SHIP STRESSES DUE TO WAVE MOTION

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(Received May 10, 1974)

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Development in ship building has drawn the attention, particularly in recent years, to the strength problems of vessels as the construction of more and more economic units with increasing dimensions and a relatively lighter weight is endeavoured. The latest experiences, however, show an increasing number of wrecks as well, which may be correlated to these increased dimensions.

Thus the establishment of the connections between the geometric dimensions and mechanical properties of the vessels seems to be fully justified.

In these studies a ship is modelled as a bar of constant cross section and homogeneous mass distribution. Such a model may be considered as absolutely corresponding to the actual situation, particularly in the case of large tankers (see Fig. 1). Thus in the definition of the stresses a definite distinction is to be made between bar force, shear force, bending and torsional moments, as usual in bar testing.

With respect to the longitudinal strength of the ship the bending moment and the shear force must be known for the stress calculations and, this is what we are going to discuss in this paper.

The hull, floating on the water surface and loaded by weight and buoyancy, is studied as a bent bar. The longitudinal weight distribution is uniform, just like that of the buoyancy due to water displacement, in the case of a smooth water surface. In such cases no stress or load will be produced.

In the case of a wavy water surface, however, the buoyancy distribution may become variable in time and space. In addition, the ship will perform, because of the waves, certain oscillating and vibrating movements.

According to the interpretation generally accepted in literature such a swaying means the oscillating motion of the ship as a rigid body. The hull can perform several different movements simultaneously:

(a) sway (vertical oscillation of the vessel),

(b) pitching (horizontal oscillation of the vessel about its centre of gravity axis, perpendicular to the longitudinal axis of the hull),

(c) rolling (oscillatory motion about the longitudinal axis of the ship),

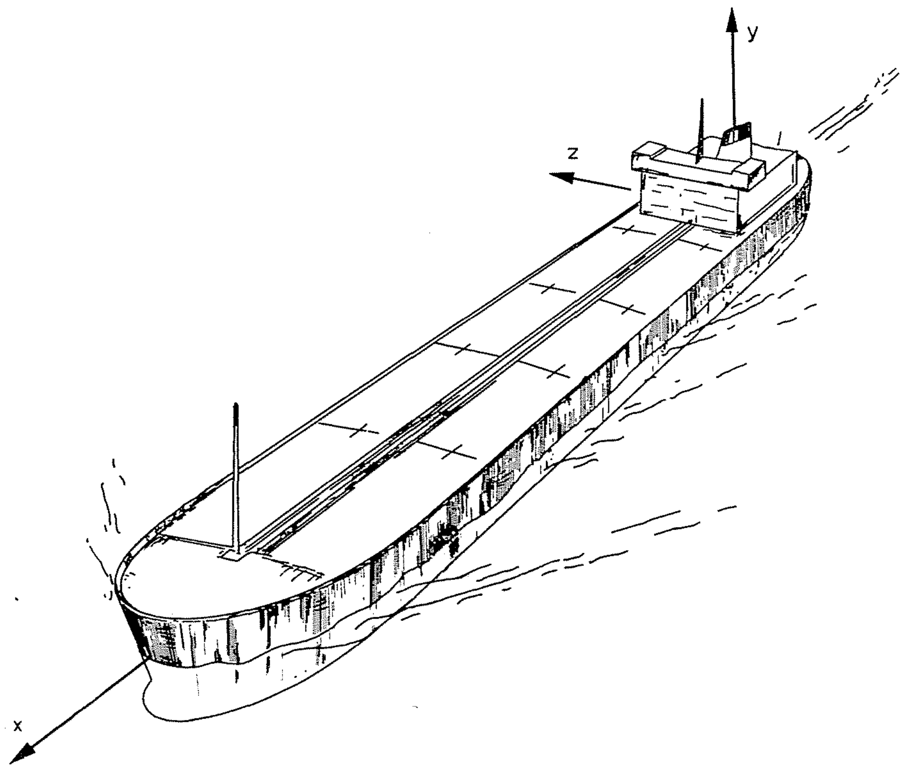

Fig. 1

(d) stroll (oscillation of the vessel about the vertical center of gravity axis, perpendicular to its longitudinal axis).

Vibration means the different vibratory motions of the ship as a flexible beam, with the usual distinctions between longitudinal, bending, and torsional vibration, respectively.

