2\lambda} \quad s = \frac{k \cdot p}{\lambda \cdot q} \quad d_n = n \frac{2\pi}{T} \frac{\rho \cdot c}{\lambda}$$

After extracting roots in the exponents of (14) we obtain:

$$u(x, t) = u_k + \sum_{n=-\infty}^{\infty} C_n^* e^{-E_n x + i(\omega_n t - F_n x)} \quad (15)$$

where

$$\omega_n = n \frac{2\pi}{T}$$

$$-E_n = r - \sqrt[4]{(s+r^2)^2 + d_n^2} \cdot \cos\left(\frac{1}{2} \text{Arctg} \frac{d_n}{s+r^2}\right)$$

$$-F_n = -\sqrt[4]{(s+r^2)^2 + d_n^2} \cdot \sin\left(\frac{1}{2} \text{Arctg} \frac{d_n}{s+r^2}\right)$$

Obviously

$$\text{Lim } E_n = 0$$

since, for $n \rightarrow \infty$

$$\begin{aligned} \sqrt[4]{(s+r^2)^2 + d_n^2} \cdot \cos\left(\frac{1}{2} \text{Arctg} \frac{d_n}{s+r^2}\right) &> \sqrt{d_n} \cos\left(\frac{1}{2} \text{Arc tg } \delta d_n\right) = \\ \sqrt{d_n} \sqrt{\frac{1 + \cos(\text{Arc tg } \delta d_n)}{2}} &= \frac{\sqrt{2}}{2} \sqrt{d_n} \cdot \sqrt{1 + \frac{1}{\sqrt{1 + (\delta \cdot d_n)^2}}} \rightarrow \frac{\sqrt{2}}{2} \sqrt{d_n} \end{aligned}$$

Writing function (15) in real form for ease of calculation:

$$\begin{aligned} u(x, t) = u_k + (A_0 - u_k) e^{-E_0 x} + \sum_{n=1}^{\infty} A_n \cos(\omega_n t - F_n x) + \\ + B_n \cdot \sin(\omega_n t - F_n x) \cdot e^{-E_n x} \end{aligned} \quad (16)$$

As an example, consider differential equation (1) with the initial condition:

$$u(0, t) = g(t) = A_0 + A_1 \cos \frac{2\pi}{T} t$$

solved according to Eq. (16) as:

$$\vartheta = (A_0 - u_k) \cdot e^{-E_0 x} + A_1 \cdot e^{-E_1 x} \cdot \cos\left(\frac{2\pi}{T} t - F_1 x\right) \quad (17)$$

3. Numerical results

Let us take the data encountered in our previous paper:

$$\begin{aligned} W &= 3.0 \cdot 10^3 \text{ [m/h]} \\ c \cdot \rho &= 10^3 \text{ [kcal/m}^3, \text{ }^\circ\text{C]} \\ \lambda &= 0.5 \text{ [kcal/m, h, }^\circ\text{C]} \\ c \cdot \rho \cdot w &= 3.0 \cdot 10^6 \text{ [kcal/m}^2, \text{ h, }^\circ\text{C]} \\ \frac{k \cdot p}{q} &= 2 \cdot 10^2 \text{ [kcal/m}^3, \text{ h, }^\circ\text{C]} \\ \omega &= 0.2616 \text{ [1/h]} \\ t &= 0 \quad 24 \text{ [h]} \\ x &= 0 \quad 5000 \text{ [m]} \end{aligned}$$

Calculating exponents:

$$\begin{aligned} -E_0 &= r - \sqrt{r^2 + s} \\ r &= \frac{c \cdot \rho \cdot w}{2\lambda} = \frac{3 \cdot 10^6}{2 \cdot 0.5} = 3 \cdot 10^6 \\ s &= \frac{k \cdot p}{\lambda \cdot q} = \frac{2 \cdot 10^2}{0.5} = 4 \cdot 10^2 \\ -E_0 &= 3 \cdot 10^6 - \sqrt{(3 \cdot 10^6)^2 + (2 \cdot 10^2)^2} = 3 \cdot 10^6 \\ &\quad \cdot \left\{ 1 - \left[1 + \left(\frac{2 \cdot 10^2}{3 \cdot 10^6} \right)^2 \right] \right\} \\ &= 3 \cdot 10^6 \left[1 - \sqrt{1 + \frac{4}{9 \cdot 10^{10}}} \right] \end{aligned}$$

