SEPARATION OF DUST PARTICLES IN CYCLONE SEPARATORS

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1. Introduction

SAFFMAN [1] derived the governing equations of the incompressible viscous flow of a dusty gas in which the dust is given in terms of a number density of small particles with very small volume concentration but appreciable mass concentration and discussed the stability of plane parallel flows to small disturbances. Since then, the flow problem in dustladen gas with reference to various configurations has been studied by several authors [2—5]. These studies can be of great help in constructing a model which can be used to design a centrifuge or a cyclone for separating the dust particles from the gas. This problem was studied by STAIRMAND [6] who found experimentally that the number of turns made by the particle before leaving the cyclone ranged between 0.5 and 3.

In the present analysis, a simple model of the cyclone has been presented for separating the dust particles from the gas. The cyclone or centrifuge consists of two circular cylinders: one rotating and the other stationary (Fig. 1). The radius of the rotating cylinder is less than that of the stationary cylinder. The dustladen gas passes through the rotating cylinder under the influence of a small axial pressure gradient and enters into the stationary cylinder where the dust particles are separated after a few rotations. It may be noted that a small axial pressure gradient is maintained for continuous supply of gas and the cylinder is rotated for imparting tangential velocity to the gas when it enters the stationary cylinder.

2. Governing equations

Let us consider the unsteady flow of an incompressible viscous dusty gas through the rotating cylinder of radius a under the influence of a time-dependent axial pressure gradient. Assume that the cylinder is rotating at time-dependent angular velocity ω_1 and the dust is uniformly distributed in the gas. We also assume that the dusty gas has a large number density of very small particles and the bulk concentration of the particles is negligible. The density of the dust material is taken to be large compared to the gas density. The particles of the dust are approximately spherical, equal in size and so small that the STOKES law of resistance between the particles and the gas holds good. Direct interactions between the particles have been neglected. The flow is

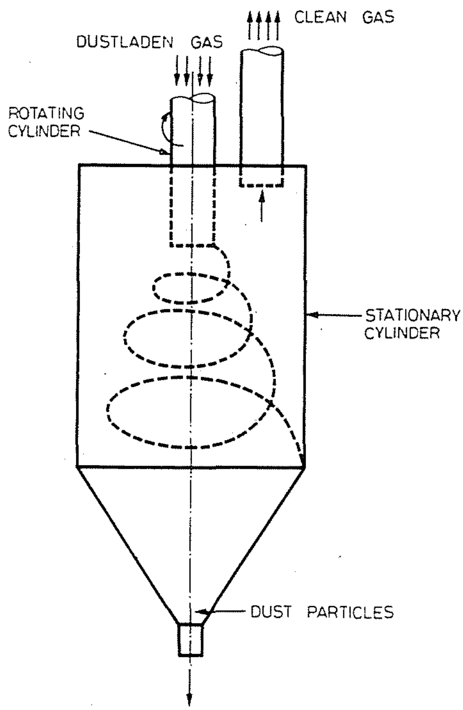

Fig. 1. A simplified diagram of a cyclone separator

assumed to be fully developed and laminar. The cylindrical co-ordinates (r, θ, z) have their origin at a convenient point on the common axis of both cylinders. Under these assumptions, the velocity distributions of the gas and the dust particles in the rotating cylinder can be expressed as:

$$u_r = 0, \quad u_\theta = u_\theta(r, t), \quad u_z = u_z(r, t)$$
$$v_r = 0, \quad v_\theta = v_\theta(r, t), \quad v_z = v_z(r, t)$$
$$N = N_0 = \text{constant}$$

where (u_r, u_θ, u_z) and (v_r, v_θ, v_z) are the velocity components of the gas and the dust particles, respectively, and N is the number density of the dust particle. Introduce the dimensionless variables:

$$\bar{u}_\theta = u_\theta/\omega b, \quad \bar{u}_z = u_z/\omega b, \quad \bar{v}_\theta = v_\theta/\omega b,$$
$$\bar{v}_z = v_z/\omega b, \quad \bar{z} = z/b, \quad R = r/b,$$
$$\bar{p} = p/2\varrho\,\omega^2 b^2, \quad \bar{t} = t/T, \quad \bar{\tau} = \bar{\tau}/T = m/kT$$

where ω is the angular velocity at $t = 0$, r is the radial distance from the axis, b is the radius of the stationary cylinder, p is the pressure, ϱ is the density, t is the time, T is the characteristic time, $\tau = m/k$ is the relaxation time, m is the mass of a dust particle, and k is the Stokes resistance coefficient.