Bending moment studies consider sway, pitching, and bending vibrations. Thus the effect of the waves on the stresses of the ship should be studied through its relations under dynamic conditions, with the ship sway and vibrations due to flexibility fully taken into account.

The movement equation is written now for the bar substituting the ship, which is considered not to move horizontally i.e. stationary in space, but the waves are assumed to travel as related to the vessel. Thus the velocity c of the wave is interpreted as relative to the ship.

The origin of the co-ordinate system stationary in space is in the mass centre of the bar resting on the smooth water surface. Axis x coincides with the longitudinal axis of the resting bar, while axis y is vertical, with its positive direction pointing downwards. Axis z is normal to the other two. The length of the bar is l, its width is b, its weight is denoted by Q, and its mass by m. The centre of gravity is at the half point of this bar of length l. The movement is related to the rest position.

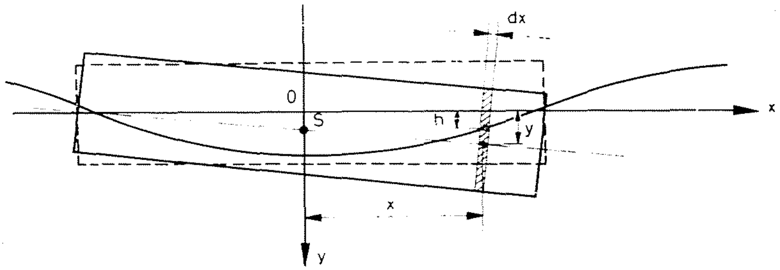

Fig. 2

The displaced position is illustrated in Fig. 2 as in the selected, in-space stationary, co-ordinate system. The shape of the water surface is expressed by the equation

$$h = \frac{H}{2} \cos (Kx - \omega t) \tag{1}$$

where

$$K = \frac{2\pi}{\lambda}$$

Owing to the wave formation the mass centre of the bar will be displaced in direction y, and rotate about axis z, while the curvature of the bar centre lime originally coincident with axis x, will change.

During motion, the originally horizontal centre line, coincident with axis x, will assume the form

$$y = (x, t)$$

provided the bar will neither emerge from nor sink into the water.

1. Expression of the movement equation

The movement equation is expressed on the basis of the Hamilton principle. Accordingly, the time integral of the Lagrange function $\mathfrak{L}$ (kinetic potential) is an extreme value and, therefore, its first variation is zero. The equation, where the integral will assume an extreme value, is the Euler equation of the variation problem, that is, in the present case the movement equation. Let us have equation

$$\mathfrak{L} = T - U \tag{2}$$

where T is the kinetic while U the total potential energy of the system. Potential U can be obtained as the sum of the internal and external potentials:

$$U = U_b + U_k$$

U_k may be expressed by the work of the external forces:

$$U_k = -L$$

whereby the potential may be written as

$$U = U_b - L$$

Now let us study a dx size and dm mass elementary part of the bar of length l. For the elementary prism the kinetic energy, if that due to the rotation of the mass dm is neglected, may be expressed as

$$\mathrm{d}T = \frac{1}{2} \dot{y}^2 \, \mathrm{d}m$$

where

$$\dot{y} = \frac{\partial y\,(xt)}{\partial t}$$

is the momentary speed of the centre of gravity of the elementary prism under test. Introducing mass μ per unit length, the kinetic energy for the whole bar will assume the form

$$T = \frac{1}{2} \int_{-\frac{l}{2}}^{+\frac{l}{2}} \mu \dot{y}^2 \, \mathrm{d}x$$

In calculating U_b, the internal energy accumulated because of the bending moment is to be reckoned with:

$$U_b = \frac{1}{2} \int_{-\frac{l}{2}}^{+\frac{l}{2}} IEy''^2 \, \mathrm{d}x$$

if the symbol

$$y'' = \frac{\partial^2 y\,(xt)}{\partial x^2}$$

is introduced. The external force means the buoyancy produced in a water of γ specific gravity:

$$\mathrm{d}F = -\gamma b\,(y - h)\,\mathrm{d}x$$

whose work for path y is

$$\mathrm{d}L = -\int_y \gamma b\,(y - h)\,\mathrm{d}x\mathrm{d}y$$

or, for the whole bar, the following equation is obtained:

$$L = -\frac{1}{2}\int_{-\frac{l}{2}}^{+\frac{l}{2}} \gamma b\,(y^2 - 2\,hy)\,\mathrm{d}x$$

