THERMAL CONDITIONS IN GRAVITY FLOW HEAT EXCHANGERS

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Heat exchangers where motion of heated liquid is caused by the variation in density due to temperature difference, are termed gravity flow heat exchangers. Hence, heat transmission rate depends on gravity flow in the heated liquid.

For instance, the overall heat transfer coefficient of a flat wall can be written in the well-known form as:

$$k = \frac{1}{\frac{1}{\alpha_1} + \frac{v}{\lambda} + \frac{1}{\alpha_2}} \tag{1}$$

where:

α_1 and α_2 — heat transfer coefficients from heating medium to the wall (1), and from the wall to the heated medium (2), respectively;

v — wall thickness;

λ — thermal conductivity of the wall material.

With water in forced convection or steam as heating medium and in case of the practically applied metal heat exchangers, the respective thermal resistances can be neglected. Namely, the resistance to heat transfer towards the heated medium (low rate water or air) prevails over the former.

Thus

$$k \cong \alpha_2. \tag{2/a}$$

For natural convection

$$N_{\mathrm{Nu}} = \frac{\alpha l}{\lambda} = f(N_{\mathrm{Gr}};\ \ N_{\mathrm{Pr}}) \tag{3}$$

and

$$N_{\mathrm{Gr}} = f(\Delta t;\ l;\ \mathrm{etc}) \tag{4}$$

where:

Δt — temperature difference between the wall and the medium;

l — characteristic length of heat flow path.

Obviously:

$$\alpha_2 = f(\Delta t) \tag{5}$$

and

$$k \cong f(\Delta t) \tag{2/b}$$

or in details:

$$k = k_0(t_1 - t_2)_k^M = k_0 \Delta t_k^M \tag{2/c}$$

where:

k_0 — constant depending on the size of the heat exchanger and the medium type (or its Nusselt, Grashof and Prandtl numbers);

M — exponent varying with the temperature-dependent Nusselt, Grashof and Prandtl numbers;

Δt_k — mean temperature difference between heating and heated media.

Eqs (2/a) and (2/c) mainly concern to liquid–gas heat exchangers, but the temperature difference may govern heat transfer in certain types of liquid–liquid or gas–gas heat exchangers.

The heat exchangers of gravity flow of different designs are practically applied in two characteristic operating conditions.

1. A typical case for unsteady state of natural convection is to heat a given volume of liquid in a tank to a definite temperature, such as certain heat exchangers for warm water supply. In this case the overall heat transfer coefficient is a function of the heat transfer coefficient at the heated side but it is also time-dependent.

2. Heat exchangers in steady state of natural convection convey media of about constant inlet and outlet temperature and are accommodated in the heated air space, such as heaters (radiators, convectors, pipe-registers).

Now, cases of steady-state heat transmission and freely flowing heated medium of identical direction or in countercurrent will be considered.

A) Heat transmission for k = constant

Heat transmission is determined by three fundamental equations, such as, in the case of counterflow:

heat loss of heating liquid:

$$\dot{Q} = \dot{m}_1 c_1 (t_{1e} - t_{1v}) \tag{6}$$

heat gain of heated medium:

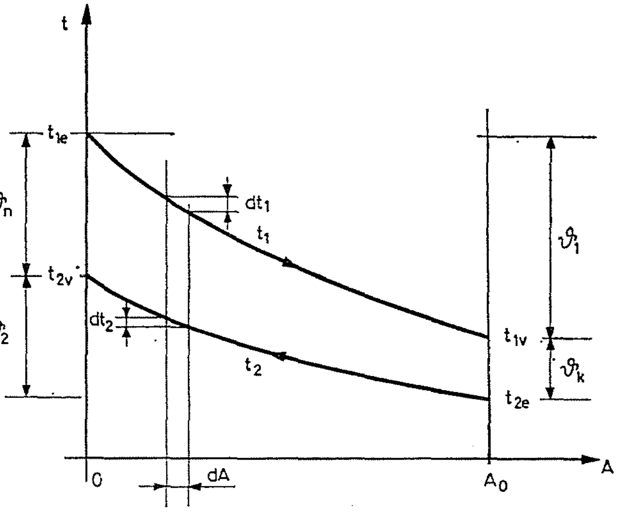
$$\dot{Q} = \dot{m}_2 c_2 (t_{2v} - t_{2e}) \tag{7}$$

heat flow through the wall between the media:

$$\dot{Q} = A_0 k \Delta t_k \tag{8}$$

where:

$\dot{m}_1$ and $\dot{m}_2$ — mass flow of media;
c_1 and c_2 — specific heat of media;
A_0 — heat transfer area;
medium temperatures being shown in Fig. 1 along the surface of the heat exchanger.



Actually, mass flow, specific heat and overall heat transfer coefficient are introduced with constant values in the calculations.

In practice, two solutions are known.

1. Grashof's relationship for the mean temperature difference

$$\Delta t_k = \frac{\vartheta_n - \vartheta_k}{\ln \dfrac{\vartheta_n}{\vartheta_k}} \tag{9}$$

and for the ratio of overtemperatures

$$\frac{\vartheta_k}{\vartheta_n} = e^{-\frac{kA_0}{\dot{W}_1}\left(1 - \frac{\dot{W}_1}{\dot{W}_2}\right)} . \tag{10}$$

Accordingly, the liquid temperature variation along the heating surface is described by an exponential (natural logarithm-based) power function.

2. Bosnjakovic introduced a new function:

$$\Phi = \frac{\dot{Q}}{\dot{W}_1(t_{1e} - t_{2e})} \tag{11}$$

but he also applied Eq. (9) for the mean temperature difference.

As a conclusion, the temperature of liquids in a heat exchanger of gravity flow changes according to a natural logarithm-based power function in steady state with the overall heat transfer coefficient taken as constant.

B) Heat transmission for $k \neq$ constant

The condition k = constant is not valid for heat exchangers of gravity flow. The technical literature is known to contain but a simple form for $k \neq$ constant:

$$\Delta t_k = \frac{k_1\vartheta_n - k_2\vartheta_k}{\ln \frac{k_1\vartheta_n}{k_2\vartheta_k}} \tag{12}$$

where:

k_1 and k_2 — overall heat transfer coefficients at the beginning and the end of heat exchange, still the knowledge of function "k" is needed.

a) Determination of mean temperature difference

Let us apply Eqs (2/c); (6); (7) and (8) for the case in Fig. 1 (counter-flow, $\vartheta_1 > \vartheta_2$).

With notations in Fig. 1, take an elementary surface dA of the heat exchanger with temperature variation of the liquids dt_1 and dt_2 along it, resulting in an elementary heat flow $d\dot{Q}$:

$$d\dot{Q} = -\dot{W}_1 dt_1 = -\dot{W}_2 dt_2 \tag{13}$$

$$d\dot{Q} = k(t_1 - t_2)dA = k_0(t_1 - t_2)^{1+M} dA. \tag{14}$$

From Eq. (13), applying (14):

$$d(t_1 - t_2) = -d\dot{Q}\left(\frac{1}{\dot{W}_1} - \frac{1}{\dot{W}_2}\right) = -k_0(t_1 - t_2)^{1+M}\left(\frac{1}{\dot{W}_1} - \frac{1}{\dot{W}_2}\right) dA. \tag{15}$$

Separating the variables and integrating:

$$\int_{t_{1e}-t_{2v}}^{t_{1v}-t_{2e}} \frac{d(t_1 - t_2)}{(t_1 - t_2)^{1+M}} = -k_0\left(\frac{1}{\dot{W}_1} - \frac{1}{\dot{W}_2}\right)\int_0^A dA \tag{16}$$

hence

$$- \frac{1}{M(t_1 - t_2)^M} \Bigg|_{t_{1e} - t_{2v}}^{t_{1v} - t_{2e}} = - k_0 \left(\frac{1}{\dot{W}_1} - \frac{1}{\dot{W}_2} \right) A \Bigg|_0^{A_0}$$

and substituting the limits:

$$\frac{1}{(t_{1v} - t_{2e})^M} - \frac{1}{(t_{1e} - t_{2v})^M} = k_0 M \left(\frac{1}{\dot{W}_1} - \frac{1}{\dot{W}_2} \right) A_0 \,. \tag{17}$$

The temperature variation along the area is seen to be described by a hyperbolic function.