Under the above assumptions, the governing equations for the present case can be expressed according to [1] as:

$$\frac{\partial \bar{p}}{\partial R} = \frac{1}{2}\frac{\bar{u}_\theta^2}{R} \tag{1}$$

$$\frac{\partial u_\theta}{\partial t} = \lambda_1^2\left[\frac{\partial^2 \bar{u}_\theta}{\partial R^2} + \frac{1}{R}\frac{\partial \bar{u}_\theta}{\partial R} - \frac{\bar{u}_\theta}{R^2}\right] + \frac{l}{\bar{\tau}}(\bar{v}_\theta - \bar{u}_z) \tag{2}$$

$$\frac{\partial \bar{v}_\theta}{\partial \bar{t}} = \frac{1}{\bar{\tau}}(\bar{u}_\theta - \bar{v}_\theta) \tag{3}$$

$$\frac{\partial \bar{u}_z}{\partial \bar{t}} = -\alpha_1\frac{\partial \bar{p}}{\partial \bar{z}} + \lambda_1^2\left(\frac{\partial^2 \bar{u}_z}{\partial R^2} + \frac{1}{R}\frac{\partial \bar{u}_z}{\partial R}\right) + \frac{l}{\bar{\tau}}(\bar{v}_z - \bar{u}_z) \tag{4}$$

$$\frac{\partial \bar{v}_z}{\partial \bar{t}} = \frac{1}{\bar{\tau}}(\bar{u}_z - \bar{v}_z) \tag{5}$$

where $l = mN_0/\varrho$ the mass concentration of the dust particles, $\lambda_1^2 = \nu T/b^2$ and $\alpha_1 = 2\omega T$ are constants, and ν is the kinematic viscosity.

3. Solutions of governing equations

Assume that the angular velocity of the cylinder (with radius a) and the axial pressure gradient decrease exponentially with time. In order to reduce the governing Eqs (2 to 5) to ordinary differential equations, assume that

$$\begin{aligned} \bar{u}_\theta &= f(R)\exp(-\lambda^2\bar{t}); \quad \bar{v}_\theta = g(R)\exp(-\lambda^2\bar{t}) \\ \bar{u}_z &= F(R)\exp(-\lambda^2\bar{t}); \quad \bar{v}_z = G(R)\exp(-\lambda^2\bar{t}) \\ -\frac{\partial \bar{p}}{\partial \bar{z}} &= \beta\exp(-\lambda^2\bar{t}); \quad \omega_1 = \omega\exp(-\lambda^2\bar{t}) \end{aligned} \tag{6}$$

where λ and β are real constants, ω_1 is the angular velocity at time $\bar{t} > 0$, and f, F, g and G are functions of R alone.

Substituting for $\bar{u}_\theta$, $\bar{v}_\theta$, $\bar{u}_z$ and $\bar{v}_z$ from Eq. (6) in Eqs (2 to 5) and simplifying, we get

$$\frac{d^2f}{dR^2}+\frac{1}{R}\frac{df}{dR}+\left(M^2-\frac{1}{R^2}\right)f=0 \tag{7}$$

$$g=\frac{f}{1-\bar{\tau}\lambda^2} \tag{8}$$

$$\frac{d^2F}{dR^2}+\frac{1}{R}\frac{dF}{dR}+Y\ M^2(F+\Omega) \tag{9}$$

$$G=\frac{F}{1-\bar{\tau}\lambda^2} \tag{10}$$

where

$$M^2=\frac{\lambda^2}{\lambda_1^2}\left[1+\frac{l}{1-\bar{\tau}\lambda^2}\right]$$

$$\Omega^2=\frac{\alpha}{\lambda^2}\left[\frac{1-\bar{\tau}\lambda^2}{1-\bar{\tau}\lambda^2+l}\right]$$

$$\alpha=\alpha_1\beta$$

Here M, Ω and α are real constants. The boundary conditions can be expressed as

$$\text{at } R=0,\ f=\text{finite},\ F=\text{finite}$$

$$\text{at } R=a/b=S,\ f=S,\ F=0 \tag{11}$$

The solutions of Eqs (7) and (9) under the boundary conditions given by Eq. (11) can be expressed as

$$f(R)=SJ_1(MR)/J_1(MS) \tag{12}$$

$$F(R)=\Omega\left[\frac{J_0(MR)}{J_0(MS)}-1\right] \tag{13}$$

where $J_0(MR)$ and $J_1(MR)$ are the Bessel functions of the first kind of argument MR and order zero and one, respectively. Hence from Eqs (8), (10), (12) and (13), we get

$$g(R)=SJ_1(MR)/[(1-\bar{\tau}\lambda^2)J_1(MS)] \tag{14}$$

$$G(R)=\Omega\left[\frac{J_0(MR)}{J_0(MS)}-1\right]\Big/(1-\bar{\tau}\lambda^2) \tag{15}$$

Therefore from Eqs (6) and (12 to 15) the tangential and axial velocities of the gas and the dust particles in the rotating cylinder are given by

$$\bar{u}_\theta=S\left[\frac{J_1(MR)}{J_1(MS)}\right]\exp(-\lambda^2 t) \tag{16}$$

$$\bar{v}_\theta=\left[\frac{SJ_1(MR)}{(1-\bar{\tau}\lambda^2)J_1(MS)}\right]\exp(-\lambda^2 t) \tag{17}$$

$$\bar{u}_z = \Omega\left[\frac{J_0(MR)}{J_0(MS)}\,1 - \right]\exp\,(-\lambda^2\,\bar{t}) \tag{18}$$

$$\bar{v}_z = \left[\frac{\Omega}{1 - \bar{\tau}\,\lambda^2}\right]\left[\frac{J_0(MR)}{J_0(MS)} - 1\right]\exp\,(-\lambda^2\,\bar{t}) \tag{19}$$

It is evident that for zero relaxation time, i.e. when $\tau = 0$; $\bar{u}_\theta = \bar{v}_\theta$ and $\bar{u}_z = \bar{v}_z$. Since it is assumed that the axial pressure gradient is small, hence the axial velocity $\bar{u}_z = \bar{v}_z$ will be small and it can further be assumed that the gas enters the stationary cylinder at an average axial velocity say $(V_z)_{\text{avg.}}$ without introducing much error in the solution.

