PROPAGATION OF ERRORS IN ROCKWELL HARDNESS-STANDARDIZING MEASUREMENTS

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I. Introduction

Internationally agreed standard specifications are not sufficient to ensure uniformity and compatibility of hardness values measured by different hardness-testing machines. To overcome this difficulty, hardness standard-testing machines were developed, suited to control test parameters within close tolerances. Thus, the lever or hydraulic transmission loading of industrial hardness testers is replaced by dead-weight loading, lever transmission dial-gauge reading for penetration depth is substituted by a measurement according to the Abbe principle by means of a reading microscope on the standard-testing machine. Penetration speed and duration are controlled by appropriate devices. Hardness standard-testing machines are installed at the national metrological institutions of many industrial countries. Hardness values measured on these machines constitute the National Hardness-Reference Scale (NHRS) of the respective country. Hardness reference-blocks calibrated on the standard-testing machine are employed to check hardness testers used in industry. This method is suited to ensure uniformity within a given country. The Hardness-Reference Scales of different countries may differ from each other on account of the tolerance ranges of testing equipment and method. These systematic differences are established by international hardness-comparison measurements [1]. The desirable aim of the relevant research work is to establish an international hardness-standard equipment which would supply the values of the International Hardness-Reference Scale (IHRS) [2].

The possibilities and limitations of the described system of ensuring uniformity of hardness measurements are often misunderstood. There is a trend of thinking over-optimistically, omitting inherent measurement uncertainties which are propagating when there are various stages in the link-up system of measuring instruments. The present paper tries to give a realistic evaluation of uncertainties of Rockwell C-hardness measurements, with special emphasis on values obtainable with industrial hardness testers, since the aim of hardness-standardizing work is to ensure uniformity of measurements at the level of industrial practice.

II. The Hierarchic Order of Hardness Measurements

The values determined by all measuring instruments, thus, also by material-testing machines, must be traceable to the standards of SI units. The connection between the international and national primary standards, and the instruments used in everyday measurement practice is ensured by the

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3.

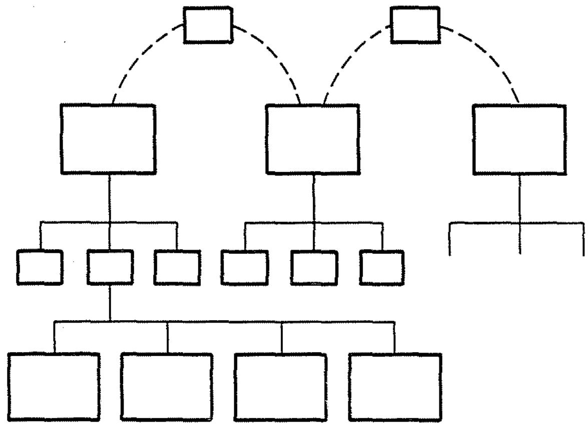

Fig. 1. Hierarchic order of hardness measurements. Present state. *1* National hardness-standard-testing machines; *2* Hardness reference blocks; *3* Hardness testers in the industry

metrological institutions of the various countries by the methods indicated in the link-up diagrams [3,4]. This block diagram is a useful means of indicating the various classes of accuracy. Instruments standing in a lower row have a lower accuracy than those standing higher, because each instrument is checked by the instrument standing in the immediately higher row. The present state of the hierarchic order of hardness-measuring devices is shown in Fig. 1. Large blocks indicate the hardness-measuring apparatus, while the small ones the hardness-reference blocks. The national standard-testing machines stand in row 1. These are maintained by the moment independently of each other, the only connection between them being the international comparisons performed with the help of hardness reference blocks. On the national hardness standard-testing machines blocks *2* are calibrated which are employed in turn, for checking hardness testers used in industry designated by *3*.