The integral of L with respect to time is produced according to the Hamilton principle:

$$J\,(y) = \int_0^{t_1} (T - U)\,\mathrm{d}t$$

which, after substitution, may be written as

$$J\,(y) = \int_0^{t_1}\int_{-\frac{l}{2}}^{+\frac{l}{2}} [\mu\dot{y}^2 - IEy''^2 - \gamma b\,(y^2 - 2\,hy)\,\mathrm{d}x\,\mathrm{d}t \tag{3}$$

Determination of the function $y = y(x, t)$ representing the extreme of the double integral $J(y)$ leads to the Euler equation of the variation problem:

$$IEy^{\mathrm{IV}} + \mu\ddot{y} + \gamma by = \gamma bh \tag{4}$$

where to the following peripheral and initial conditions are associated:

$$y''\left(\frac{l}{2},\, t\right) = 0; \qquad y''\left(-\frac{l}{2},\, t\right) = 0$$

$$y'''\left(\frac{l}{2},\, t\right) = 0; \qquad y'''\left(-\frac{l}{2},\, t\right) = 0$$

$$y(x, 0) = 0; \qquad \dot{y}(x, 0) = 0$$

The above relations reveal that the bending moment M and shear force V are always zero at both ends of the bar, and that the movement is related to the resting position (smooth water surface), whereby both displacement and velocity are zero at the time $t = 0$.

2. Determination of the bending moment

Since the original objective has been the determination of the bending moment, it is best to transform the problem on the basis of the equation

$$\frac{\partial^2 y}{\partial x^2} = -\frac{M}{IE}$$

valid for flexible strands. Such a transformation has the advantage of making the relations expressing the peripheral conditions assume a simpler form, not to mention that although the direct methods of the variation calculus render the function sought for with a good approximation, but this does not apply to the derivatives of that function.

After transformation, (4) gives the differential equation

$$IEM^{IV} + \mu \ddot{M} + \gamma b M = -\gamma b I E h'' \tag{5}$$

which contains the bending moment function $M = M(x, t)$ looked for. In this case the peripheral and initial conditions will read

$$M\left(\frac{l}{2}, t\right) = 0; \qquad M\left(-\frac{l}{2}, t\right) = 0$$

$$M'\left(\frac{l}{2}, t\right) = 0; \qquad M'\left(-\frac{l}{2}, t\right) = 0$$

$$M(x, 0) = 0; \qquad \dot{M}(x, 0) = 0$$

Accordingly, variation equation (3), too, can be transformed:

$$J(M) = \frac{1}{2}\int_0^{t_1}\int_{-\frac{l}{2}}^{+\frac{l}{2}} [\mu \dot{M}^2 - IEM'' - \gamma b M^2 - 2\, IE\gamma b M h'']\, \mathrm{d}x \mathrm{d}t \tag{6}$$

whereby we arrive to the problem of how to determine the function $M = M(x, t)$, supplying the extreme value of the integral $J(M)$, under the given peripheral and initial conditions.

3. Solution of the problem

The problem of the extreme value of a double integral can be reduced, on the basis of the method introduced by L. V. KANTOROVITCH, to the solution of an ordinary differential equation related to the extreme of a single integral. The solution is sought for in the form of expressions containing singlevariable indeterminate functions. Only that part of the function looked for is assumed in advance which satisfies a priori the peripheral conditions, whereas its second part is determined in accordance with the character of the problem.

Let us plot the function $M(x, t)$ searched for in the form of a function series consisting of the products of functions $\varphi(x)$ depending only on x, and $f(t)$ dependent only on t where, as a basic system, the bending moment function, corresponding to the character of the oscillation patterns associated with the various natural oscillations, is selected.

If the function $\varphi(x)$ representing the solution is written in the form

$$\varphi(x) = \left[x^2 - \left(\frac{l}{2}\right)^2\right]^2 \qquad (7)$$

then it will always satisfy the peripheral conditions. Hence only the function $f(t)$ will have to be determined, so as to supply the extreme of the integral $J(M)$ given by Eq. (6).