Using the basic equations again, it may be written:

$$\frac{1}{(t_{1v} - t_{2e})^M} - \frac{1}{(t_{1e} - t_{2v})^M} = \frac{M \dot{Q}}{\Delta t_k^{1+M}} \left(\frac{t_{1e} - t_{1v}}{\dot{Q}} - \frac{t_{2v} - t_{2e}}{\dot{Q}} \right) .$$

Rearranging the right side and using symbols in Fig. 1:

$$\frac{1}{\vartheta_k^M} - \frac{1}{\vartheta_n^M} - \frac{M}{\Delta t_k^{1+M}} (\vartheta_n - \vartheta_k) \,.$$

Expanding the mean temperature difference:

$$\Delta t_k^{1+M} = \frac{M(\vartheta_n - \vartheta_k)}{\dfrac{1}{\vartheta_k^M} - \dfrac{1}{\vartheta_n^M}} = M \vartheta_k^M \vartheta_n^M \frac{\vartheta_n - \vartheta_k}{\vartheta_n^M - \vartheta_k^M} \,. \tag{18}$$

The deduction started by stating that for a counterflow heat exchanger $\vartheta_1 > \vartheta_2$, i.e. $\vartheta_n > \vartheta_k$. From Eq. (17) it is obvious, that Eq. (18) is valid for parallel flow ($\vartheta_2 > \vartheta_1$ and $\vartheta_n < \vartheta_k$), for the sense, its absolute value being unchanged.

Eq. (18) is not valid for $\vartheta_1 = \vartheta_2$ i.e. $\vartheta_k = \vartheta_n$. In this case

$$\Delta t_k = \vartheta_k = \vartheta_n; \tag{19}$$

it is possible only if

$$M = 1. \tag{20}$$

Transforming Eq. (18) to:

$$\Delta t_k^2 = \vartheta_k \vartheta_n \frac{\vartheta_n - \vartheta_k}{\vartheta_n - \vartheta_k} = \vartheta_k \vartheta_n = \vartheta_k^2 = \vartheta_n^2$$

meeting Eq. (19).

Thus, equality in Eq. (20) indicates the condition for linear variation of both media along the heating surface.

b) *Comparison of mean temperature differences obtained in different ways*

There are different approximative formulae for practically calculating the mean temperature difference.

For $\vartheta_k = \vartheta_n = \Delta t_k$

$$\Delta t_{ka} = \frac{\vartheta_k + \vartheta_n}{2}$$

an arithmetical mean temperature difference (subscript a), or in another form as:

$$\frac{\Delta t_{ka}}{\vartheta_n} = \frac{1 + \dfrac{\vartheta_k}{\vartheta_n}}{2} \tag{21}$$

The logarithmic mean temperature difference (subscript ln) Eq. (9) rearranged:

$$\frac{\Delta t_{k\,\ln}}{\vartheta_n} = \frac{1 - \dfrac{\vartheta_k}{\vartheta_n}}{\ln \dfrac{1}{\dfrac{\vartheta_k}{\vartheta_n}}} \tag{22}$$

Eqs (21) and (22) help to examine the variation of mean temperature differences referred to ϑ_n vs. ϑ_k/ϑ_n (Fig. 2) and to decide over their field of application with a view of accuracy requirements. For $\vartheta_k/\vartheta_n > 0{,}5$, the arithmetical mean temperature difference is seen to closely approximate the logarithmic one.

