OPTIMUM DESIGN OF AXIAL FLOW FANS WITH CAMBERED BLADES OF CONSTANT THICKNESS

By

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Introduction

When developed into plane, the blading of axial flow fans gives a cascade having a chord pitch ratio l/t (see Fig. 1) usually diminishing in radial direction. At the tip the blades may be regarded as single airfoils. This same assumption at the blade root is impermissible and dimensioning must be performed on the basis of cascade measurements.

A characteristic parameter of cascade measurements is the angle β_1 included between the approach velocity and the normal of the cascade. If the approach angle is small, it is a steep cascade. Steep cascades are capable of a greater deflection and their losses are smaller than those of flat cascades. At the same time a steeper cascade attains a higher axial velocity at the fan impeller than the circumferential velocity and this fact mostly affects the economy of fan operation (static efficiency). High axial velocity in the annulus area diminishes partly through the sudden increase of the cross-section beyond the hub, partly in the diffuser — if a diffuser is applied. But in either case, a considerable loss occurs which increases rapidly (approximately quadratic-

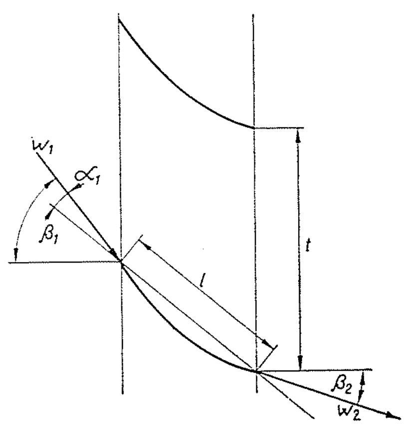

Fig. 1. Flow deflected by a cascade

ally) with higher axial velocity. In case of delivery into the atmosphere or into a large space, even the kinetic energy of outlet gets lost. With a fan of given geometry the loss is also proportional to the square of the axial velocity. Under these conditions it seems worth-while to study the losses of cascades in conjunction with the outlet losses and seek for a common optimum.

As to the blade cross-section, due to their simple structure, this study concentrates on cambered blades, most frequently applied in fans.

Optimum camber

Blade efficiency is generally characterized by the lift to drag ratio: c_f/c_e. In fans a so-called secondary drag is associated to the profile drag. Like in the case of the induced drag of single finite blades,

$$c_{e\ sz} = k\, c_f^2.$$

According to H. Wallis [1] in a cambered blade $k = 0{,}025$.
The lift to drag ratios calculated with this secondary drag:

$$\varepsilon = \frac{c_f}{c_e + 0{,}025\, c_f^2}$$

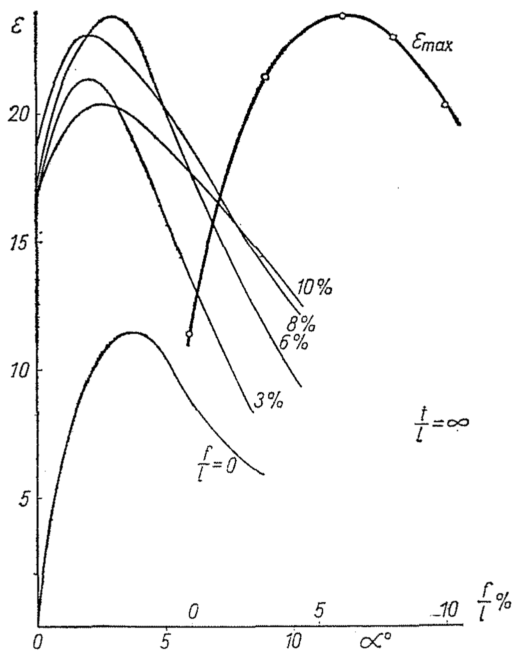

Fig. 2. Lift to drag ratio of single cambered plates vs. camber

computed from the measurements of single blades ($t/l = \infty$) are shown in Fig. 2. The diagram is based on H. Wallis' [1] measurements. The set of curves having a cambering parameter f/l yields the lift to drag ratio as a function of the angle α of attack. In the lower part of the chart their maximum is indicated vs. camber f/l. The peak value the ε_{max} curve (23.8) lies at a camber of $f/l = 6\%$.

