ERRORS OF THE GECKELER APPROXIMATION FOR CONICAL SHELLS

By

M. ÖRVÖS, L. LEGEZA, and P. REUSS

Department of Chemical Machines and Agricultural Industries, Technical University, Budapest

(Received May 15, 1975)

Presented by Prof. Dr. S. SZENTGYÖRGYI

Introduction

Stress analysis of axisymmetric conical shells is rather tedious by exact methods. Approximations of different accuracies may mean a simplification. For the analysis of stresses and deformations in axisymmetric conical shells, one of the most widely extended approximations is that by Geckeler.

Outputs from a Geckeler approximation will be compared to those from a numerical method likely to be exact. Varying the wall thickness of the conical shell, and the half cone apex angle as parameters, the relative error of the approximation is determined, and geometry limits of its applicability are suggested. A case of an isotropic, homogeneous conical shell under axisymmetric edge force and edge moment is analysed.

Edge influence coefficients

Relationship based on the Geckeler approximation for edge forces and edge moments lending themselves for examining the variation of stresses and deformations along the generatrix have been published in [1].

Stress and deformation maxima are at the edge, therefore, often it is sufficient to restrict analyses there. The edge influence coefficients are [2]:

$$w_H = \frac{\cos^2\alpha}{2B\,k^3} \cdot H\,,$$

$$w_M = \frac{\cos\alpha}{2B\,k^2} \cdot M,$$

$$\beta_H = \frac{\cos\alpha}{2Bk^2} \cdot H\,,$$

$$\beta_M = \frac{1}{B\,k} \cdot M\,,$$

where

$$B = \frac{E\delta^3}{12(1-\mu^2)},$$

$$k = \frac{\sqrt[4]{3(1-\mu^2)}}{\sqrt{R\delta}}.$$

Assuming $\mu = 0{,}3$, edge influence coefficients can be written in dimensionless form:

$$\frac{w_H}{R} = 2{,}57\left(\frac{R}{\delta}\right)^{3/2}\sqrt{\cos\alpha}\cdot\frac{H}{ER} = \bar{w}_H\cdot\bar{H},$$

$$\frac{w_M}{R} = 3{,}30\left(\frac{R}{\delta}\right)^{2}\cdot\frac{M}{ER^2} = \bar{w}_M\cdot\bar{M},$$

$$\beta_H = 3{,}30\left(\frac{R}{\delta}\right)^{2}\cdot\frac{H}{ER} = \bar{\beta}_H\cdot\bar{H},$$

$$\beta_M = 8{,}48\left(\frac{R}{\delta}\right)^{3/2}\frac{1}{\sqrt{\cos\alpha}}\cdot\frac{M}{ER^2} = \bar{\beta}_M\cdot\bar{M}.$$

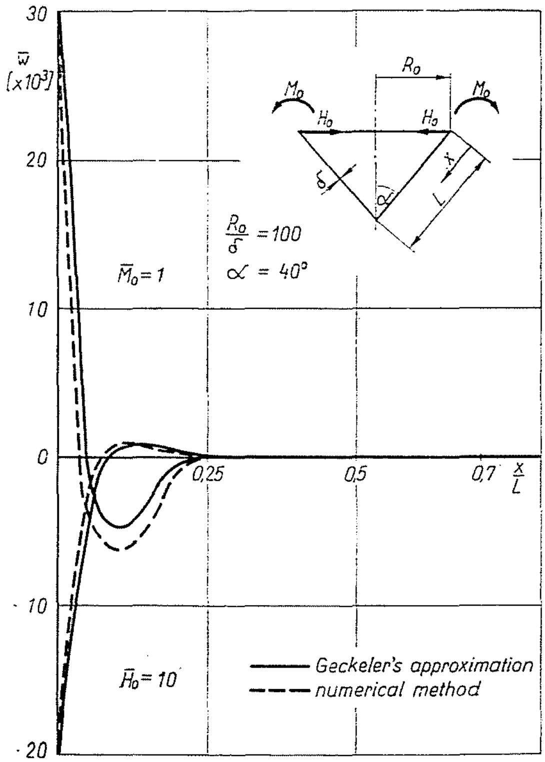
The calculation method for shells of revolution, serving as basis of comparison, starts from the numerical solution of the couple of differential equations deduced by REISSNER [3], based on the method of finite differences that will not be described here. For further applications see [4].

Discussion of results

Deformations obtained by the numerical method and by the Geckeler approximation are compared in diagrams plotting the course of deformations along the cone generatrix (Figs 1, 2, 3, 4). Results from both methods are displayed on the same diagram.

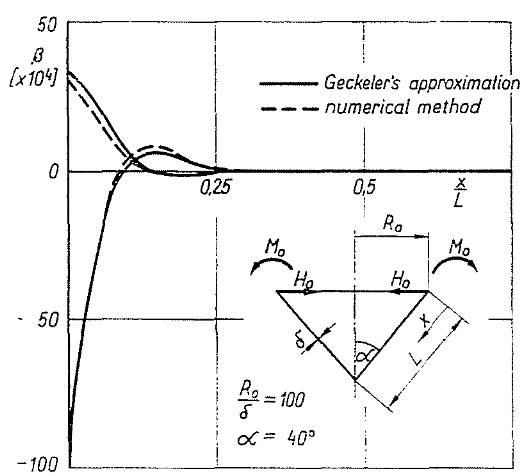
Deformations are linearily dependent on edge loads, and subject to the principle of superposition. For the sake of simplicity, effect of unit edge force or unit edge moment has been examined. Courses are seen to be of identical character, and the approximation is irrelevant to the damping length.