Rearranging Eq. (18):

$$\frac{\Delta t_{kM}}{\vartheta_n} = \left(M \vartheta_k^M \frac{\vartheta_n^M}{\vartheta_n^{1+M}} \frac{\vartheta_n - \vartheta_k}{\vartheta_n^M - \vartheta_k^M} \right)^{\frac{1}{1+M}} =$$

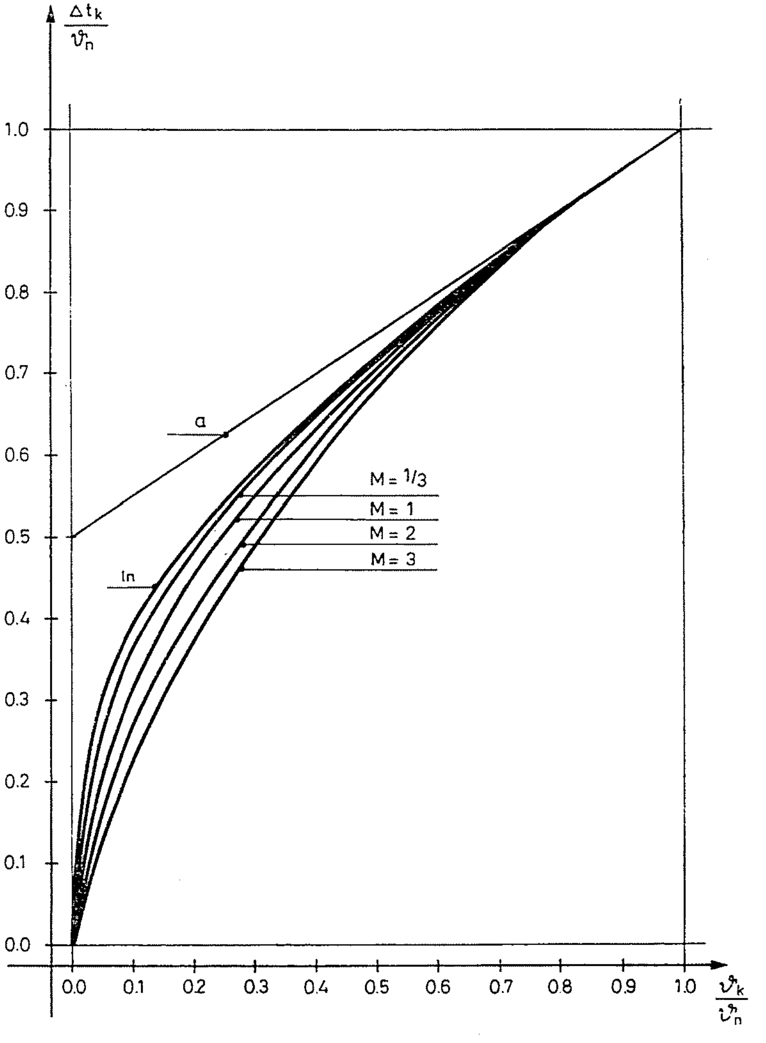
$$= \left[M \left(\frac{\vartheta_k}{\vartheta_n} \right)^M \frac{1 - \dfrac{\vartheta_k}{\vartheta_n}}{1 - \left(\dfrac{\vartheta_k}{\vartheta_n} \right)^M} \right]^{\frac{1}{1+M}} . \tag{23}$$

The extreme values of this function are easy to establish.

For $\vartheta_k/\vartheta_n = 0$ the function is zeroed.

For $\vartheta_k/\vartheta_n = 1$ the function is undefined. Derivating both the numerator and the denominator in Eq. (23):

$$\frac{-1}{-M \left(\dfrac{\vartheta_k}{\vartheta_n} \right)^{M-1}} = \frac{1}{M}$$



Thus:

$$\left.\frac{\Delta t_{kM}}{\vartheta_n}\right|_{\frac{\vartheta_k}{\vartheta_n}=1} = \left(M1^M \frac{1}{M}\right)^{\frac{1}{1+M}} = 1$$

The results of Eq. (23) are shown in Fig. 2 for different M values.

c) *Connection between characteristic overtemperature values*

In practice it is necessary to know the inlet and outlet temperatures of the liquids.

For given mass flow and specific heat only three of the temperatures t_{1e}, t_{1v}, t_{2e} and t_{2v} are arbitrary, the fourth one is determined. Thus, the direct connection between ϑ_n and ϑ_k is needed.