Measurement results on cascades composed of cambered blades have been published by Ikui, Inoue, Kaneko [2]. In their paper they give a chart of the drag coefficient for a cascade of density $t/l = 1$ and approach angle $\beta_1 = 40°$, from which the lift and drag coefficients were calculated with the formulas:

$$c_e = \frac{t}{l} \frac{\cos \beta_\infty}{\cos^2 \beta_1} \xi$$

$$c_f = 2 \frac{t}{l} (\mathrm{tg}\, \beta_1 - \mathrm{tg}\, \beta_2) \frac{\cos^2 \beta_\infty}{\cos (\beta_\infty - \delta)},$$

where $\delta = c_e/c_f \cdot c_f$ being no explicit formula, it may reasonably be calculated by iteration. For the derivation of the formulas, see Appendix.

The calculation method implies that secondary losses appear only among the fan losses rather than in cascade measurements.

The so obtained lift drag ratios ε for a cascade density $l/t = 1$ and an approach angle $\beta_1 = 40°$ is shown in Fig. 3, the attack angle α_1 being the one included between the approach velocity and the chord.

The ε_{max} curve has its maximum at a camber of 10%. This maximum is 16.1 — considerably lower than that of single airfoils.

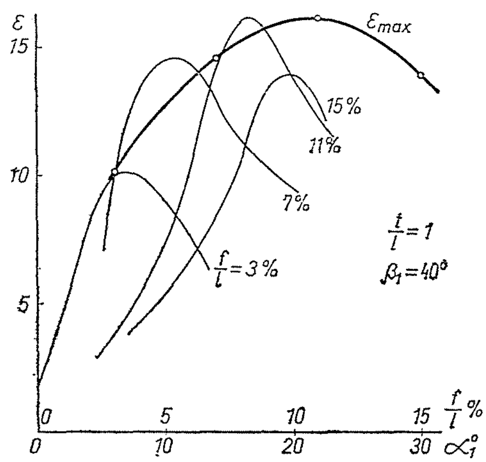

Fig. 3. Lift to drag ratio of cascades function vs. camber

Correlation between optimum lift to drag ratio and cascade density

Ikui, Inoune and Kaneko published drag factors of cascades with different densities only for blades of a 15% camber. The lift to drag ratios calculated with this value are shown in Fig. 4. The lacking monotony in the variation of the ε_{max} values as a function of t/l may be explained solely by uncertainties in the measurements. As a general tendency, with increasingly smaller densities of the cascade ($t/l \to \infty$) ε_{max} increases, since, however, with this camber no coefficients of single airfoils have been measured, we do not know the location of the asymptote of the curve, either.

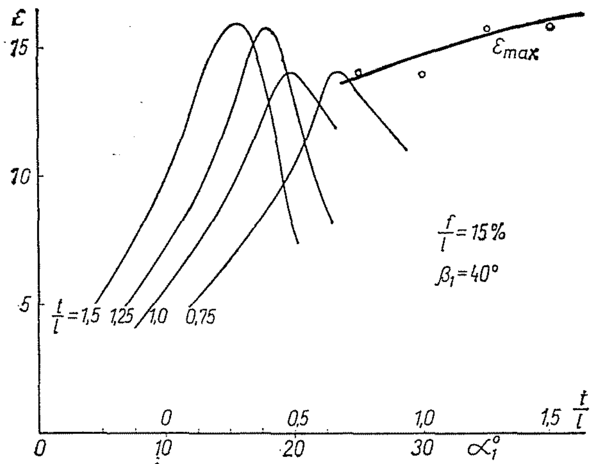

Fig. 4. Lift to drag ratio of cascades vs. density

The changes seem to be less than in the case of smaller cambers. Namely, according to Fig. 2 and 3,

	$t/l = 1$	$t/l = \infty$
for a camber of 6% ε_{max} is	13.7	23.8
for a camber of 10% ε_{max} is	16.1	20.3

With a greater camber, accordingly, the higher density of the cascade will cause a smaller drop in the value of ε_{max}.

