INVESTIGATION OF THE EDGE - INFLUENCE ON SPHERICAL SHELLS

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Practical application of the theory of shells is often faced by the problem of joints.

Tables and diagrams containing strain coefficients ease computations, especially for short shells, where, the strains produced by stresses in one edge of the shell cannot be neglected on the other edge.

Below axisymmetric edge-influence in constant-thickness, spherical shells will be considered. The system of differential equations of shells is solved by approximate formulae obtained by using Legendre functions.

The solution of the system of differential equations

The differential equation for thin, constant-thickness, spherical shells subject to zero inner pressure is of the form [1], [4]:

$$\ddot{Q}_\vartheta + \dot{Q}_\vartheta \operatorname{ctg}\vartheta - Q_\vartheta(\operatorname{ctg}^2\vartheta - \mu) = - Eh\chi_\vartheta$$

$$\ddot{\chi}_\vartheta + \dot{\chi}_\vartheta \operatorname{ctg}\vartheta - \chi_\vartheta(\operatorname{ctg}^2\vartheta + \mu) = \frac{R^2}{D} Q_\vartheta$$

Introducing the notations:

$$V = Q_\vartheta R$$

$$\frac{d}{d\vartheta}(x) = (x)^{\cdot}$$

$$L(x) = (x)^{\cdot\cdot} + (x)^{\cdot} \operatorname{ctg}\vartheta - (x) \operatorname{ctg}\vartheta$$

The system of equations

$$L\,L(V) + \mu L(V) = - EhR \cdot L(\chi_\vartheta)$$

$$L\,L(\chi_\vartheta) - \mu L(\chi_\vartheta) = \frac{R}{D} L(V)$$

is obtained.

The V and χ_ϑ functions can be separated:

$$L\,L(V) + \lambda^2 V = 0$$

$$L\,L(\chi_\vartheta) + \lambda^2 \chi_\vartheta = 0$$

The solution of this system of equations according to [3]:

$$V = C_1 X_1(\vartheta, \lambda) + C_2 X_2(\vartheta, \lambda) + C_3 Y_1(\vartheta, \lambda) + C_4 Y_2(\vartheta, \lambda)$$

$$\chi_\vartheta = \frac{1}{EhR}\Big\{X_1(C_3\lambda + \mu C_1) + X_2(C_4\lambda + \mu C_2) + Y_1(-C_1\lambda + \mu C_3) + Y_2(-C_2\lambda + \mu C_4)\Big\}$$

$$Q_\vartheta = \frac{V}{R}$$

$$N_\vartheta = -\operatorname{ctg}\vartheta \cdot Q_\vartheta$$

$$N_\varphi = -\dot{Q}_\vartheta$$

$$M_\vartheta = -\frac{D}{R}\left(\dot{\chi}_\vartheta + \mu\chi_\vartheta \operatorname{ctg}\vartheta\right)$$

$$w = \frac{R\sin\vartheta}{-Eh}\left(N_\varphi - \mu N_\vartheta\right)$$

For functions X_1, X_2, Y_1 and Y_2 and their derivatives see [3].

Comparison with other procedures

The above method will be compared with other procedures according to [2].

The differential equation for thin constant-thickness spherical shells has different ways of solution such as methodes by Geckeler, Esslinger and Love.

These methods are numerically compared in [2] too, for the loading cases shown on Fig. 1.

Data published in [2] and obtained by the above approximate method are compared in Table 1.

From the table, the accuracy of the above method is apparent. Its closed formulae permit computerization.

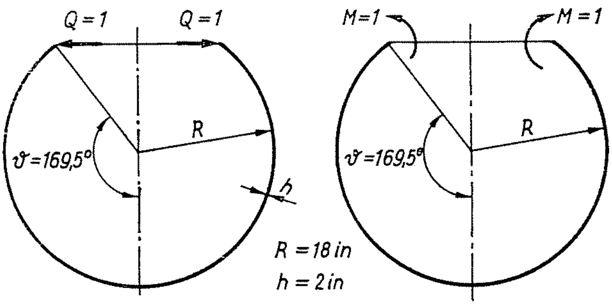


Fig. 1.

Reference	Method	Unit force on the edge		Unit moment on the edge	
		Ehw	$Eh\chi_\vartheta$	Ehw	$Eh\chi_\vartheta$
[2]	Geckeler	147.01	54.50	54.50	40.41
	Esslinger	163.45	46.62	46.62	35.25
	Love *a)*	166.16	50.05	49.95	37.27
	b)	164.66	47.71	47.64	36.24
[3]	The above approximate method	154.80	48.75	48.75	37.17

Analysis of the edge — influence

Data were obtained by the approximate solution for the cases shown in Fig. 2.