4. Motion of the particles in the stationary cylinder

The separation of dust particles (with zero relaxation time, i.e., when the dust particles are very fine) from a gas in a cyclone is caused by the centrifugal force which is derived from the kinetic energy possessed by the dusty gas as it enters the cyclone at tangential velocity $\bar{u}_\theta = \bar{v}_\theta$ and at average axial velocity $(V_z)_{\text{avg.}}$. This energy makes the particle-laden gas rotate inside the stationary cylinder as shown in Fig. 1, and in doing so, it sustains the centrifugal force necessary for the radial motion of the particles. The gravitational force and the axial component of the velocity provide the downward component of the resultant and this makes the particles follow a spiral path of increasing radius. If the number of turns of the gas stream before leaving the cyclone is sufficient for the particles to reach the wall of the cylinder, they will separate. Otherwise they will be carried away with the clean gas.

The governing equation in non-dimensional form for the motion of the particles inside the stationary cylinder is given by [7]

$$R\,dR = (S_2\bar{u}_\theta^2 d\bar{t})/2 \tag{20}$$

where

$$S_2 = \bar{D}_p^2 S_1 \bar{R}e,\ \ S_1 = (1 - \varepsilon)/q\varepsilon,$$
$$Re = \omega b^2/\nu, \quad \bar{R}e = Re\,\omega T.$$

Here $\bar{D}_p = D_p/2b$ is the non-dimensional diameter of the particle, $\varepsilon = \varrho/\varrho_p$ the gas to particle density ratio, Re is the Reynolds number. Since the radial motion is slow and the distance travelled is short, no appreciable error is introduced by taking the magnitude of $\bar{u}_\theta$ as $S\exp\,(-\lambda^2\bar{t})$ i.e., the value of $\bar{u}_\theta$ at radius S. Substituting for $\bar{u}_\theta$ from Eq. (16) in Eq. (20) and integrating, we have

$$(1 - S^2) = S_2 S^2[1 - \exp\,(-2\lambda^2\,\bar{t})]/\lambda^2 \tag{21}$$

(Here the lower and upper limits for R have been taken as S and 1, respectively. Similarly for $\bar{t}$, they have been taken as 0 and $\bar{t}$, respectively.) Eq. (21) gives the time $\bar{t}$ required for the particle to reach the wall.

Let a number n of turns required for the particle to travel the distance across the gas stream and separate from it. The approximate relationship between the time required and the number of turns is [7]:

$$2\pi n = \int_0^{\bar{t}} \bar{u}_0 \, d\bar{t} \tag{22}$$

where $\bar{u}_\theta(1, \bar{t})$ is the tangential velocity of the particle at the stationary cylinder, assumed to approximate that at the radius S, i.e., $\bar{u}_\theta(1, \bar{t}) = \bar{u}_\theta(S, \bar{t})$. Integrating (22):

$$2\pi n = S[1 - \exp(-\lambda^2 \bar{t})]/\lambda^2 \tag{23}$$

Eliminating $\bar{t}$ from Eqs (21) and (23), we get

$$n = \left[S - \left\{ S^2 - \frac{\lambda^2(1 - S^2)}{S_2} \right\}^{1/2} \right] \Big/ 2\pi \lambda^2 \tag{24}$$

It is evident from Eq. (24) that $n = 0$ for equal radii of both cylinders, i.e., for $S = 1$, then. In order that n be real, $\lambda^2/S_2 \leq S^2/(1 - S^2)$. This relationship provides a useful criterion for the choice of λ^2 if S_2 and S are prescribed or of λ^2/S_2 if S is only given. It may be remarked that with proper choice of λ, the number of turns rquired for the separation of the dust particles from the gas can be reduced. Hence the present model may prove to be more efficient than the existing ones. It is to be noted that the effect of the relaxation time on the dust particle has been omitted in calculating the number of turns required for the separation of the dust particles from the gas as its effect has been assumed to be small. Its effect can, however, easily be included in the above analysis.

5. Conclusions

The number of turns required for the particle to separate from the gas decreases for greater particle diameters, but it increases with increasing ratio of fluid to particle density. It becomes zero when the radii of the rotating and the stationary cylinder are equal.

Summary

The unsteady flow of an incompressible viscous dusty gas through cylinders has been studied and a simple model of a cyclone separator has been suggested.

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