Variation of the optimum lift to drag ratio as a function of the approach angle

Ikui—Inoue—Kaneko published a chart for different approach angles β_1 for a cascade density $t/l = 1$ and a camber $f/l = 7\%$. Its conversion is illustrated in Fig. 5.

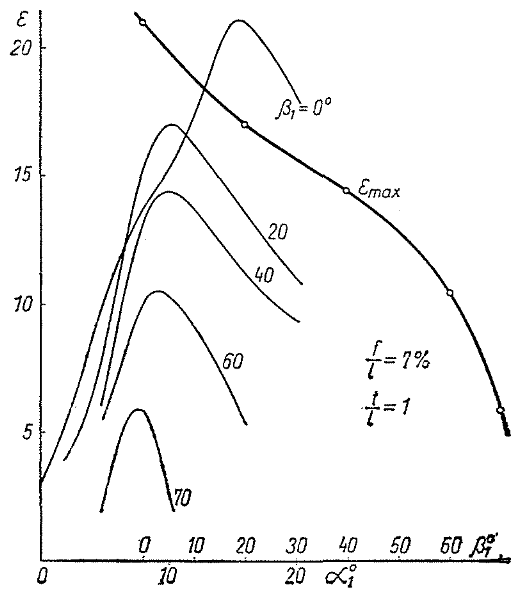

Fig. 5. Lift drag ratio of cascades vs. approach angle

The values of ε_{max} are seen to diminish rapidly, depending on the approach angle β_1. The value of $\varepsilon_{max} = 21$ obtained for $\beta_1 = 0°$, viz. with the flow perpendicular to the cascade, will drop to 10.5 for $\beta_1 = 60°$ and diminish still more rapidly above this value to be zeroed for about 80°. This is why in axial flow fans the optimum approach angle is critical. Namely, to minimize the outlet loss, a flat type of cascade is required.

With respect to the parameters of this curve it may be stated that a 7% camber suitably characterizes fan blades. On the mid-radius where losses are generally calculated, a density of $t/l = 1$ is greater than usual. Blading mostly approximates or attains the density t/l at the hub while on the mid-radius usually $t/l \approx 1.5$. Due to the scant material available for the time being, in our examinations we relied on Fig. 5. Accordingly, our findings will hold true for fans with a relatively dense spacing of the blades.

Losses in axial flow fans

In what follows two different axial flow fans will be examined:

a) one with a straightener

b) one with a prerotator.

Since in both fans axial outflow had been assumed, there will be no rotation losses.

The impeller and secondary losses are calculated on the mid-radius with the following formula:

$$\frac{\Delta p_{j+sz}}{\Delta p_{\bar{o}id}} = \frac{w_\infty}{u} \frac{1}{\cos(\beta_\infty - \delta)} \frac{1}{\varepsilon_{\max.}},$$

where u means the circumferential velocity at the mid-radius. With $\varepsilon_{\max}$ it was assumed that the blades had been adjusted to the optimum angle of attack.

Neglecting δ and without prerotation, Fig. 6 is arrived at

$$\frac{w_\infty}{u} \frac{1}{\cos\beta_\infty} = \frac{c_a}{n} \frac{1}{\cos^2\beta_\infty} = \frac{c_a}{u} \frac{\left(u - \frac{\Delta c_u}{2}\right)^2 + c_a^2}{c_a^2} =$$

$$= \frac{c_a}{n} \frac{\left(1 - \frac{1}{2}\frac{\Delta c_u}{u}\right)^2 + \left(\frac{c_a}{u}\right)^2}{\left(\frac{c_a}{u}\right)^2} = \frac{\left(1 - \frac{1}{2}\frac{\Delta c_u}{u}\right)^2 + \operatorname{ctg}^2\beta_1}{\operatorname{ctg}\beta_1},$$