Deflections normal to the axis of rotation and angular displacements due to the indicated loads.

Positive directions are those in Fig. 3.

Values of α_1, α_2 and R/h varied according to Table 2.

Fig. 4 through 7 contain the obtained data. Figs 4 and 5 show ERw and $ER^2\chi_\vartheta$ values characteristic of the deflection and angular displacement, respectively, of the loaded edge as functions of α_1, α_2 and R/h.

α_1 degrees	α_2 degrees	R/h
90	70, 50, 30, 10	
70	50, 30, 10	30—450
50	30, 10	
30	10	

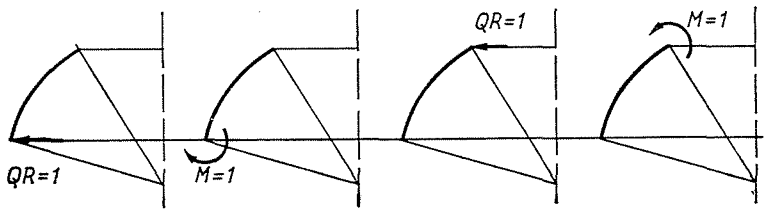


Fig. 2.

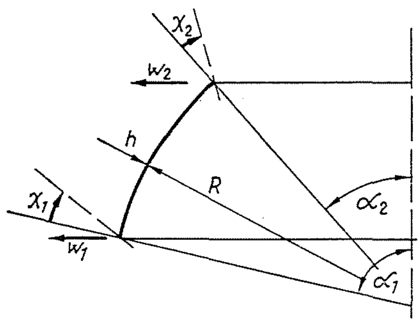


Fig. 3.

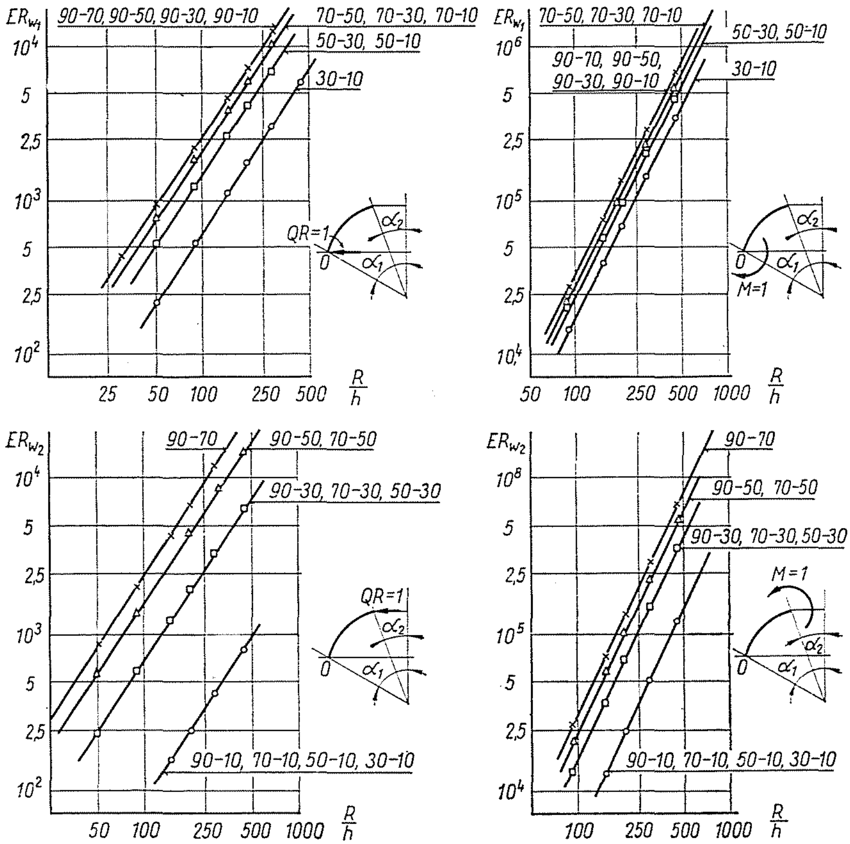
For R/h 40 (and $\Delta\alpha = \alpha_1 - \alpha_2 = 20°$) the deflection and the angular displacement of the loaded edge are seen to differ by max. 5% from data obtained on closed shells, thus practically these diagrams are identical. They differ only for low R/h values.

Figs 6 and 7 show data characteristic of the deflection and angular displacement of the unloaded edge. According to the data, by varying the value R/h the behaviour of the unloaded edge can significantly be influenced.

Data obtained for $\alpha_1 - \alpha_2 > 40$ slightly differ from zero and decrease quickly with the increase of R/h.

This is illustrated by means of cases $\alpha_1 = 90°$ and $\alpha_2 = 50°$.

