ELASTIC STRESSES IN TORISPHERICAL PRESSURE VESSEL HEADS

By

P. REUSS

Department of Chemical Machines, Technical University, Budapest

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Presented by Prof. Dr. S. SZENTGYÖRGYI

Introduction

Economical construction of pressure vessels and chemical equipment is preconditioned by exact design, depending on the knowledge of the stress conditions. Previously, design had been based exclusively on empirical or experimental formulae, namely, in spite of the knowledge of basic relationships of the theory of shells, practical calculations were rather intricate. Because of the high expenses and time demand of experimental work, the advent of computers was essential for the development of design methods. Pressure vessels have often torispherical heads, subject of several publications. Data refer, however, to heads of determined size ratios, although designs vary for each country. This fact imposed to develop a computer program for the analysis of vessel heads conform to Hungarian standards.

Published results

Rather than to give an overall survey of the profusion of relevant literature, let us refer to papers by FESSLER and STANLEY [6], FINDLEY et al. [7], [(8]). Essential research results will be quoted below.

Several research workers have been concerned with the design of toroidal shells, important elements of vessel heads, it became obvious, however, that no solution was possible by classic methods — e.g. using Bessel's functions — but only by numerical methods.

GALLETLY [4] calculated toroidal shells by the Runge—Kutta method based on the linear theory of shells. Application of the edge influence coefficients was presented also on torispherical shell with the proportions: $t/D = 0.002083$, $r/D = 0.0625$, $R/D = 0.9375$ (see Fig. 1). His paper [3] presented also stress curves in the toroidal part (knuckle). His calculations are valid in the elastic range. An inherent drawback of Galletly's method is the increase of sources of error in case of "long" shells.

MARCAL and PILGRIM [5] analysed a torispherical shell in both the elastic and the plastic range, applying a numerical method and comparing

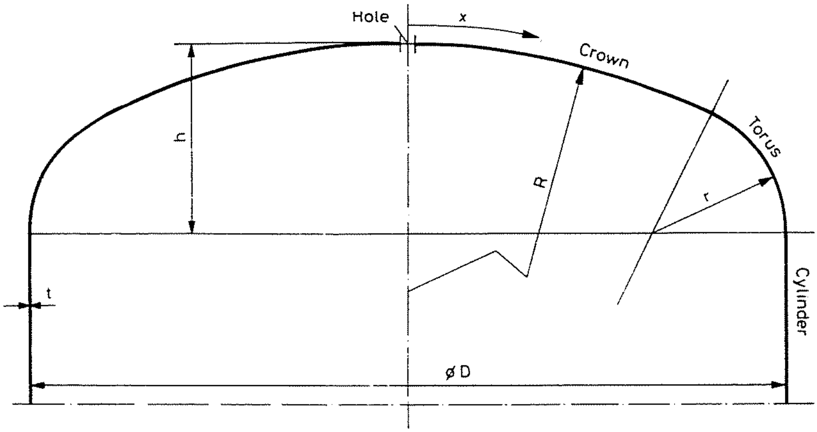

Fig. 1. Torispherical vessel head geometry

the results to those obtained by STODDART using strain gauges. Shell data were: $t/D = 0.0208$, $r/D = 0.167$, $R/D = 1.0$. Calculated and measured results were in fair agreement throughout.

FESSLER, STANLEY, FINDLAY and MOFFAT experimented over ten years [6], [7], [8]. Stresses in four steel test vessel heads have been determined by strain gauges. Stresses in the elastic range have been determined numerically, by two computer programs [7] developed by KENDRICK, MCKEEMAN and PILGRIM, CHEUNG, MARCAL. For instance, data of vessel head No. 32 were: $t/D = 0.015$, $r/D = 0.070$, $R/D = 0.813$. Computed and test stresses were in fair agreement. Tests were continued in the plastic range.

The quoted results show the linear theory of thin shells to suit description of the elastic stress state and to determine the initial plastic deformation in the size range of practical importance. By the time, however, no test results on very thin shells — $t/D \leqq 0.002$ — are available.

Computer program and numerical results

The method of finite differences has been applied, as described by SEPETOSKI et al. [1]. Computations are valid for elastic, linear deformations, hence they are not valid in the plastic range or for large deformations likely to occur in very thin shells.

The selected method is advantageous by being valid for long shells in the range of membrane stresses.