For the function $M(x, t)$ a number of approximative solutions can be obtained:

$$M_1 = \varphi(x) \cdot f_1(t)$$
$$M_2 = \varphi(x) \cdot f_1(t) + x \cdot \varphi(x) \cdot f_2(t)$$
$$M_3 = \varphi(x) \cdot f_1(t) + x \cdot \varphi(x) \cdot f_2(t) + \varphi^2(x) \cdot f_3(t)$$

The more terms are reckoned with in the solution, the more accurate result will be obtained.

First approximation of the solution

Owing to the rather lengthy calculations, only the solution of the first approximation will be dealt with restricting the discussion, even so, to nothing but the explanation of the results.

After substituting (6) the function $f_1(t)$ is determined, whereby the function

$$M_1(x, t) = \varphi(x) \cdot K_1 (\cos \omega t - \cos \alpha_1 t) \qquad (8)$$

will be obtained for the bending moment.

For the amplitude K_1 in (8) the equation

$$K_1 = \frac{D_1}{\alpha_1^2 - \omega^2} \tag{9}$$

is obtained, where D_1 is a constant depending on the geometrical dimensions and the elasticity of the material, ω is the angular frequency of wave production, and α_1 is the projective angular velocity of the natural oscillation calculated from the first approximation.

The frequency of the natural oscillation is expressed by the equation

$$\alpha_1^2 = \frac{504\, IE}{\mu l^4} + \frac{\gamma^b}{\mu} \tag{10}$$

consisting of two terms. The first one supplies, with good approximation, the square of the angular frequency pertaining to the basic vibration of an elastic bar of length l and constant cross section, under the given peripheral conditions, whose accurate value is known from the literature:

$$\beta_1^2 = \frac{500{,}546\, IE}{\mu l^4} \tag{11}$$

Eq. (10) is the square of the projection angular velocity of the sway motion of a bar of mass m and length l, as a rigid body, and at the same time, of pitching motion as well, since the bar is symmetrical.

If

$$\alpha_0^2 = \frac{\gamma\, b}{\mu} \tag{12}$$

is introduced, Eq. (10) may be written in the following form:

$$\alpha_1^2 = \beta_1^2 + \alpha_0^2 \tag{13}$$

Similar results would be obtained by further approximations where, for the subsequent natural oscillations, the following expressions could be arrived at:

$$\alpha_2^2 = \beta_2^2 + \alpha_0^2$$

$$\alpha_3^2 = \beta_3^2 + \alpha_0^2$$

where $\beta_1, \beta_2, \beta_3 \ldots$ etc. lead to the corresponding natural frequency values known from literature, while α_0 is always the same quantity that had been obtained with the basic oscillation.

In steady state the natural oscillations will cease to exist because of the various external and internal attenuations which had not been reckoned with in the definition of the problem, and only the generated oscillation is maintained. From (8), thereby, the bending moment may be written in the form

$$M_1(x, t) = \varphi(x)\, K_1 \cos \omega t \tag{15}$$

where K_1 is the value determined from the resonance function (9), which can be rather high around the resonance level. It can be verified that

$$\lim_{IE \to \infty} K_1 = \frac{D_0}{\frac{4}{5} l^5} \tag{16}$$

where D_0 contains only the geometrical dimensions. Thus it may be stated that the resonance phenomenon can only occur in the case of an elastic body.

4. Application of the results

A general analysis of the results thus obtained would be a very much sophisticated and lengthy job and, therefore, it might be much more reasonable to perform the calculations for a given practical case demonstrating these test results.

Calculation will be based on a vessel the data of which are known from the literature: No 2 of the 25 vessels introduced by the comprehensive work of F. H. TODD, known as S. S. TANKER.

The principal dimensions of this ship are:

$l = 134$ m,

$b = 18$ m,

water displacement: $\Delta = 15\,433$ t,

$I = 28.4$ m^4

$E = 2.1 \times 10^7$ Mp/m^2

$\alpha_{cal} = 7.77$ lit/sec

$\alpha_{meas} = 8.16$ lit/sec

According to TODD, the maximum bending moment in the central cross section is

$$M_{max.} = 78\,476 \text{ Mpm}$$

which is a static value by assuming smooth water surface, due to the uneven distribution of the loading forces and buoyancy.

With these data, the natural frequency and the bending moment can be readily calculated on the basis of the above relations.

From (10), corresponding to the first approximation, the natural frequency will be

$$\alpha_1 = 8.98 \text{ lit/sec}$$

but the calculation of the bending moment requires the knowledge of further data.