The desirable future state of the link-up diagram for hardness measurements is shown in Fig. 2. Here the international standard-testing machine *1*, the national standard-testing machines *3*, and the hardness testers used in industry *5*, are connected by hardness-reference blocks *2* and *4*. It should be mentioned that in countries having extended calibration services another stage, namely the secondary standard-testing machines may be inserted between the national standard *3* and the machines of industry *5*.

III. Propagation of Errors

The calculation of propagation of errors [5] according to the hierarchic order shown in Fig. 1 is facilitated by the fact that both systematic and random errors arising at the individual stages are independent of each other. Accordingly, systematic errors are additive, while in the case of random errors, the variances (squares of standard deviations) are additive.

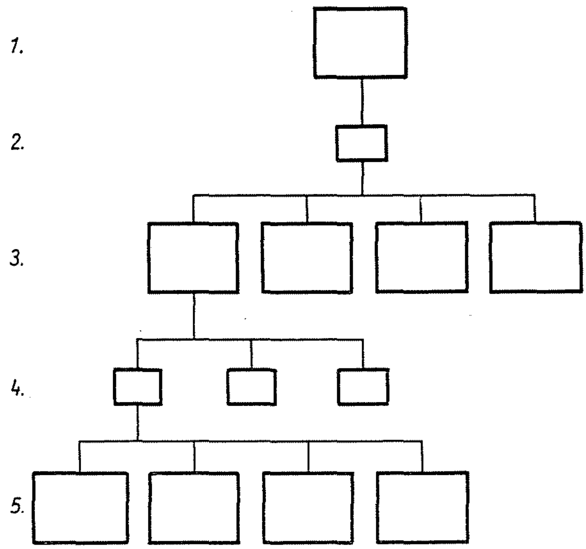

Fig. 2. Hierarchic order of hardness measurements according to a future international reference standard. *1* International hardness-reference standard-testing machine; *2* and *4* Hardness-reference blocks; *3* National hardness standard-testing machines; *5* Hardness testers in the industry

Standard deviations at the various stages of the link-up diagram were determined either experimentally, or estimations were made on the basis of standard specifications [6]. These can be regarded as the condensed outcome of many years of experience, thus supplying fairly reliable estimations. In standard specifications frequently tolerance ranges are indicated. To obtain standard deviation values the estimation formula [7]

$$\sigma_R = A(n)\, R$$

will be employed in the following,

where σ_R denotes the estimated standard deviation,
n the number of values forming the range,
$A(n)$ a tabulated coefficient, and
R the range of values (maximum less minimum).