Expanding the radical into series:

$$\begin{aligned} -E_0 &= -\frac{2}{3} \cdot 10^{-4} \\ -E_1 &= r - \sqrt[4]{(r^2 + s)^2 + d_1^2} \cos \left(\frac{1}{2} \text{Arctg} \frac{d_1}{r^2 + s} \right) \end{aligned}$$

since

$$d_1 = \frac{c \cdot \rho \cdot \omega}{\lambda} = \frac{10^3 \cdot \omega}{0.5} = 2 \cdot 10^3 \omega$$

thus

$$\begin{aligned} -E_1 &= 3 \cdot 10^6 - \sqrt[4]{(3^2 \cdot 10^{12} + 4 \cdot 10^2)^2 + 2^2 \cdot 10^6 \omega^2} \cdot \\ &\quad \cos \left(\frac{1}{2} \text{Arc tg} \frac{2 \cdot 10^3 \cdot \omega}{(3 \cdot 10^6)^2 + (2 \cdot 10^2)^2} \right) \end{aligned}$$

In the same way we have:

$$-E_1 = -\frac{2}{3} \cdot 10^4$$

$$-F_1 = -\sqrt[4]{(r^2 + s)^2 + d_1^2} \sin\left(\frac{1}{2} \text{Arc tg} \frac{d_1}{r^2 + s}\right)$$

$$\sin\left(\frac{1}{2} \text{Arc tg} \frac{d_1}{r^2 + s}\right) = \sin\left(\frac{1}{2} \text{Arc tg} \frac{2 \cdot 10^3 \omega}{3^2 \cdot 10^{12} + 4 \cdot 10^2}\right) \approx$$

$$\approx \sin \frac{1}{2} \frac{2 \cdot 10^3 \omega}{9 \cdot 10^{12}} \approx \frac{10^3 \omega}{9 \cdot 10^{12}} = \frac{\omega}{9 \cdot 10^9}$$

$$-F_1 = -\left(3 \cdot 10^6 + \frac{2}{3} \cdot 10^{-4}\right) \frac{\omega}{9 \cdot 10^9} = -\left(\frac{\omega}{3 \cdot 10^3} + \frac{2\omega}{3 \cdot 9 \cdot 10^{13}}\right) \approx -\frac{\omega}{3} \cdot 10^{-3}$$

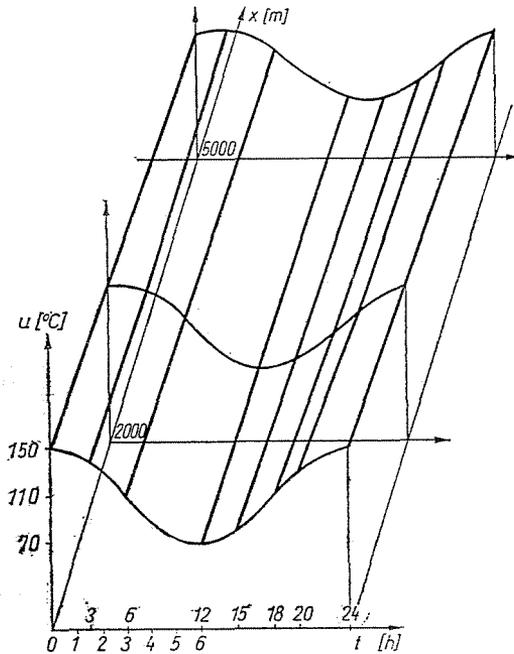


Fig. 2. The distribution of temperature as a function of pipeline length and time

Thus:

$$-E_0 = \frac{2}{3} \cdot 10^{-4}$$

$$-E_1 = -\frac{2}{3} \cdot 10^{-4}$$

$$-F_1 = -\frac{\omega}{3} \cdot 10^{-3}$$

and the initial parameters:

$$\begin{aligned} u_k &= 0 \\ A_0 &= 110 \text{ }^\circ\text{C} \\ A_1 &= 40 \text{ }^\circ\text{C} \end{aligned}$$

to yield:

$$\vartheta(x, t) = 110 \cdot e^{-\frac{2}{3} \cdot 10^{-4} x} + 40 e^{-\frac{2}{3} \cdot 10^{-4} x} \cdot \cos \left(\omega t - \frac{\omega}{3} \cdot 10^{-3} \cdot x \right) \quad (17a)$$

4. Calculation of the temperature distribution omitting the second-order term [1]

$$c \cdot \rho \frac{\partial u}{\partial t} = -c \cdot \rho \cdot w \cdot \frac{\partial u}{\partial x} - \frac{k \cdot p}{q} (u - u_k)$$

With $x = 0$, the initial condition is:

$$\begin{aligned} \vartheta(0, t) &= A_0 + A_1 \cos \omega t \\ \omega &= \frac{2\pi}{T}; \quad T = 24[\text{h}] \end{aligned}$$

