STRENGTH OF THICK-WALLED TUBES AND CYLINDRICAL VESSELS EXPOSED TO AXISYMMETRICAL LINE LOADS, (PART II)

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First part of this paper [1] has been concerned with a method for determining additional stresses due to axisymmetrical line loads acting on thick-walled tubes and cylindrical vessels. The modified method based on the principle of elastically bedded beams permits to deduce relationships for the design of vessels, tube walls, wall layers of the most common ration of radii

$$k_0 = \frac{r_1}{r_2} \geq 0{,}4$$

In this part design formulae developed by analogy to thin shell relationships will be described, primarily aiming at easing use of expressions for thick-walled vessels and tubes.

In addition to the design formulae, comparison with other methods, and test results will be presented.

Similarly as before, terms "cylindrical vessel", "vessel body" etc. will be uniformly replaced by "thick-walled tube".

Analysis of thick-walled tube stresses

The previously described stress factors lend themselves for the analysis of thick-walled tube stresses due to both line forces Q_0 and to line moments M_0.

Numerical determination of stresses requires a concrete expression.

Additional stresses due to edge load Q_0 at an arbitrary point along the radius of the thick-walled tube:

axially:

$$\sigma_{xQ} = \beta Q_0 \qquad \bar{\sigma}_{xQ} = \beta Q_0 k_M \, (k_s - k) \, H_2 \tag{1}$$

tangentially:

$$\sigma_{\varphi Q} = \beta Q_0 \bar{\sigma}_{\varphi Q} = \beta Q_0 \frac{k_N}{k} H_1 + \mu \sigma_{xQ} \tag{2}$$

Additional stresses due to edge moment M_0 at an arbitrary point along the radius of the thick-walled tube:

axially:

$$\sigma_{xM} = \beta^2 M_0 \quad \bar{\sigma}_{xM} = \beta^2 M_0 k_M (k_s - k) H_3 \tag{3}$$

tangentially:

$$\sigma_{\varphi M} = \beta^2 M_0 \bar{\sigma}_{\varphi M} = \beta^2 M_0 \frac{k_N}{k} H_4 + \mu \sigma_{xM} \tag{4}$$

where k_s, k_N, k_M are defined as, and determined from diagrams in. Part. I. β is a shell constant for thick-walled tubes calculated simply from thin shell constant β_{vh}:

$$\beta = k_\beta \beta_{vh} \tag{5}$$

For a Poisson's ratio $\mu = 0.3$:

$$\beta = k_\beta \frac{1}{r_2} \frac{1{,}82}{\sqrt{1 - k_0^2}} \tag{6}$$

where k_β is again defined and represented as in Part I.

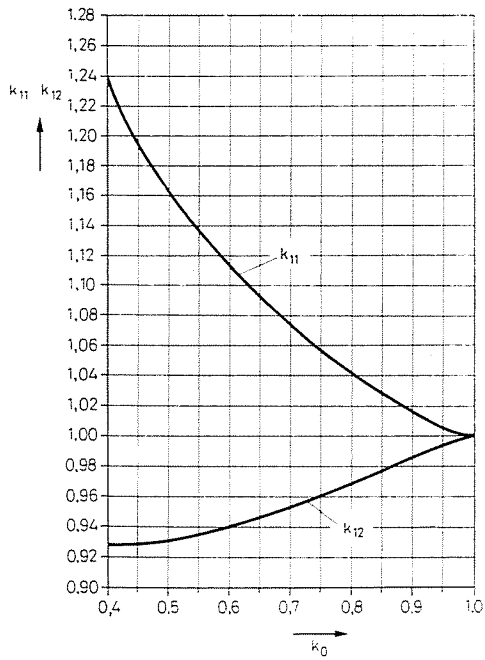
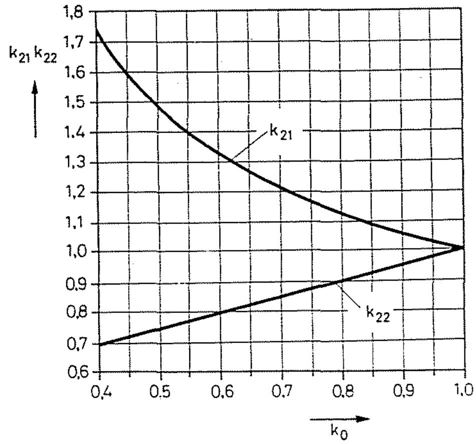