Eqs (7); (8) and (18) lead to

$$\dot{W}_2 \vartheta_2 = A_0 k_0 M \vartheta_n^M \vartheta_k^M \frac{\vartheta_n - \vartheta_k}{\vartheta_n^M - \vartheta_k^M} . \tag{24}$$

From Eqs (7) and (6)

$$\dot{W}_1 \vartheta_1 = \dot{W}_2 \vartheta_2$$

hence

$$\vartheta_1 = \frac{\dot{W}_2}{\dot{W}_1} \vartheta_2 = w_0 \vartheta_2 . \tag{25}$$

It is seen in Fig. 1 that

$$\vartheta_n + \vartheta_2 = \vartheta_k + \vartheta_1. \tag{26}$$

Replacing Eq. (25) into Eq. (26)

$$\vartheta_n + \vartheta_2 = \vartheta_k + w_0 \vartheta_2.$$

After arranging:

$$\vartheta_2 = \frac{\vartheta_n - \vartheta_k}{w_0 - 1} . \tag{27}$$

Replaced into Eq. (24):

$$\frac{\dot{W}_2}{w_0 - 1} (\vartheta_n - \vartheta_k) = A_0 k_0 M \vartheta_n^M \vartheta_k^M \frac{\vartheta_n - \vartheta_k}{\vartheta_n^M - \vartheta_k^M} .$$

With a simple notation:

$$\frac{\dot{W}_2}{w_0 - 1} = \frac{\dot{W}_2 \dot{W}_1}{\dot{W}_2 - \dot{W}} = w . \tag{28}$$

Eq. (24) rearranged:

$$\vartheta_n^M \vartheta_k^M \frac{1}{\vartheta_n^M - \vartheta_k^M} = \frac{w}{A_0 k_0 M} = B \tag{29}$$

and

$$\vartheta_n = \left(\frac{B \vartheta_k^M}{B - \vartheta_k^M} \right)^{\frac{1}{M}} . \tag{30}$$

The result can be checked by:

$$\frac{\vartheta_n}{\vartheta_k} = \left(\frac{B}{B - \vartheta_k^M} \right)^{\frac{1}{M}} .$$

1. For $\vartheta_n > \vartheta_k$, $\vartheta_1 > \vartheta_2$ and $\dot{W}_2 > \dot{W}_1$.

Since

$$A_0 k_0 M > 0; \text{ and } w > 1; \text{ thus } B > 0.$$

For

$$B > \vartheta_k^M$$

$$\left(\frac{B}{B - \vartheta_k^M}\right)^{\frac{1}{M}} = \frac{\vartheta_n}{\vartheta_k} > 1$$

i.e. the initial condition.

2. For $\vartheta_k > \vartheta_n$, i.e. $\vartheta_2 > \vartheta_1$ and $\dot{W}_1 > \dot{W}_2$:

$$w < 0 \qquad \text{and} \qquad B < 0.$$

Thus

$$\left(\frac{-B}{-B - \vartheta_k^M}\right)^{\frac{1}{M}} = \left(\frac{B}{B + \vartheta_k^M}\right)^{\frac{1}{M}} = \frac{\vartheta_n}{\vartheta_k} < 1$$

again the initial condition.

In the first case the assumption

$$B > \vartheta_k^M$$

concludes the possibility of $\vartheta_n > \vartheta_k$.

d) *Other considerations*

The actual investigation referred to the steady state of the gravity flow heat exchanger alone. Fig. 2 and Eqs (22) and (23) permit to select the accuracy of the mean temperature difference obtained by applying either a constant or a variable value for the overall heat transfer coefficient.

The same method may be used in the case of unsteady state for gravity flow heat exchangers.

Summary

Thermal conditions in gravity flow heat exchangers determined — as a contrary to the present practice — by taking into consideration the overall heat transfer coefficient due to the temperature difference, have been investigated.

Results for steady state show the temperature of the liquids to vary along the surface according to a hyperbolic functions rather than logarithmically, and the very factor modifying the heat transfer coefficient to affect the mean temperature difference.

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