where

$$\beta_1 = \frac{u}{c_a}.$$

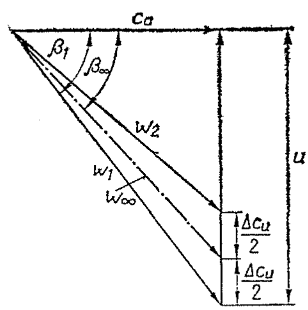

Fig. 6. Velocity pattern of axial flow fan without prerotation

With prerotation (Fig. 7), based on the same assumption

$$\frac{w}{u_\infty} \frac{1}{\sin\gamma_\infty} = \frac{\left(1 + \frac{1}{2}\frac{\Delta c_u}{u}\right)^2 + \left(\frac{c_a}{u}\right)^2}{\frac{c_a}{u}},$$

but in this case

$$\beta_1 = \operatorname{arctg} \frac{u + \Delta c_u}{c_a} = \operatorname{arctg} \frac{1 + \dfrac{\Delta c_u}{u}}{\dfrac{c_a}{u}}$$

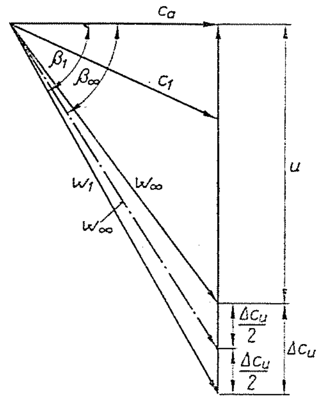

Fig. 7. Velocity pattern of axial flow fan with prerotation

Thus, this loss is, in both cases, expressed by the characteristic ratios $\Delta c_u/u$ and c_a/u, respectively.

The losses regarded to be constant are as follows:

The annulus loss	$\dfrac{\Delta p_{gy}}{\Delta p_{\ddot{o}\,id}}$	$= 0.03$
The tip clearance loss	$\dfrac{\Delta p_r}{\Delta p_{\ddot{o}\,id}}$	$= 0.03$
The deflector loss	$\dfrac{\Delta p_t}{\Delta p_{\ddot{o}\,id}}$	$= 0.03$
Total:		0.09

The diffuser and the outlet losses are expressed in terms of the dynamic pressure of the axial velocity:

$$\Delta p_k = k \frac{\varrho}{2} c_a^2$$

whereby:

$$\frac{\Delta p_k}{\Delta p_{\ddot{o}\,id}} = k \frac{\dfrac{\varrho}{2} c_a^2}{\varrho u \Delta c_u} = \frac{k}{2} \left(\frac{c_a}{u}\right)^2 \frac{u}{\Delta c_u} = \frac{k}{2} \operatorname{ctg}^2 \beta_1 \frac{1}{\dfrac{\Delta c_u}{u}}$$

The sum of the losses calculated according to the above is indicated in Fig. 8 for a fan with a straightener.

The figure shows two sets of curves. The upper set stands for a mid-radius deflection $\Delta c_u/u = 0.25$, the lower one for $\Delta c_u/u = 0.5$. A further parameter is the factor k of the outlet loss which varies between 0 and 1.