Fig. 1


k21 k22
1,8
1,7
1,6
1,5
1,4
1,3
1,2
1,1
1,0
0,9
0,8
0,7
0,6
k21
k22
0,4
0,5
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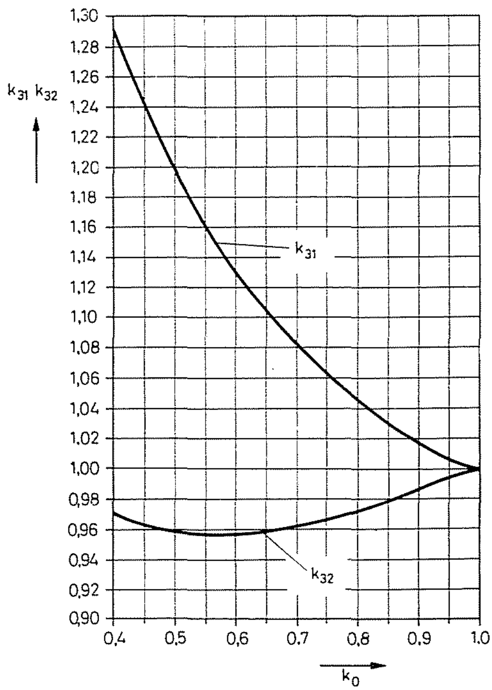
0,8
0,9
1,0
k0

Fig. 2


k31 k32
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1,28
1,26
1,24
1,22
1,20
1,18
1,16
1,14
1,12
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1,08
1,06
1,04
1,02
1,00
0,98
0,96
0,94
0,92
0,90
k31
k32
0,4
0,5
0,6
0,7
0,8
0,9
1,0
k0

Fig. 3

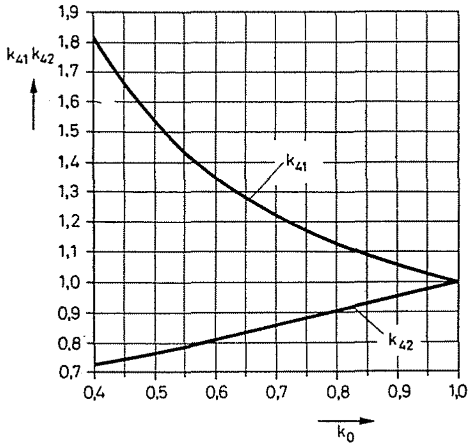

Fig. 4

The presented modified method permits the thin shell design method to be directly applied for thick-walled tubes, intermediating given factors. This treatment of stress expressions permits to illustratively compare additional stresses in thick-walled and thin-walled tubes.

Stress expressions are written — as before — for edge force Q_0 and edge moment M_0.

Additional stresses due to edge load Q_0 at an arbitrary point along the radius of the thick-walled tube:

axially:

$$\sigma_{xQ} = \frac{2}{k_\beta k_B (1 - k_0)} (k_s - k) \sigma_{xQv} \tag{7}$$

introducing the factor

$$k_1 = \frac{2}{k_\beta k_B (1 - k_0)} \tag{8}$$

$$\sigma_{xQ} = k_1 (k_s - k) \sigma_{xQv} \tag{9}$$

tangentially:

$$\sigma_{\varphi Q} = \frac{1 + k_0}{2 k_\beta^3 k_B} \frac{1}{k} \frac{N_{\varphi Qv}}{s} + \mu \sigma_{xQ} \tag{10}$$

introducing the factor

$$k_2 = \frac{1 + k_0}{2 k_\beta^3 k_B}$$

and substituting (9):

$$\sigma_{\varphi Q} = \frac{k_2}{k} \frac{N_{\varphi Qv}}{s} + \mu k_1 (k_s - k) \sigma_{xQv} \tag{11}$$