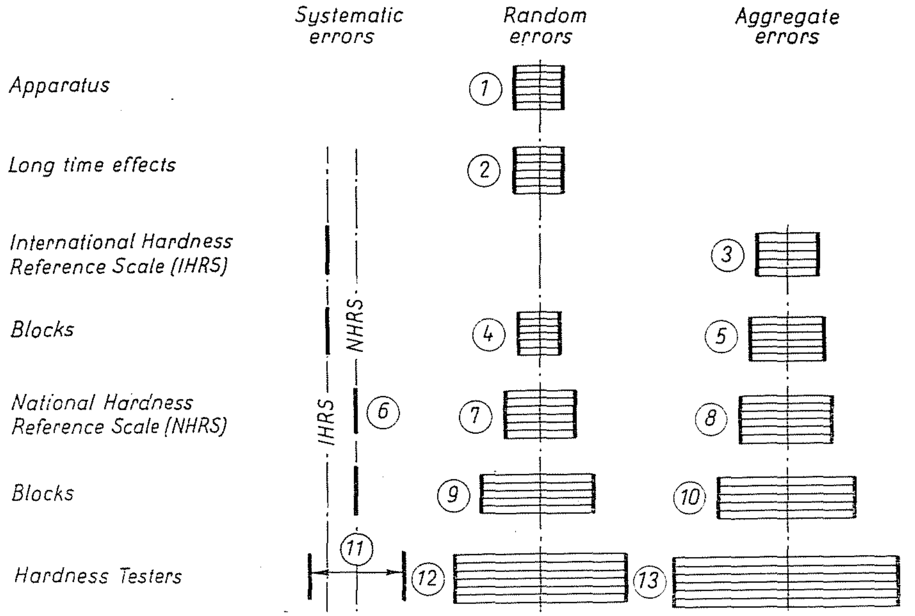

Fig. 3. Scheme of error propagation

In the calculations the cases $n = 5$ and 10 occur, for which we have the values

$$A(5) = 0{,}43,$$
$$A(10) = 0{,}32.$$

Since standard-deviation values used in the calculations were determined on the basis of a high number of measurements (several hundreds or thousands), confidence limits of hardness values can be calculated by using the formula

$$\tau = \frac{\mu}{\sqrt{n}}\,\sigma,$$

where $\mu_{0,95} = 1{,}96$ for 95% probability and
$\mu_{0,99} = 2{,}58$ for 99% probability.

To facilitate the discussion of the individual components entering the propagation of errors, Fig. 3 gives a general view. The stages correspond to those given in Fig. 2. The systematic errors, the random errors, and the aggregate errors are illustrated in three columns. The index numbers in circles correspond to the points of the following discussion and to indices used in the formulae.

Various research workers tried to establish the inaccuracies of hardness-standard-testing machines [2, 8, 9]. Several approaches are possible, giving final results of fairly good compliance. The author has tried to decompose errors to momentary and long-time uncertainties [2, 10, 11].

1. By employing the experimental arrangement of Graeco-Roman Squares on the surface of hardness-reference blocks, the effects of different penetrators and of the disuniformity of hardness along the surface can be separated by the analysis of variance. The value of residual variance gives a

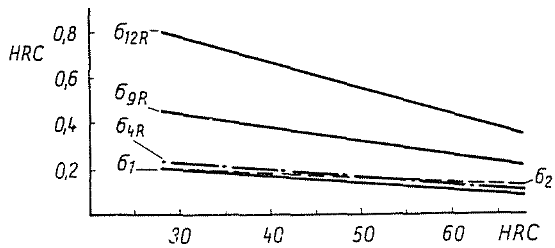

Fig. 4. Standard deviations of error components in Rockwell C measurements, in function of hardness. σ_1, measuring process and apparatus; σ_2, long-time changes; σ_{4R} and σ_{9R}, different points on block surface; σ_{12R}, reproducibility of measurements on industrial hardness testers

good estimation for uncertainties caused by the measuring equipment and for subjective errors. This can be called the standard deviation of the measuring process and apparatus (σ_1). If there were ideal hardness blocks, with exactly identical hardness in all the points, the standard deviation of repeated measurements would correspond to this value σ_1. Its magnitude has been determined on the basis of the evaluation of a high number of roman squares on blocks of various hardness grades. Numerical values are indicated in Fig. 4, where also all the error components discussed later are shown.

2. The values measured on the hardness-standard-testing machine have a long-time fluctuation. To establish the magnitude of fluctuations, a set of five blocks are measured once in each month at the National Office of Measures (OMH), Budapest. Standard deviation of all measured values was determined as a characteristic of long-time changes (σ_2). Values are given in Fig. 4. The experimental method did not permit to separate long-time changes of the machine and those of the blocks. Thus the value σ_2 includes also the effect of long-time block changes. This unavoidably causes too high apparent values for the standard deviation of the hardness-reference scale. This will be compensated, however, in the course of error propagation calculations when long-time block changes will not be taken into consideration at their proper place, namely at point 4. The same applies to points 7 and 9.