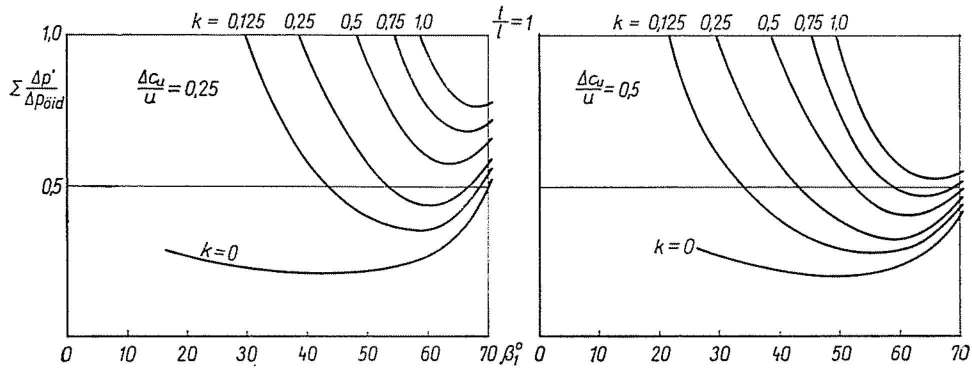

Fig. 8. Total loss of axial flow fan without prerotation

The value of $k = 0$ means that no outlet loss is taken into consideration; in other words the total pressure difference is regarded to be useful. The value of $k = 0.5$, for instance, corresponds to a diffuser extending to twice its original cross-section outlet velocity ($c_{ki} = c_a/2$) and to a combined efficiency $\eta_d = 0.67$ of the diffuser and the Borda—Carnot transition.

Evidently, for $k = 0$, that is with no outlet loss, an approach angle $\beta_1 = 40$ to $50°$ seems to be optimum, practically independently of the value of $\Delta c_u/u$ and with a very flat minimum. The value of the minimum is 0.2 which corresponds to a fan efficiency $\eta = 0.8$. This seems to be realistic, although with a less densely spaced or an airfoil blading, a higher efficiency can be attained.

Taking consideration also of the outlet loss up to approximately $\beta = 68°$ the minima shift towards flatter cascades. With greater deflections ($\Delta c_u/u = 0.5$) the optimum approach angle is around 60 to 65° and the static efficiency may drop even below 50%. With smaller deflection ($\Delta c_u/u = 0.25$) a still flatter cascade is more favourable and the static efficiency will be less than 30%.

With prerotation (Fig. 9) the overall efficiency is around 74%. Thus, from this aspect prerotation is unfavourable. The $\beta_1 = 40$ to $50°$ which yields the optimum, corresponds to a cascade which is considerably steeper than usual. However, such flatter cascades will be justified if we study the static efficiency curves.

With great deflection ($\Delta c_u/u = 0.5$) and small outlet loss ($k = 0{,}25$) the optimum, will be at $\beta_1 = 60$ to $65°$ but even this optimum gives a very low static efficiency. In the case of $\Delta c_u/u = 0.25$ the static efficiency is even worse.

A comparison of Fig. 8 and 9 will indicate that for static efficiency a fan with straightener is the choice of preference.

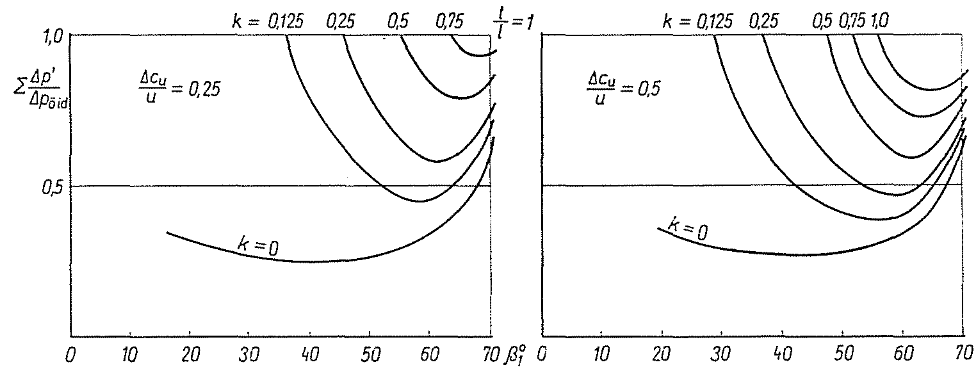

Fig. 9. Total loss of axial flow fan with prerotation

APPENDIX

Calculation of force factors in cascades

Fig. 10 shows the forces acting upon a length dr of one element of a cascade. The pressure loss $\Delta p'$ is caused by the drag dF_e:

$$\Delta p' = \frac{1}{t\,\mathrm{d}r}\,\mathrm{d}\,F_e \cos\beta_\infty = \frac{\varrho}{2}\,w_\infty^2\,\frac{l}{t}\,c_e \cos\beta_\infty\,.$$