Additional stresses due to edge moment M_0 at an arbitrary point along the thick-walled tube radius:

$$\sigma_{xM} = \frac{2}{k_B(1-k_0)}(k_s - k)\,\sigma_{xMv} \tag{12}$$

Introducing the factor

$$k_3 = \frac{2}{k_B(1-k_0)}$$

$$\sigma_{xM} = k_3(k_s - k)\,\sigma_{xMv} \tag{13}$$

tangentially:

$$\sigma_{\varphi M} = \frac{1+k_0}{2k_\beta^2 k_B}\,\frac{1}{k}\,\frac{N_{\varphi Mv}}{s} + \mu\sigma_{xM} \tag{14}$$

introducing the factor

$$k_4 = \frac{1+k_0}{2k_\beta^2 k_B}$$

and substituting (13):

$$\sigma_{\varphi M} = \frac{k_4}{k}\,\frac{N_{\varphi Mv}}{s} + \mu k_3(k_s - k)\,\sigma_{xMv} \tag{15}$$

where stresses and line forces σ_{xQv}, $N_{\varphi Qv}$, σ_{xMv}, $N_{\varphi Mv}$ are valid for thin shells, to be calculated according to [2], [3].

In design, primarily the stress maxima due to bending developing in extreme fibres of thick-walled tubes are needed. On the inner and outer tube surface, $k = k_0$, and $k = 1$, respectively. Replacing the ratio of radii k for an arbitrary radius by extreme fibre values results in stress maxima in the extreme fibre:

$k_{11}, k_{21}, k_{31}, k_{41}$ are factors for the inner surface.

$$k_{i1} = k_i(k_s - k_0); \qquad [i = 1, 3] \tag{16.a, b}$$

$$k_{i1} = k_i\frac{1}{k_0}; \qquad [i = 2, 4]$$

$k_{12}, k_{22} = k_2, k_{32}, k_{42} = k_4$ are factors for the outer surface.

$$k_{i2} = k_i(1 - k_s); \qquad [i = 1, 3] \tag{17.a, b}$$

$$k_{i2} = k_i; \qquad [i = 2, 4]$$

Factors plotted in diagrams are shown in Figs 1, through 4.

Design relationships

Comparative evaluation of the modified method has been based on stress maxima. Design stresses being essentially stress maxima, this comparison is a guide for developing design formulae.

Comparative examination results of the presented modified method have been plotted in Figs 5 through 8.

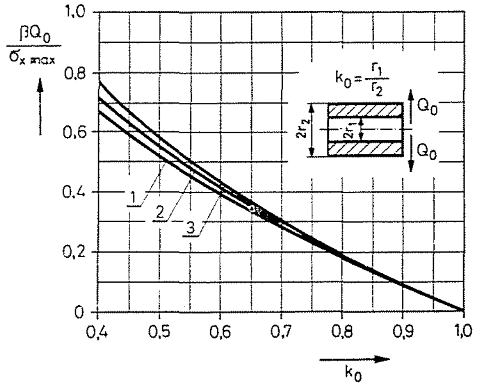

Fig. 5. Axial stress maxima at the inner side

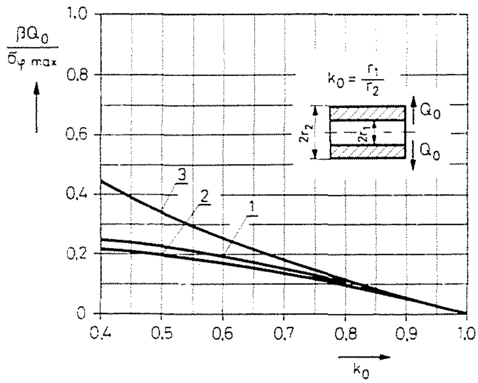

Fig. 6. Axial stress maxima at the outer side

Stresses resulting from the modified method are represented by curve 1, while those obtained by Biederman's method [3] are shown by curve 2, and those from the shell theory by curve 3.

For the sake of conciseness, the comparison will only be presented for edge force Q_0, the result was, however, similar for edge moment M_0. Diagrams in Figs 5 through 8 show the presented method to exhibit a close agreement with the Biederman method in the tested range.

