DESIGN OF RADIAL PIPE NETWORKS*

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1. Introduction

The problems related to the analysis and design of networks can be classified as follows:

According to the character of the problem:

a) design of new networks,

b) analysis of existing networks.

According to the structure of the network:

a) problems of radial networks,

b) problems of looped networks.

According to the type of flow:

a) steady-flow pattern (consumption constant in time),

b) transient-flow pattern (consumption changing in time).

In the present paper, a question is picked out at random from the complex sphere of problems, viz., *the design of new radial networks for steady flow*. It should be noted that this problem is one of the most important and most common one, especially for district heating networks.

2. Formulation of the problem

Conditions governing the design are the following:

a) The structure of the network is given. This is not an essential restriction because in the overwhelming majority of practical cases the local features permit little variation in the structure of the network. In these cases network can be individually dimensioned and optimized, eliminating the need for an extremely complex topological optimization, of rather theoretical interest.

b) The physical characteristics and flow equations of the flowing medium, as well the physical (frictional) characteristics of the pipelines are given.

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c) The place and magnitude of consumption are given. The relationships for the consumption in a given unit (dwelling unit, building, housing estate) and its expected growth are known.

d) As it appears from items *a*, *b* and *c*, exact values and expected variations in time of the input data (pipe friction coefficient, internal roughness of pipe walls, variation of consumption in time) are not known, and cannot be exactly determined by mathematical or even computer means.

3. Topology of the network

In computer designing pipe networks drawings cannot be used directly. The topology of the network has to be formulated in a computer language, hence by digits. Mapping the network design is facilitated by the graph theory. Graph is the geometric representation of the set Y of objects of any nature with their relationships V. The individual objects are the elements or nodes of the graph, their pairswise relationships are called branches. The symbol of the graph is: $G = (Y, V)$. Branches with a direction constitute a directed graph. The graph representation of pipe networks consists in regarding the individual pipe sections as the branches of the graph; the places of consumption and the branch points as the nodes of the graph. The graph obtained in this way is an abstraction of the network.

Accordingly, the concept of a radial network is defined as a connected graph where every node is connected of another subject to the relationship:

$$n = m + 1$$

(n and m being the number of nodes and branches of the graph, respectively). Several methods are used for the numerical formulation of the relationships expressed by the graph, depending on the character of the problem.

The node-to-node description defines an n by n matrix, of element a_{ij}:

$$a_{ij} = \begin{cases} 0, & \text{if} \quad (y_i, y_j) \in V \\ 1, & \text{if} \quad (y_i, y_j) \notin V \end{cases}$$

The node-to-branch description defines an n by m matrix, of element a_{ij}:

$$a_{ij} = \begin{cases} 0, & \text{if branch } j \text{ is not incident to node } i \\ 1, & \text{if branch } j \text{ is incident to node } i \end{cases}$$

The Pair's characteristic series describes the graph by a series of length $2n - 1$ of the code of nodes (by a vector containing $2n - 1$ elements).

The "from . . . to . . ." description defines a 2 by m matrix with elements $a_{1,j}$ and $a_{2,j}$ being the two end points of branch j.

By properly numbering the branches and nodes of the graph, the "from... to . . ." matrix can be made replaceable by a single vector of m elements. In this description, branch j connects the j-th and a_j-th nodes.

This is called the "simplified from . . . to . . ." description. It should be noted that the described methods in their actual form, are suitable to describe radial graphs, and they are easy to extend to directed graphs containing loops.

The node-to-branch matrix permits unambiguous formulation the two Kirchhoff's laws and besides for the calculation of the compression -work, needed in the network.

4. Hydraulic conditions

The pressure drop in individual pipe sections is given by relationships

$$\lambda = f(Re, d/k)$$

and

$$\Delta p = f(\lambda, L, V, d, k)$$

due to PRANDTL, COLEBROOK, KÁRMÁN and NIKURADZE.

In models with pipe diameters as continuous variables, the pipe friction coefficient λ is determined on the basis of the diameter estimated for the given section.

Discrete models permit to exactly determine λ.

The shape resistances are expressed by equivalent pipe lengths. Our model is valid for a plain ground, thus the pressure drops are uniform from the feed point to the end point of the network

$$\sum_{\Omega_j} p_i = p_0 \tag{1}$$

5. The mathematical model

To design the pipe network means to determine the pipe section diameters. Indicating a system of diameters by vector symbol d:

$$d = (d_1, d_2 \ldots d_n)^* \in E_n$$

From among vectors d, however, only those meeting the hydraulic conditions enter into consideration. The totality of possible solutions is set L. ($L \subset E_n$). For $L = 0$, there is no solution.