Again, the general solution:

$$\vartheta(x, t) = A_0 e^{-\frac{b}{w} x} + A_1 e^{-\frac{b}{w} x} \cos \left(\omega t - \omega \frac{x}{w} \right)$$

where A_0 and A_1 are constants of the initial condition, and

$$b = \frac{k \cdot p}{c \rho q}$$

$w = 3 \cdot 10^3 \text{ m/h} = \text{flow rate}$

Calculation of constants

Initial conditions being identical, material constants are also the same:

$$\begin{aligned} -\frac{b}{w} &= -\frac{k \cdot p}{c \cdot \rho \cdot q \cdot w} = -\frac{2 \cdot 10^2}{10^3 \cdot 3 \cdot 10^3} = -\frac{2}{3} \cdot 10^{-4} \\ -\frac{\omega}{w} &= -\frac{\omega}{3 \cdot 10^3} = -\frac{\omega}{3} \cdot 10^{-3} \\ A_0 &= 110 \text{ }^\circ\text{C} \\ A_1 &= 40 \text{ }^\circ\text{C} \end{aligned}$$

The equation being in *final* form:

$$\vartheta(x, t) = 110e^{-\frac{2}{3} \cdot 10^{-4}x} + 40e^{-\frac{2}{3} \cdot 10^{-4}x} \cos \left(\omega t - \frac{\omega}{3} \cdot 10^{-3} x \right) \quad (17 \text{ b})$$

The two alternatives (17a involving the second-order term) and (17b) show complete identity.

In spite of different initial conditions, identity of the final form is due to neglects permissible in engineering practice:

a) neglect of terms of higher than second order in expanding the radicals into series;

b) application of the principle of sine value of a small angle equals the angle itself known for trigonometric functions.

Thus, negligibility of the heat conduction term $\left(\lambda \cdot \frac{\partial^2 t}{\partial x^2} \right)$ is proven.

Similarly, our former assumption on the temperature of the absorbent (the heated building) at the pipeline end not to react on the temperature distribution of the water, seems to be justified.

Water-temperature distribution behaves as if in a semi-infinite pipeline. The mains and the return pipes are dealt with separately, taking the "heated building" or other heat-consumer appliance as "heat source" for the return pipe.

5. Numerical example

Calculation of temperature along an $x = 5000$ m pipeline. The equation of temperature variation derived above is

$$\vartheta(x, t) = 110 \cdot e^{-\frac{2}{3} \cdot 10^{-4}x} + 40 \cdot e^{-\frac{2}{3} \cdot 10^{-4}x} \cos \left(\omega t - \frac{\omega}{3} \cdot 10^{-3} x \right) \quad (17 \text{ a,b})$$

For a better understanding, the detailed calculation results have been compiled in the following 7 tables:

Table 1

x [m]	0	50	100	200	500	1000	2000	5000
$-\frac{2}{3} \cdot 10^{-4} x = C$	0	$-\frac{1}{3} \cdot 10^{-2}$	$-\frac{2}{3} \cdot 10^{-2}$	$-\frac{4}{3} \cdot 10^{-2}$	$-\frac{1}{3} \cdot 10^{-1}$	$-\frac{2}{3} \cdot 10^{-1}$	$-\frac{4}{3} \cdot 10^{-1}$	$-\frac{1}{3}$
$e^{-\frac{2}{3} \cdot 10^{-4} x} = e^c$	1		0.994	0.986	0.967	0.936	0.875	0.716
$110 e^c$	110	109.56	109.34	108.46	106.37	102.96	96.25	78.76
$40 e^c$	40	39.84	39.76	39.44	38.68	37.44	35.00	28.64
$10^{-3} x$	0	0.05	0.1	0.2	0.5	1	2	5

Table 2

t	$3t$	$x \rightarrow$	0	50	100	200	500	1000	2000	5000
0	0	0	0	-0.05	-0.1	-0.2	-0.5	-1	-2	5
3	9	9	9	8.95	8.9	8.8	8.5	8	7	4
6	18	18	18	17.95	17.9	17.8	17.5	17	16	13
12	36	36	36	36.95	35.9	35.8	35.5	35	34	31
15	45	45	45	44.95	44.9	44.8	44.5	44	43	40
18	54	54	54	53.95	43.9	53.8	53.5	53	52	49
20	60	60	60	59.95	59.9	58.8	59.5	59	58	55
24	72	72	72	71.95	71.9	71.8	71.5	71	70	67

Table 3

$$\frac{\omega}{3}(3t - 10^{-3}x) \text{ [rad]}$$

0	50	100	200	500	1000	2000	5000
0	0.004	-0.08	-0.017	-0.043	-0.087	-0.17	-0.43
0.786	0.771	0.777	0.769	0.742	0.699	0.611	0.349
1.571	1.568	1.562	1.552	1.528	1.484	1.398	1.135
3.14	2.14	3.135	3.126	3.1	3.056	2.97	2.71
3.93	3.928	3.92	3.917	3.886	3.84	3.76	3.49
4.71	4.71	4.706	4.7	4.67	4.63	4.54	4.28
5.24	5.24	5.23	5.2	5.195	5.15	5.07	4.8
6.28	6.28	6.278	6.24	6.24	6.2	6.11	5.85