The above computing method is of use for investigating axisymmetrical holes in more or less convex bottoms of pots to be considered as spherical shells.



ER_{w_1}
90-70, 90-50, 90-30, 90-10
70-50, 70-30, 70-10
50-30, 50-10
30-10
QR=1
α_1
α_2
0
25 50 100 250 500
$\frac{R}{h}$
ER_{w_1}
70-50, 70-30, 70-10
50-30, 50-10
90-70, 90-50,
90-30, 90-10
30-10
M=1
50 100 250 500 1000
ER_{w_2}
90-70
90-50, 70-50
90-30, 70-30, 50-30
90-10, 70-10, 50-10, 30-10
QR=1
50 100 250 500 1000
ER_{w_2}
90-70
90-50, 70-50
90-30, 70-30, 50-30
90-10, 70-10, 50-10, 30-10
M=1
100 250 500 1000

Fig. 4.

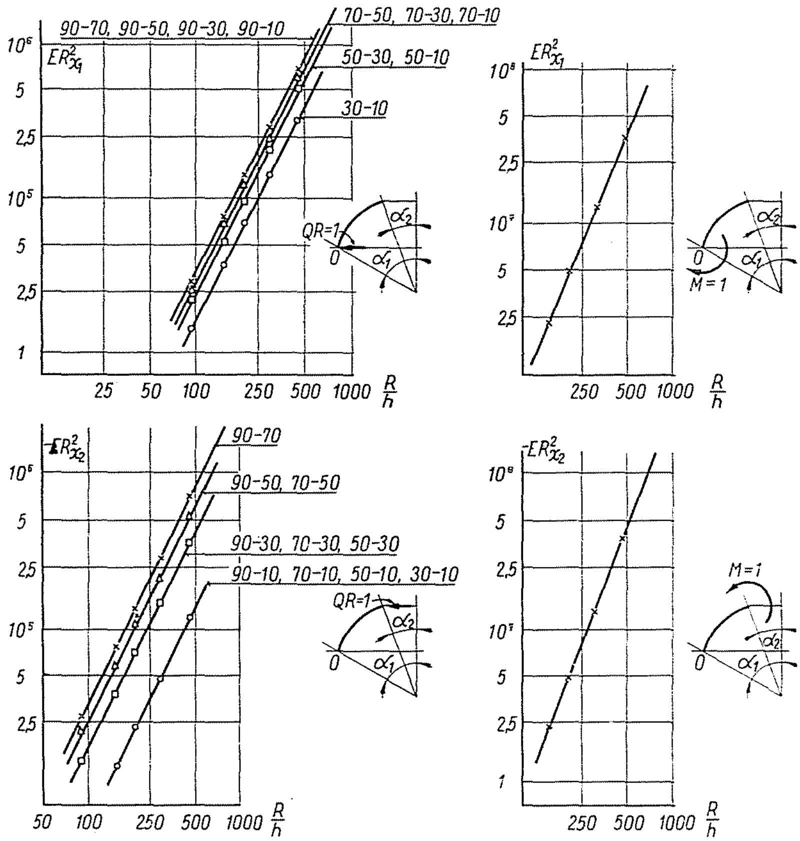
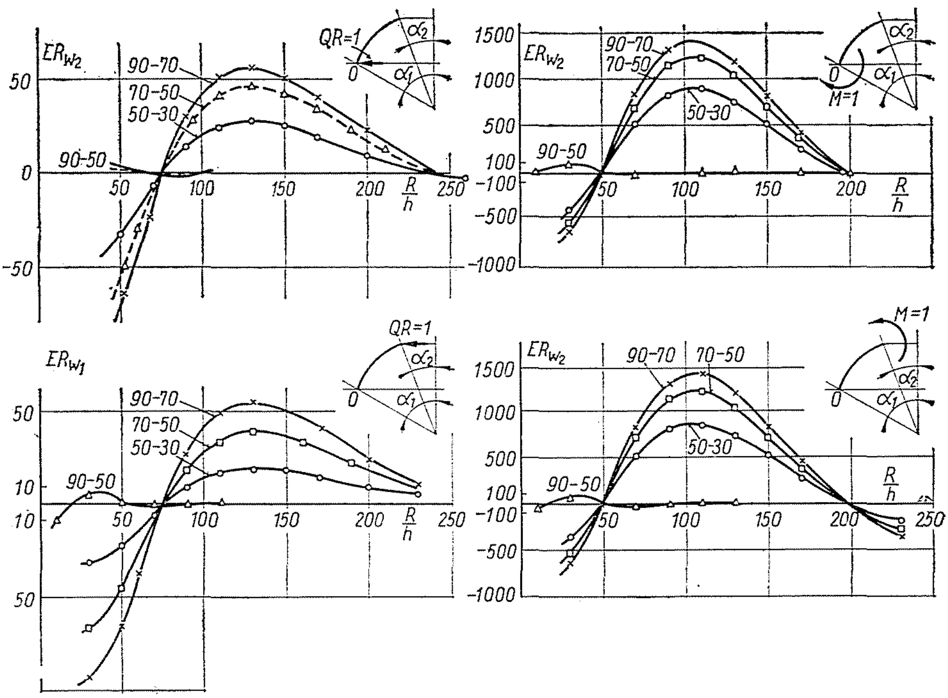