Figs 5 and 6 show the percentage relative error of the Geckeler approximation. The numerical method can practically be considered as exact, therefore, its results can be taken as references. Diagrams show edge deformation errors, with curve family parameters of wall thickness ratio R/δ and half cone apex angle α, respectively, where α ranges from 10° to 85° and the wall thickness ratio from 20 to 200.



30
$\bar{w}$ [x10³]
20
10
0
-10
-20
0,25
0,5
0,7
x/L
R_o
M_o
H_o
δ
α
x
L
$R_o/\delta = 100$
$\alpha = 40°$
$\bar{M}_o = 1$
$\bar{H}_o = 10$
Geckeler's approximation
numerical method

Fig. 1



50
β [x10⁴]
0
-50
-100
0,25
0,5
x/L
Geckeler's approximation
numerical method
R_o
M_o
H_o
δ
α
x
L
$R_o/\delta = 100$
$\alpha = 40°$

Fig. 2

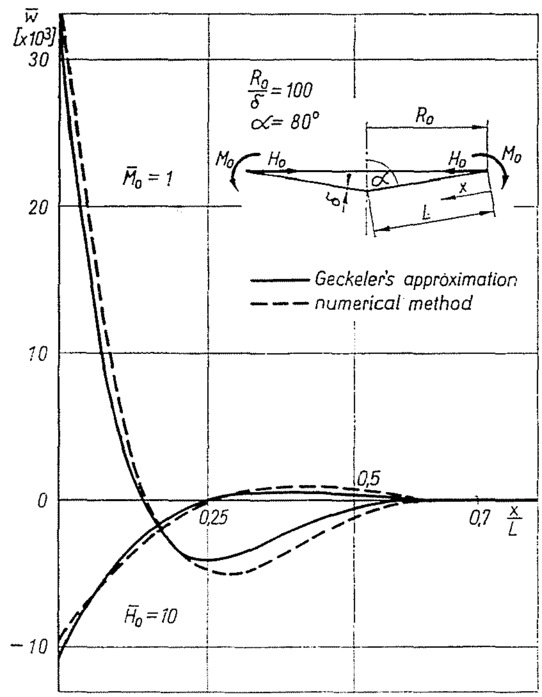

$\bar{w}$ [x10³]
30
20
10
0
−10
$\frac{R_0}{\delta}$ = 100
α = 80°
R_0
M_0
H_0
$\bar{M}_0$ = 1
$\bar{H}_0$ = 10
Geckeler's approximation
numerical method
0,25
0,5
0,7
$\frac{x}{L}$

Fig. 3

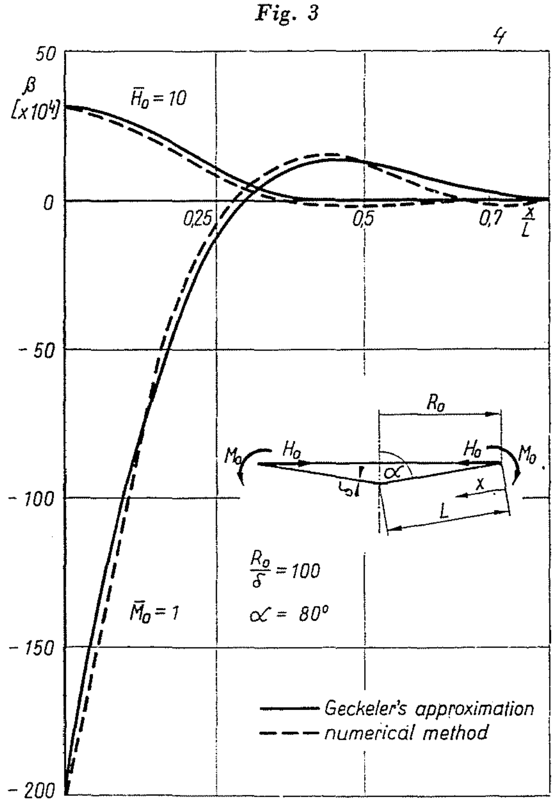

50
β [x10⁴]
0
−50
−100
−150
−200
$\bar{H}_0$ = 10
$\bar{M}_0$ = 1
0,25
0,5
0,7
$\frac{x}{L}$
R_0
M_0
H_0
$\frac{R_0}{\delta}$ = 100
α = 80°
Geckeler's approximation
numerical method