Table 4

$$\frac{\omega}{3}(3t - 10^{-3}x) \text{ [}^\circ\text{]}$$

0	50	100	200	500	1000	2000	5000
0	-0.23	-0.46	-0.97	-2.46	-4.98	-9.74	-24.6
45	44.7	44.5	44	42.5	41.2	35	20
90	89.8	89.6	89	87.6	85.1	80.1	65
180	180	179.5	179	177.5	175	170	155
225	224	224.1	224	222.8	220	215.6	200
270	270	269.6	269	267.5	265.2	206	245
300	300	299.7	298	297.5	295	200.5	275
360	360	359.5	358.5	358	355.5	350	335

Table 5

$$\cos \frac{\omega}{3}(3t - 10^{-3}x)$$

t	$\frac{e^{j\pi x}}{3}$	0	50	100	200	500	1000	2000	5000
0	+	1	1	1	0.9998	0.999	0.996	0.9858	0.91
3	+	0.7071	0.71	0.7133	0.7193	0.7373	0.751	0.8192	0.9397
6	+	0	0.005	0.006	0.0175	0.042	0.085	0.171	0.4226
12	-	1	1	1	0.9998	0.999	0.9962	0.9848	0.9063
15	-	0.7071	0.7150	0.7152	0.719	0.734	0.7660	0.813	0.9397
18	-	0	0	0.005	0.0175	0.0436	0.082	0.1736	0.4226
20	+	0.5	0.5	0.495	0.4695	0.4617	0.4426	0.3502	0.0872
24	+	1	1	1	0.9997	0.9994	0.9969	0.9448	0.9063

Table 6

$$40 \cdot e^{-\frac{2}{3} \cdot 10^{-4} x} \left[\cos \frac{\omega}{3} (3t - 10^{-3} x) \right]$$

$x=$	0	50	100	200	500	1000	2000	5000
$t = 0[h]$	40	39.8	39.8	39.4	39.6	37.4	34.5	26.1
3	28.3	28.2	28.3	28.3	28.4	28.2	28.4	27.9
6	0	0.2	0.24	0.69	1.63	3.2	6.00	12.1
12	40	39.8	39.8	39.4	38.6	37.6	34.4	26
15	28.3	28.3	28.3	28.3	28.3	28.8	28.4	27.9
18	0	0	0.2	0.69	1.69	3.08	6.1	12.1
20	20	19.9	19.7	18.55	17.9	15.9	12.3	2.04
24	40	39.8	39.8	39.4	38.7	37.5	33.1	26

Table 7

Sum

$$110e^{-\frac{2}{3} \cdot 10^{-4} x}$$

t	x	0	50	100	200	500	1000	2000	5000
0	$110e^{-\frac{2}{3} \cdot 10^{-4} x}$	110	109.56	109.34	108.46	106.37	102.96	96.25	78.76
		150	149.36	149.14	147.86	144.97	140.36	130.75	104.86
3		138.3	138.86	137.64	136.76	134.77	131.16	124.65	106.66
6		110	109.76	109.68	109.15	108	106.16	102.25	90.86
12		070	69.76	67.54	69.06	67.77	65.36	61.85	52.76
15		81.7	81.26	81.04	80.16	78.07	74.16	67.85	60.86
18		110	110	109.8	107.77	104.68	99.88	90.15	66.66
20		130	129.46	129.04	127.01	124.27	118.86	108.55	80.8
24		150	149.8	149.14	147.86	145.07	140.46	129.35	104.76

Fig. 1. shows the field of temperature complying with numerical results. We took the liberty to repeat our former statement that the simplifications made earlier were fully justified, and the simplified solution of the partial differential equation suits calculation of temperature conditions.

variables being:

$$t = 0, 3, 6, 12, 15, 28, 20, 24 \text{ (h)}$$

$$x = 0, 50, 100, 200, 500, 1000, 2000, 5000 \text{ (m)}$$

In calculating the cooling of the fluid along pipelines, the second-order term in the differential equation was neglected. Now, the correctness of the neglect is analysed and justified.

Summary

Our previous paper dealt with heat losses in pipelines during hours of peak operation. The developed computation method to determine temperature drop and heat losses in hot water networks during cyclic temperature variation was checked by measurements. The computation method involved some approximations, neglects those justified empirically and by measurements. In this paper, the admissibility of the neglect will be theoretically analysed and justified.

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