90-70, 90-50, 90-30, 90-10
70-50, 70-30, 70-10
50-30, 50-10
30-10
QR=1
M=1
90-70
90-50, 70-50
90-30, 70-30, 50-30
90-10, 70-10, 50-10, 30-10

Fig. 5.



ER_{w_2}
90-70
70-50
50-30
90-50
QR=1
α_2
α_1
0
50
0
-50
100
150
200
250
R/h
1500
1000
500
100
-100
-500
-1000
M=1
ER_{w_1}
10

Fig. 6.

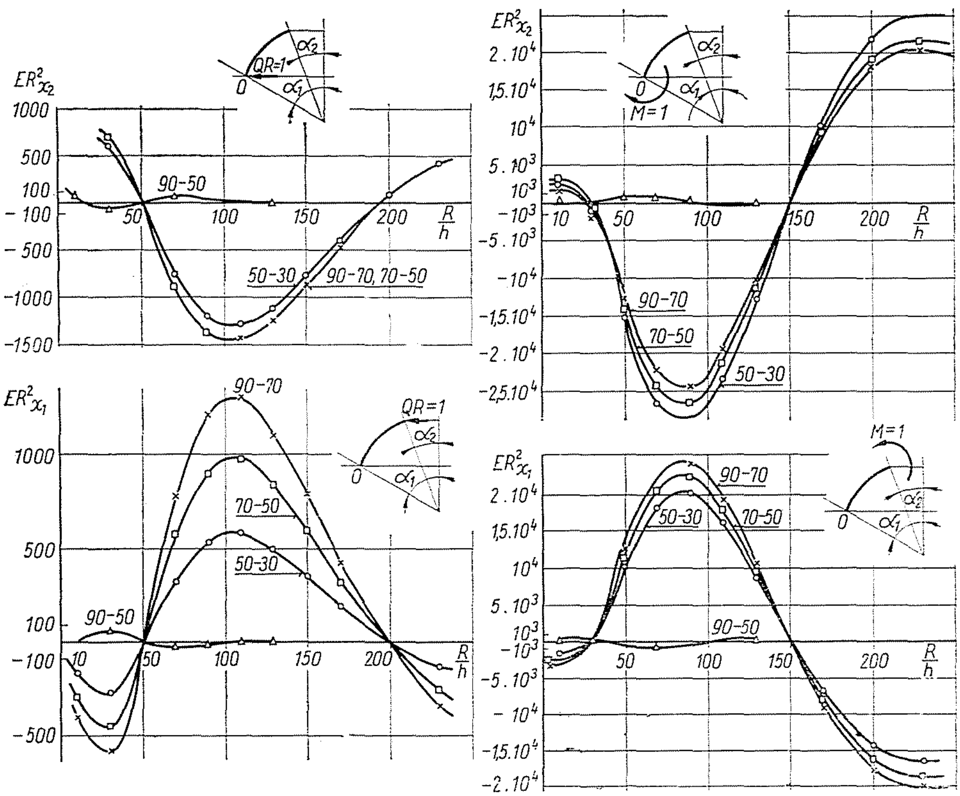


Fig. 7.

Summary

An approximate solution of the system of differential equations is applied to investigate the edge-influence in spherical shells.

Numerical data delivered by the approximate solution are compared with those from other solutions and plotted in diagrams. The computing method permits the numerical analysis of axisymmetrical holes in spherical shells.

Notations

R	shell radius
h	shell thickness
X_1, X_2, Y_1, Y_2	formulae derived from Legendre — polynomials
$D = Eh^3/12(1-\mu^2)$	flexural rigidity constant of shell
M_ϑ	meridional bending moment (unit length)
N_ϑ	stress resultant in ϑ-direction (unit length)
N_φ	stress resultant in φ-direction (unit length)
Q_ϑ	transverse meridional shearing force (unit length)
ϑ	angle between the normal of the meridian section and the axis of rotation

$\lambda^2 = EhR^2/D - \mu^2$	
E	modulus of elasticity
w	displacement of the middle surface normal to the axis of rotation
χ_ϑ	angular displacement of the tangent to the meridian curve
C_1, C_2, C_3, C_4	constants depending on initial conditions

References

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