The loss coefficient of the cascade:

$$\zeta = \frac{\Delta p'}{\frac{\varrho}{2}\,w_1^2} = \left(\frac{w_\infty}{w_1}\right)^2 \frac{l}{t}\,c_e \cos\beta_\infty =$$

$$= \frac{\cos^2\beta_1}{\cos^2\beta_\infty}\,\frac{l}{t}\,c_e \cos\beta_\infty = \frac{\cos^2\beta_1}{\cos\beta_\infty}\,\frac{l}{t}\,c_e$$

whence:

$$c_e = \xi\,\frac{t}{l}\,\frac{\cos\beta_\infty}{\cos^2\beta_1}$$

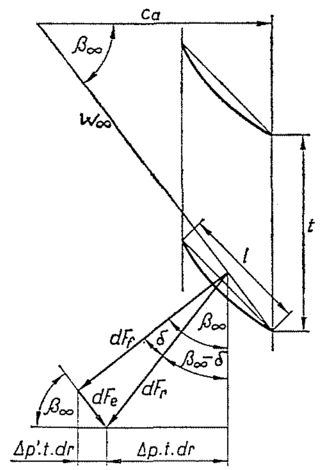

Fig. 10. Forces acting upon the blades of axial flow fans

On the basis of the momentum theory, written down in the direction of the cascade plane, and making use of the approximate equalities

$$dF_f \approx dF_r$$

and

$$\frac{\mathrm{d}\,F_e}{\mathrm{d}\,F_f} = \frac{c_e}{c_f} = \operatorname{tg}\delta \approx \delta \quad (\text{since } \delta = 1 \div 2^\circ)\,,$$

we have

$$dF_f\ \cos(\beta_\infty - \delta) = \varrho t\ dr\ c_a\,\Delta c_u,$$

where Δc_u is the velocity variation in the direction of the cascade.

Thus:

$$\frac{\varrho}{2}\,w_\infty^2\,l\,dr\,c_f\cos\left(\beta_\infty - \delta\right) = \varrho\,t\,dr\,c_a\,\Delta c_u\,,$$

whence:

$$c_f = \frac{t}{l}\,\frac{2\,c_a\,\Delta c_u}{w_\infty^2\,\cos\left(\beta_\infty - \delta\right)} = \frac{t}{l}\,\frac{2\,c_a^2\,\dfrac{\Delta c_u}{c_a}}{w_\infty^2\,\cos\left(\beta_\infty - \delta\right)} =$$

$$= 2\,\frac{t}{l}\left(\operatorname{tg}\beta_1 - \operatorname{tg}\beta_2\right)\frac{\cos^2\beta_\infty}{\cos\left(\beta_\infty - \delta\right)}$$

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Summary

To minimize the outlet losses in axial flow fans (to boost the static efficiency) large approach angles, viz. flat cascades, seem to be preferable. The lift to drag ratio of flat cascades, on the other hand, is considerably less than that of steeper cascades. The optimum between the two, depending on the layout (prerotation or straightening), deflection ($\Delta c_u/u$) and the outlet loss coefficient

$$k = \frac{\Delta p_k}{\frac{\varrho}{2} c_a^2}$$

yields the most advantageous approach angle.

Based on measurements on cascades made up of cambered blades, cascades steeper than the usual seem to offer the most favourable design.

At the Department of Fluid Mechanics, Technical University, Budapest, measurements are in progress to elucidate the still unclarified problems.

References

1. WALLIS, R. A.: Axial Flow Fans; Newnes, London, 1961.
2. IKUI, T., INOUE, M., KANEKO, K.: Two-Dimensional Cascade Performance of Circular-Arc Blades; ISME 9.

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