Since in general set L contains a large population, the economic efficiency of each solution $L_i \subset L$ is measured by an objective function: some scalar function $f(d)$ assigned to vector d. Solution L_i is more efficient than L_j if

$$f(d^i) < f(d^j).$$

A $d^0 \in L$ programme, the most efficient among the possible ones, is called the optimal programme.

Essentially, the problem consists in simply determining vector d giving the extremal value of function

$$\max \{f(d) | d \in L\} .$$

For the solution of this optimum problem, either deterministic or stochastic, static or dynamic either models can be established, with continuous or discrete variables.

Evidently, the real design problem is better approached by discrete models, since actually the diameters cannot have but standard values. On the other hand, discrete models are more difficult regarding computer treatment.

5.1. Continuous models

The expounded conditional extremal value problem is solved by the method of Lagrange multipliers.

The process minimizes the sum of the yearly investment and operation (pumping and heat loss) costs of the network (K_h annual). The problem is to find the minimum of the annual costs, provided the hydraulic conditions are met (Eq. 1).

The so-called modified cost

$$K_m = K_h \text{ annual} + \sum_j \lambda_j (\sum_{\Omega_j} \Delta p_t - p_0) \tag{2}$$

The investment and operation costs can be formulated as a power function of vector d. Taking $\Delta p = f(d)$ into consideration, $d = (d_1, d_2, \dots d_m)$ and $\lambda = (\lambda_1, \lambda_2, \dots \lambda_v)$ are unknown in the equation.

The extremal value of K_m is obtained by solving the equation system

$$\frac{\partial K_m}{\partial d_i} = 0 \, \forall_i ; \quad \frac{\partial K_m}{\partial \lambda_j} = 0 \, \forall_j .$$

Inasmuch also feed pressure p_0 is required, the optimal feed pressure is determined by

$$\frac{\partial K_m}{\partial p_0} = 0$$

The diameters obtained in this way are other then standard.

The series of standard diameters is obtained by transformation

$$T(d) = dn$$

$[dn = (dn_1, \dots, dn_k)]$.

The "merit" of the transformation process depends on how the objective function $f(dn)$ approximates the optimal solution, that is,

$$f(T(d)) - f(d) \rightarrow Min.$$

It is a result worth mentioning that in the case of a given feed pressure Δp_m the ratio of pipe diameters is constant and independent of the feed pressure

$$\frac{d_i^{(1)}}{d_i^{(2)}} = \left(\frac{p_0^{(1)}}{p_0^{(2)}}\right)^{1/5}.$$

Thus, optimizing the network for a given pressure p_0, then the optimal dimensions for any other p_0 are obtained by multiplying with a constant calculated from the ratio of two p_0 values.

5.2. *Discrete models*

Recently, discrete models have come to the foreground of interest. Of the methods of mathematical programming, Gomory's general algorithm of integer value programming, the simplex method, of linear programming the branch and bound method, the methods of assignment and of dynamic programming have been investigated. In principle, every method proved to be suitable for solving the optimization of pipe networks with discrete variables. Dynamic programming proved to be rather suitable for solving pipe network design problems in practice with a view on computer technique and running time. Pipe network design by discrete dynamic programming means to solve a multi-stage decision system.

Though the technique of dynamic programming permits to examine stochastic systems, too, the physical character of that problem is better reflected by the deterministic model. To solve a design problem is simply to determine the optimal set of decisions. Or defined according to Bellman's principle of optimality: "an optimal path leading form node Y_0 to node $Y_m(Y_0, \ldots, Y_i, \ldots, Y_m)$ may contain only such partial paths $(Y_i, \ldots, Y_j$, where $0 \leq i \leq m$, $i < j \leq m)$ which are optimal between Y_i and Y_j."

The state of the system is determined in each stage by a variable, in a given case by a scalar Z_i, the variable of state of the system. At stage i a decision is made, that is, the decision variable U_i of the system, assumes some value.

The state of the system at stage $i+1$ is determined by the transition function of the system:

$$Z_{i+1} = g_i(Z_i, U_i)$$

The outcome of the decision (stage return) is given by function $f_i(Z_i, U_i)$. The possible values of the state variables of the system are determined by the previous and subsequent states, according to the following relationship:

$$Z_i \in \Gamma_i Z_{i-1}$$

and

$$Z_i \in \Gamma_{i+1}^{-1} Z_{i+1}$$

respectively.

Accordingly, $Z_i \in \Gamma_i Z_{i-1} \cap \Gamma_{i+1}^{-1} Z_{i+1}$, where operator gives the set of state i obtainable from the stage i — 1. Thus, the intersection of the two sets means the space of possibility of state i. The general model of the decision system is shown in Figs 1 and 2 for cases of a serial and for a non-serial system, respectively.