3. The standard deviation of the International Hardness-Reference Scale (IHRS) can be given as

$$\sigma_3 = \sqrt{\sigma_1^2 + \sigma_2^2}\,.$$

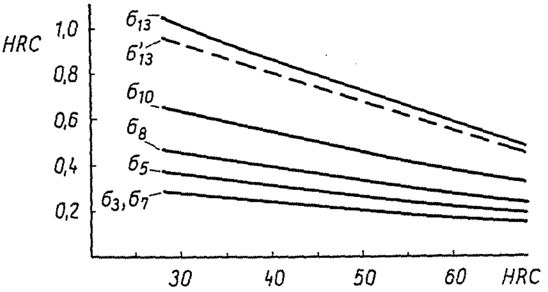

Fig. 5. Error propagation in hardness measurements. Aggregate standard-deviation values in various stages of the hierarchic order, in function of hardness. σ_3, International Hardness-Reference Scale (IHRS); σ_5, values of the IHRS as transferred by blocks; σ_7, national hardness standard testing machine; σ_8, National Hardness-Reference scale (NHRS); σ_{10}, values of the NHRS as transferred by blocks; σ_{13}, hardness values measured in the industrial laboratory; σ'_{13}, hardness values measured in the industrial laboratory, error propagation calculated only from the NHRS

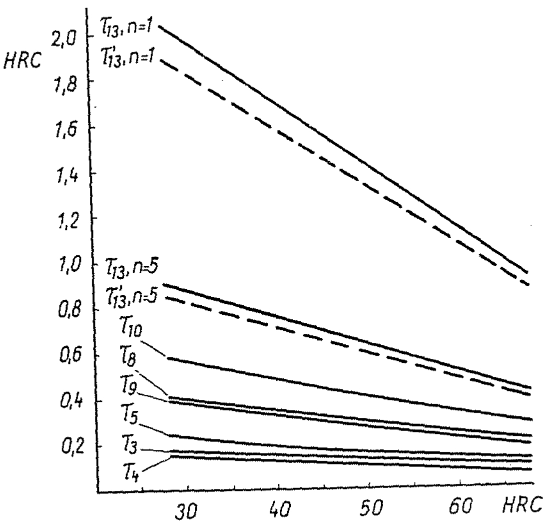

Fig. 6. 95% confidence limits of values obtained on hardness-measuring devices of various levels, as a function of hardness. τ_3, International Hardness-Reference Scale (IHRS); τ_4, ten indentations on hardness-reference blocks; τ_5, values of the IHRS as transferred by blocks; τ_8, National Hardness-Reference Scale (NHRS); τ_9, five indentations on hardness-reference blocks; τ_{10}, values of the NHRS as transferred by blocks; τ_{13}, hardness values measured in the industrial laboratory; τ'_{13}, hardness values measured in the industrial laboratory, error propagation calculated only from the NHRS

Calculated values are shown in Fig. 5, the confidence limits of averages from ten measurements in Figs 6 and 7.

4. Values of the IHRS are transferred by means of high-quality blocks having a range of measured values corresponding to 1% of penetration depth and calibrated by 10 indentations, so as to obtain a low inaccuracy. An estimation for the standard deviation of these blocks in function of hardness is

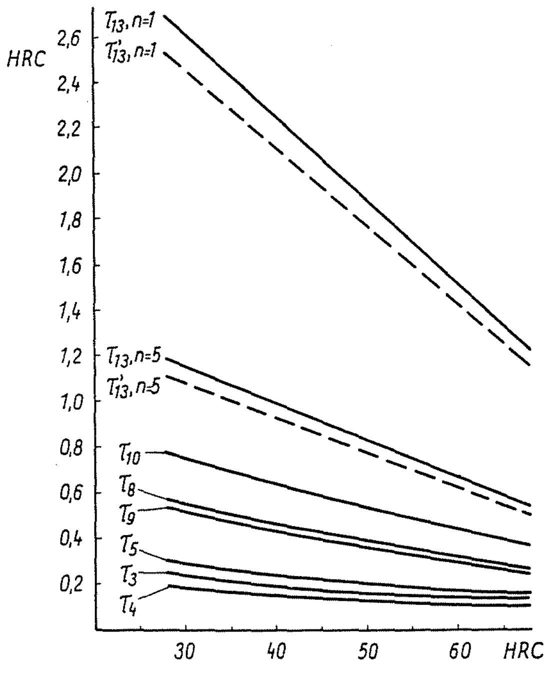

Fig. 7. 99% confidence limits of values obtained on various level hardness-measuring devices as a function of hardness.