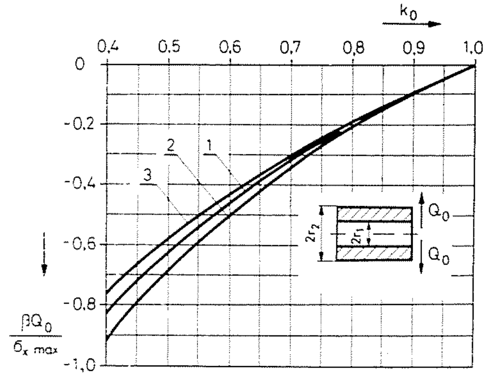

Fig. 7. Tangential stress maxima at the inner side

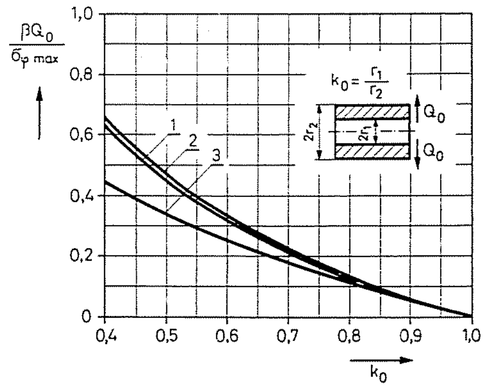

Fig. 8. Tangential stress maxima at the outer side

Axial stresses calculated by the thick-walled tube method and according to the shell theory do not differ significantly, but tangential stresses do.

Comparative examinations lead to the conclusion that for practical calculations shell theory relationships suit axial stress determination, while tangential stresses should be determined by relationships for thick-walled tubes:

$$\sigma_{xQ} = \sigma_{xQv} \tag{18}$$

$$\sigma_{xM} = \sigma_{xMv} \tag{19}$$

$$\sigma_{\varphi Q} = k_2 \frac{1}{k} \frac{N_{\varphi Qv}}{s} + \mu \sigma_{xQv} \tag{20}$$

$$\sigma_{\varphi M} = k_4 \frac{1}{k} \frac{N_{\varphi Qv}}{s} + \mu \sigma_{xMv} \tag{21}$$

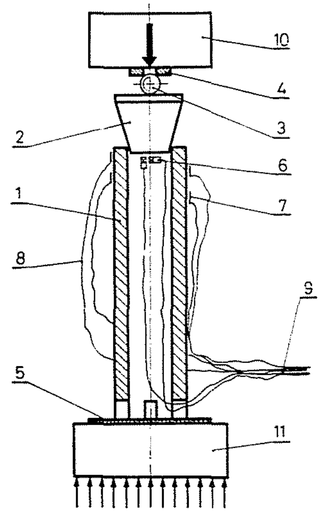

Fig. 9. Scheme of the test arrangement

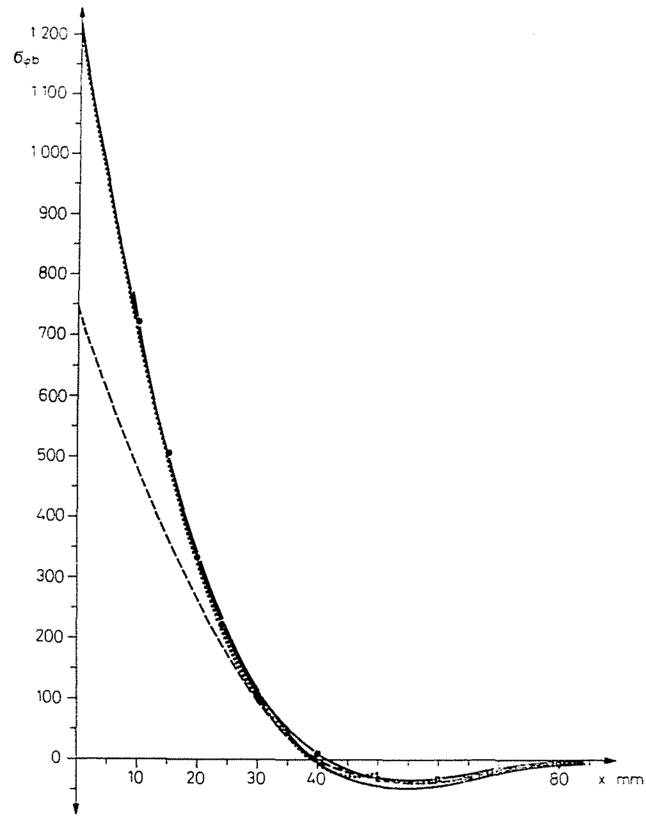

Fig. 10. **Tangential stress due to load $P = 4000$ kp at the inner tube side. According to the modified method: full line; according to the shell theory: dashed line; according to the Biederman method: dotted line; measured: discret points**