According to literature, cases when the length of the wave is identical to that of the vessel should be regarded as hazardous, so the wave length is assumed to be $\lambda = l$, while the wave height may be calculated as $H = \lambda/20$. The rate of wave propagation is $c = 16$ m/sec, whereas the speed of the vessel against the waves is $v = 10$ knots $= 5.15$ m/sec.

The period of the exciting effect of the waves is, thereby,

$$T = \frac{\lambda}{c + v} = 6.33 \text{ sec}$$

Thus the angular frequency of excitation will amount to

$$\omega = \frac{2\pi}{T} = 1.0 \text{ lit/sec}$$

After calculating the respective constants, we shall obtain for the maximum bending moment

$$M_{1\,\text{max.}} = 51\,962.7 \text{ Mpm}$$

For comparison, the value obtained after the third approximation would be

$$M_{3\,\text{max.}} = 55\,616.4 \text{ Mpm}$$

The result rendered by the first approximation equals to 93.43 per cent of $M_{3\,\text{max.}}$, which means that the very first approximation itself has already given an acceptable result. The static load of our model, pertaining to smooth water surface, is zero, thus the $M_{1\,\text{max.}}$ value obtained for the bending moment is due exclusively to dynamic effects. It follows that the standard load is the sum of Todd's static value and $M_{1\,\text{max.}}$ as calculated above.

The auxiliary load calculated exclusively from dynamic effects is seen to approximate in order of magnitude the static value calculated for the vessel, although the resonance point is still far away.

The situation might be much less favourable if the difference between ω and α decreases. The last examination is, therefore, that of the variation of the M_{max} value in the function of wave length and excitation frequency.

After introducing $w = v + c$ and the ratio $n = \lambda/l$, the magnitude of $M_{max.}$ is calculated in the function of w, at different n parameter values. The results are presented in Fig. 3.

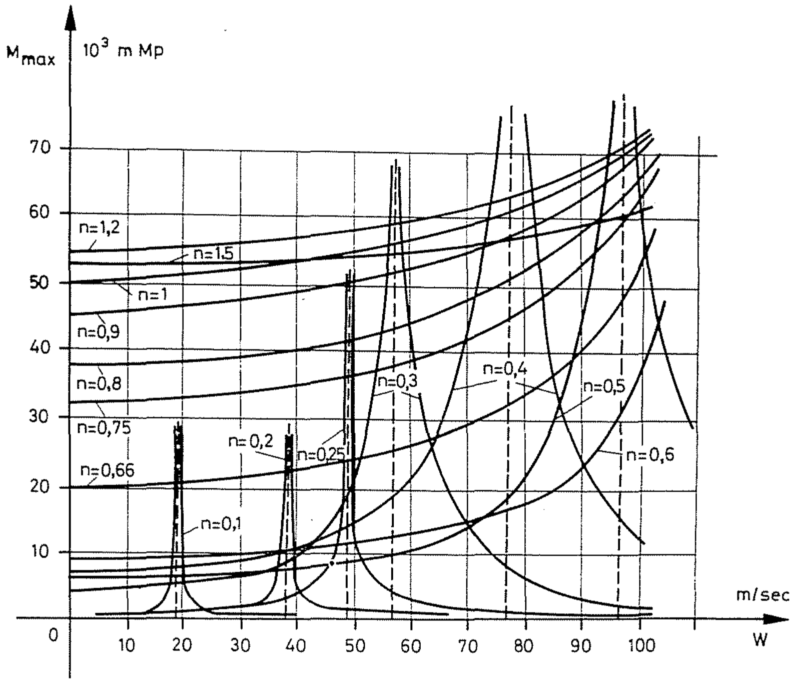

Fig. 3

This Figure reveals that each n value has such a w velocity associated whereas $M_{max.}$ approximates infinity. The resonance points are increasingly displaced to the right along the w axis with an increasing n.

It can be observed that at low n values (0.1 to 0.5) only around the point of resonance may considerable stresses and load be reckoned with, an increasing n is a priori associated therewith, although a resonance is still remote. Maximum load is obtained at $n = 1.2$ or so.

Summary

According to the presented paper it may be stated that in the determination of ship stresses the usual static methods should be completed by taking into consideration the dynamic effects caused by wave generation.

This is all the more important in modern shipbuilding as the increasing principal dimensions of the vessels increase these dynamic effects as well.

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