Fig. 4

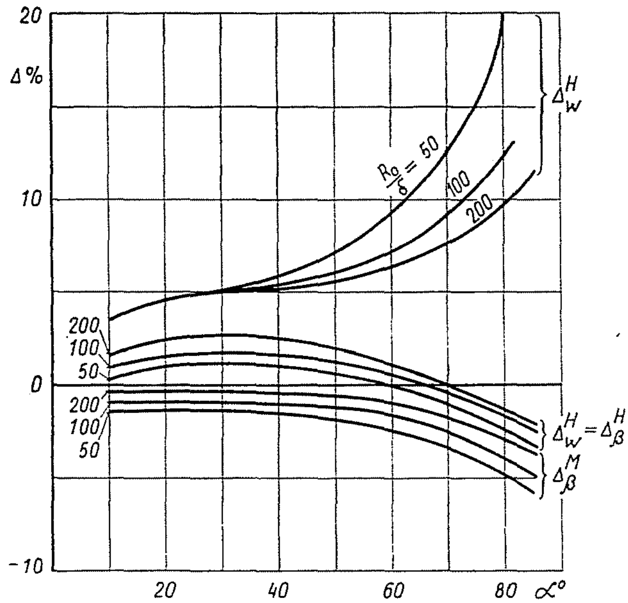

Δ%
20
10
0
-10
20
40
60
80
α°
$R_0/\delta = 50$
100
200
Δ_w^H
200
100
50
200
100
50
$\Delta_w^H = \Delta_\beta^H$
Δ_β^M

Fig. 5

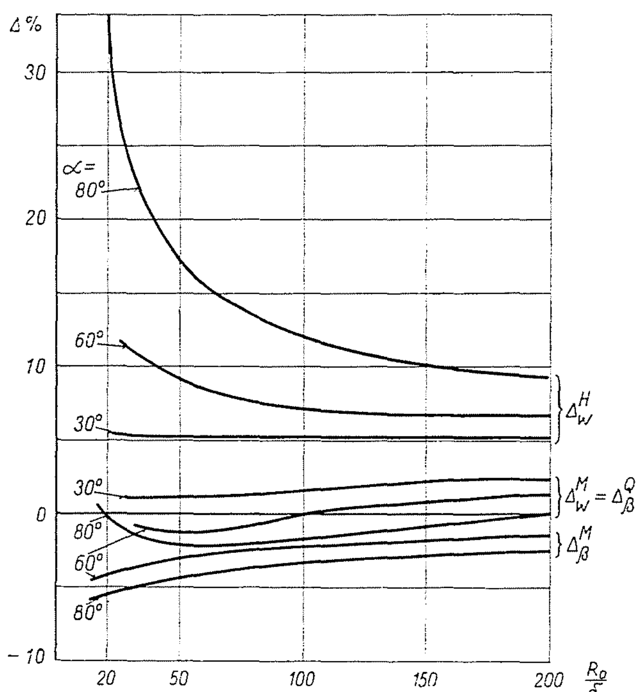

Δ%
30
20
10
0
-10
20
50
100
150
200
R_0/δ
α =
80°
60°
30°
30°
80°
60°
80°
Δ_w^H
$\Delta_w^M = \Delta_\beta^Q$
Δ_β^M

Fig. 6

Error functions show the error of the displacement due to edge force to be the greatest. In the tested range, the error of angular rotations is below 5%, hence limits of applicability of the Geckeler approximation are governed by the error of the radial displacement due to the edge force. Accordingly, it can be stated that admitting a deviation of 6%, the half apex angle can be about $\alpha \leq 40°$, and the wall thickness ratio $R/\delta \geq 50$. The same error is committed for $\alpha \leq 55°$ and $R/\delta \geq 100$. The error only exceeds 10%, if conditions $R/\delta < 150$ and $\alpha > 80°$ coincide.

Legend:

B	bending stiffness
E	modulus of elasticity
H	radial edge force
k	shell constant
L	distance of cone edge from apex
M	meridional edge moment
R	radius of the cone at the edge
w	radial displacement
x	arc length along the generatrix, from the edge
α	half cone apex angle
β	angular rotation
δ	wall thickness
Δ_w^M	error of radial displacement due to an edge moment
Δ_β^M	error of angular rotation due to an edge moment
Δ_w^H	error of radial displacement due to an edge force
Δ_β^H	error of angular rotation due to an edge force

Summary

Results from the Geckeler approximation are compared to those from a numerical method considered as exact. Varying the wall thickness ratio and the half cone apex angle as parameters, the relative error of the approximation is numerically determined, and a suggestion is made on the geometrical limitations of the approximation applicability.

References

1. VARGA, L.: Näherungsverfahren zur Bestimmung der an drehsymmetrischen randbelasteten Kegelschalen angreifenden Schnittkräfte und Schnittmomente. Per. Pol. M. E. Vol. **6**, 4 (1962).
2. HAMPE: Statik rotationssymmetrischer Flächentragwerke. Berlin, 1967.
3. REISSNER, E.: On the Theory of Thin Elastic Shells, Reissner Anniversary Volume, J. W. Edwards, Ann Arbor, Mich., 1949.
4. REUSS, P.: Elastic Stresses in Toriconical Pressure Vessel Heads. Per. Pol. M. E. Vol. **19**, 1 (1975).

Mária ÖRVÖS
László LEGEZA
Dr. Pál REUSS
H-1521 Budapest