τ_3, International Hardness-Reference scale (IHRS); τ_4, ten indentations on hardness-reference blocks; τ_5, values of the IHRS as transferred by blocks; τ_8, National Hardness-Reference Scale (NHRS); τ_9, five indentations on hardness-reference blocks; τ_{10} values of the NHRS as transferred by blocks; τ_{13}, hardness values measured in the industrial laboratory; τ'_{13}, hardness values measured in the industrial laboratory, error propagation calculated only from the NHRS

given in Fig. 4 (σ_{4R}). The subscript R indicates here, and also in the following, that the respective value is an estimation calculated from the range of values.

5. The standard deviation of hardness values transferred from the IHRS by blocks is found to be

$$\sigma_5 = \sqrt{\sigma_3^2 + \sigma_{4R}^2} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_{4R}^2} \, .$$

6. National Hardness-Reference Scales (NHRS) may have a systematic difference with respect to the IHRS. Of course, this difference and the necessary corrections are to be eliminated later, as the results of organizing standardization work on an international level.

7. The standard deviation of the national standard-testing machine can be taken as identical with that of the international standard-testing machine, thus

$$\sigma_7 = \sigma_3.$$

8. By calculating error propagation, the standard deviation of hardness values determined on the national standard-testing machine, that is, the standard deviation of the NHRS is found to be

$$\sigma_8 = \sqrt{\sigma_5^2 + \sigma_7^2} = \sqrt{\sigma_5^2 + \sigma_3^2} = \sqrt{2\sigma_1^2 + 2\sigma_2^2 + \sigma_{4R}^2} \; .$$

9. Hardness-reference blocks used to transfer the values of the NHRS to the hardness testers used in industry have a range of values corresponding to 1,5% of penetration depth, in the case of making 5 indentations, according to ISO Recommendation R 674–1968. The estimated standard deviation σ_{9R} and the confidence limits τ_9 are shown in Fig. 4, 6, and 7, respectively.

10. The standard deviation of hardness values of the NHRS transferred by the blocks is calculated as

$$\sigma_{10} = \sqrt{\sigma_8^2 + \sigma_{9R}^2} = \sqrt{2\sigma_1^2 + 2\sigma_2^2 + \sigma_{4R}^2 + \sigma_{9R}^2} \; .$$

11. The tolerance limits for systematic errors of hardness testers used in industry are specified in ISO Recommendation R 716–1968 as $\pm$ 1,5 HRC units in the hardness range of HRC 59 to HRC 65, and $\pm$ 2 HRC units in the range of HRC 20 to HRC 55.

12. It is difficult to estimate the standard deviation of hardness testers. The value can be determined experimentally for each machine by performing a high number of measurements. The results obtained in this way may differ considerably. A value applicable for the majority of hardness testers as a fair estimation can be derived from standard specifications. The permitted range of individually measured values is specified as 3% of penetration depth. This range includes also the range of values of the hardness blocks, specified as 1,5%. Thus the range remaining for the hardness tester proper is found to be

$$R = \sqrt{3{,}0^2 - 1{,}5^2} = 2{,}6\%.$$

From this value an estimation of the standard deviation of the hardness tester σ_{12R} (Fig. 4) can be made, using the previously mentioned formula.

13. The final result of the calculation, the standard deviation of hardness values determined in the industrial laboratory, including all the sources of error arising in tracing back the values to the international standard is found to be

$$\sigma_{13} = \sqrt{\sigma_{10}^2 + \sigma_{12R}^2} = \sqrt{2\sigma_1^2 + 2\sigma_2^2 + \sigma_{4R}^2 + \sigma_9^2 + \sigma_{12R}^2}.$$

Numerical values are shown in Fig. 5. Confidence limits are plotted in Figs 6 and 7 for 95%, and for 99% reliability, resp., both for the mean of five measurements and for single measurements. It is evident from this that relatively high inaccuracies may arise in consequence of error propagation. In practice these values may cause differences between hardness values determined at different places, a serious problem in international trade of machinery products.