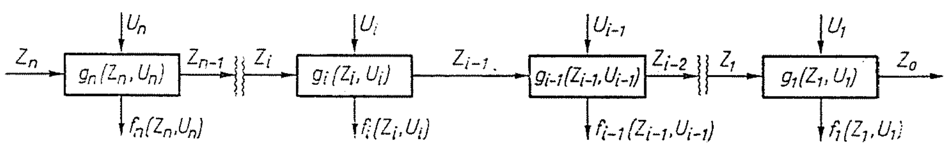

Fig. 1

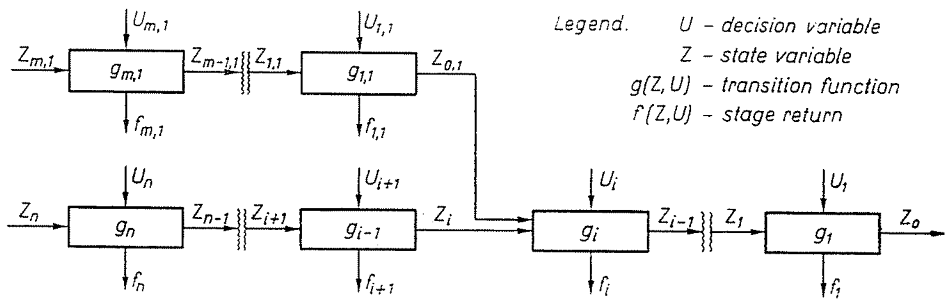

Fig. 2

A serial system represents a pipe network without branches, while a non-serial system (branching or connected system), a radial pipe network with arbitrary topology. Depending on whether the initial state Z_0, or the final state Z_n of the system, or both are given, a final state, an initial state, or an initial-final state problem, can be spoken of.

If the endpoints of the radial network are considered as the starting point and the feed point of the pipe network (the pump) as its endpoint, for a given feed pressure p_0, an initial-final state problem is involved. If p_0 is to be determined rather than given a final state problem is spoken of.

For a given network optimization problem, the concepts in the decision model correspond concretely to following:

decision variable	U	—	diameter	d
state variable	Z	—	pressure	p

transition function	$g(Z, U)$ —	pressure drop	p
stage return	$f(Z, U)$ —	cost function	$f(d, p)$

Optimization starts from the endpoints of the network and proceeds gradually towards the feed point, making use of the fundamental recursion equations of dynamic programming:

$$F_1(Z_1) = \max_{U_1} f_1(Z_1, U_1)$$

$$F_n(Z_n) = \max_{U_n} (f_n(Z_n, U_n) + F_{n-1}(Z_{n-1}))$$

provided

$$Z_{i+1} = g_i(Z_i, U_i)$$

Thus, the optimal solution of the n-stage system is determined stage-wise. The direction of progress is made unequivocal by the precedence relation existing on the graph. At the intersection of two branches the optimization equation is somewhat modified:

$$F_n(Z_n) = \max [f(Z_n, U_n) + F^1_{n-1}(Z^1_{n-1}) + F^2_{n-1}(Z^2_{n-1})]$$

where the superscript indicates the subsystems provided by the n-th node of the graph, as a root.

The developed algorithm is suitable for the optimization of radial networks with an arbitrary topology.

Initial-final state problems involve to divide the available pressure into K equal parts. In this way, the system can be both in stage m and state K. The optimal states belonging to the individual stages and the optimal standard diameters determining them can be determined by the above recursion formula of discrete dynamic programming. A final state problem can be reduced to that of finding the optimal path.

Networks of a size occurring in practice (a few hundred sections) can be designed at a moderate running time (a few minutes). The practical utility of this design process is enhanced by the optimization affecting only on standard diameter series meeting the rate limitation.

6. List of symbols

n	number of nodes in the graph
m	number of branches in the graph
λ	pipe friction coefficient or Lagrange's multiplier
d	diameter
K	roughness
Re	Reynolds number
L	pipe length
V	section flow
p	pressure
Δp	pressure drop

dn	standard diameter
K_n,	annual cost
Km	modified cost
$\subset$	$A \subset B$ A is the subset of B
$\cap$	$A \cap B$ is the common part of A and B, its intersection
$\in$	$a_1 \in A$ a_i is an element of A
$\forall i$	for every i
T	transformation operator

Summary

Computer methods for designing and optimizing radial networks have been examined, together with the data needed, and the expected accuracy of the process — compared to the "manual" calculation methods.

The network topology is described by means of the graph theory and the matrix calculus.

The mathematical model is comprised of design and hydraulic conditions. In the case of continuously changing diameters, the model is solved by Lagrange's multiplier method. In the case of standard diameters, the solution is given by discrete dynamic programming.

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