Test results

To check the validity of theoretical relationships, tests have been made with the test scheme in Fig. 9. Test tube dimensions are: ∅ 46.8 × ∅ 78 × 30, hence ratio of radii: $k_0 = 0.6$.

Edge load Q_0 is transmitted to test tube 1 by clamping cone 2. Axial, uniform loading is safeguarded by steel ball 3, washer 4 and paper 5 between platens 10 and 11. Inner and outer side strain gauge tags 6 and 7 are connected by cables 7 and 8 to the instruments.

For the determination of stresses in the tube loaded by edge force Q_0 the strain gauge method has been chosen. Twenty-two measuring spots have been applied on the inner and outer tube surface.

Figs 10 and 11 present only the most typical diagrams of tangential stresses in the outer and inner tube side these being the design stress maxima. Measured axial and tangential stresses on both the inner and the outer tube side have been processed. No direct environment of the force application point has been analysed, neither tested.

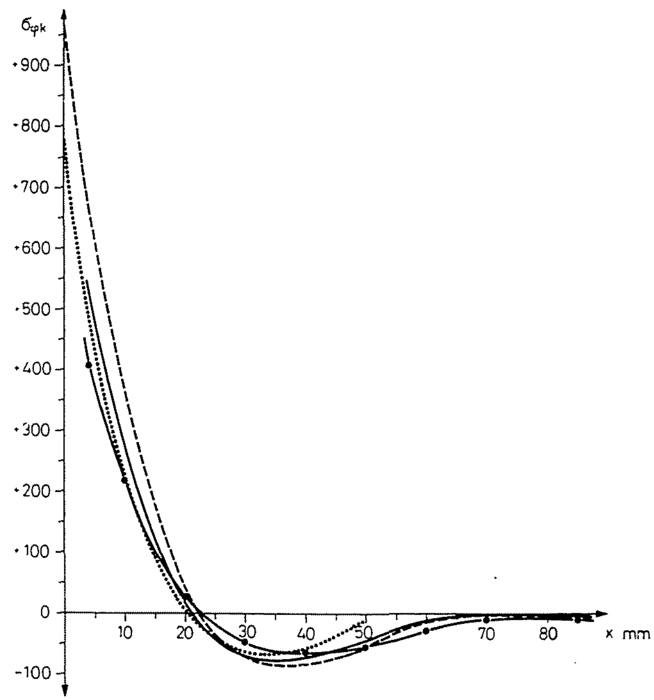

Fig. 11. Tangential stress due to load $P = 4000$ kp at the outer tube side. According to the modified method: full line; according to the shell theory: dashed line; according to the Biederman method dotted line; measured: discret points

Summary

Relationships have been written for the design of pressure vessels, cylindrical casings, tubes exposed to additional stresses due to circular symmetrical line loads. For the design of cylinders with the most common radius ratio $k_0 \geq 0{,}4$, simplified formulae can be written. The treatment by analogy to the shell theory is a further simplification.

Comparison between literature and shell theory methods valid for thick-walled cylinders shows shell theory methods to be accurate enough for the practical analysis of axial stresses, but tangential stresses need to be determined by the presented relationships valid for thick-walled tubes.

Determinations on the model tube supported theoretical relationships.

Legend

k	—	radius ratio, factors
H	—	attenuation functions
M	cmkp/cm	line moment
N	kp/cm	line force
Q	kp/cm	line load
r	cm	radius
s	cm	wall thickness
β	l/cm	shell constant
μ	—	Poisson's ratio
σ	kp/cm	stress

Subscripts:

M	in case of line moment
Q	in case of line load
x	axial
v, vh	referring to this shell
φ	tangential
o	line load

References

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