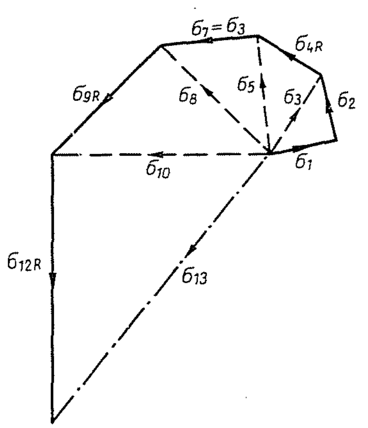

Fig. 8. Construction of error propagation from components, at HRC 30

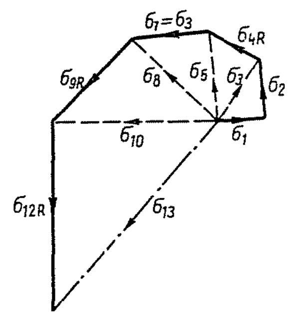

Fig. 9. Construction of error propagation from components, at HRC 60

Fig. 6 indicates that a hardness measurement made in a laboratory will be within the limits of $\pm$ 2 . . . 1 HRC from "true value", as we go from soft to hard (HRC 30 . . . 65), at a probability of 95%. This means that values measured in different laboratories may differ in an unfavourable case by 4 . . . 2 HR units. The respective values at the probability level of 99% are $\pm$ 2,5 . . . 1,5 HRC, or 5 . . . 3 HR units at maximum (Fig. 7).

Another perspicuous method of illustrating propagation of errors is shown in Figs 8 and 9, making use of the possibility that the square root of the sum of squares of two values can be represented as the hypotenuse of a right triangle having right angle sides proportional to the two values.

As mentioned above, the IHRS has not been realized so far. At present, hardness values can be traced back to the National Hardnesss-Reference Scales. When calculating error propagation for this case, values σ_1, σ_2, and σ_4 should be omitted from the formulae, whereby the standard deviation of hardness measurements in the industrial laboratory σ'_{13} is found to be

$$\sigma'_{13} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_9^2 + \sigma_{12}^2} \, .$$

The calculated values σ'_{13} are shown in Fig. 5, the corresponding confidence limit values $\tau'_{13,5}$ and $\tau'_{13,1}$ in Figs 6 and 7, respectively. It is evident that differences between σ_{13} and σ'_{13} are very small, indicating that the establishment of an international standard would not introduce further important random errors, while eliminating the sometimes high systematic errors indicated by hardness scale comparisons.

IV. Conclusions

The method of calculating error propagation at the various levels of the Rockwell C-hardness-tester-calibration system is suited for a realistic assessment of random errors arising in the practical work of material testing laboratories, and of possible differences between hardness values determined at various places, inherent in the process of measurement and in the method of tracing back hardness values. The calculation is based on various well-founded estimations for several factors. It would be desirable to reach an international agreement on the method of calculation. This could be best done by a universal metrology organisation, e.g. by the *Organisation Internationale de Métrologie Légale*, undertaking to set up an international standard of hardness.

Summary

In the introduction the outstanding problems of ensuring compatible hardness measurements throughout the world are surveyed. Discussing the calibration systems for hardness testers, the hierarchic order employed at present and the desirable future order, including an international hardness-standard-testing machine, realizing the International Hardness-Reference Scale, are described. A calculation method is given for determining aggregate random errors at various levels of the hierarchic order. Standard deviation values for the standard-testing machine were determined in a high number of experiments by the author. Other error components were estimated on the basis of standard specifications as outcomes of many years of practical experience. The calibration system of hardness-measuring instruments needs international regulation aimed at unifying methods applied in different countries.

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