Our ALGOL program in its actual form suits to design axisymmetric vessels under pressure with no break in the meridian curve and consisting

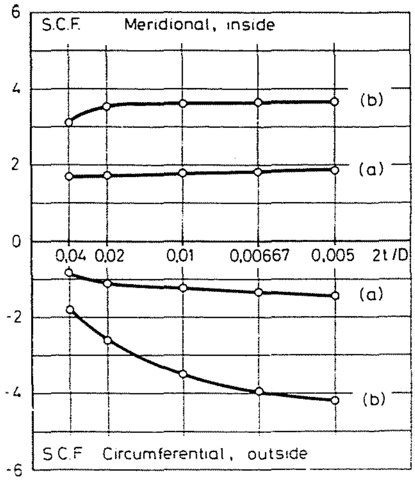

Fig. 2. Stress concentration factors

of parts of constant meridional curvature. Thus, they can be composed of cylinder, cone, torus, sphere, ring plate. Even linear thickness variation can be handled. In spite of several thousands steps likely to be involved in the computer program, running time on an ODRA-1204 computer is a few minutes. The required operative storage capacity is 16 K words — (1 word = 24 bit) — in addition a magnetic drum storage capacity of some 32 K words is needed.

The program has been checked from several aspects, both for the listed elementary shells, compared with classic solutions and literary data, and for torispherical shells. Shells tested by GALLETLY [3], MARCAL and PILGRIM [5] and by FINDLEY et al. [7] have been computed, at a slight difference in each three cases.

Size range of vessel heads according to the Hungarian standard:

a) High-rise heads:

$$\frac{h}{D} = 0{,}25, \quad \frac{R}{D} = 0{,}9, \quad \frac{r}{D} = 0{,}175\,.$$

b) Shallow heads:

$$\frac{h}{D} = 0{,}18, \quad \frac{R}{D} = 1{,}0, \quad \frac{r}{D} = 0{,}08\,.$$

The wall thickness to diameter ratio ranges from $t/D = 0.04$ to 0.0028 for the current bottoms. For both types a) and b), computer analysis concerned five wall thicknesses:

$$t/D = 0.02,\ 0.01,\ 0.005,\ 0.00333,\ 0.0025\,.$$

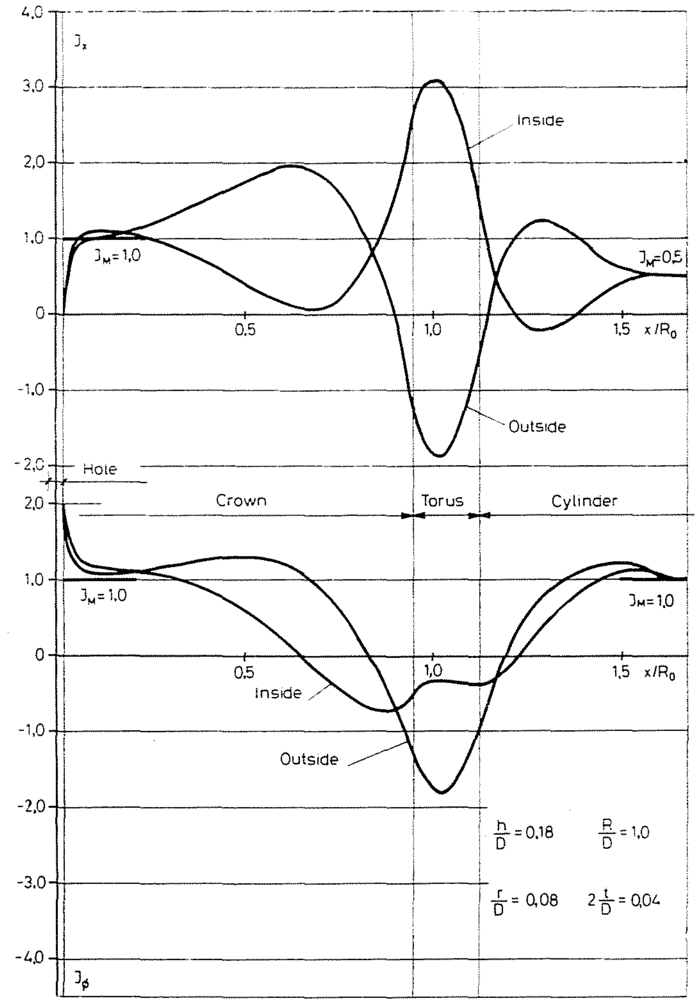

Fig. 3

Stress curves of two heads each for both types are shown in Figs 3 through 6. Fig. 2 shows the maximum stress concentration factors as a function of wall thickness. Stress concentration factors refer to the circumferential membrane stress in the cylinder, i.e.

$$\text{S. C. F.} = \frac{\text{maximum stress}}{pD/2\,t}.$$

Stresses have been computed assuming a small, unreinforced opening in the crown centre. Figs 3 and 4 show also the stresses around the hole, to be about twice the membrane stress in the crown.

The maximum meridional stress is seen in the figures to develop about mid-knuckle and to be a tension. It is interesting to see this extreme value to decompose in head a) with the greater knuckle radius into two independent maxima at crown to torus and at torus to cylinder junctions (Fig. 6), the latter being, however, less. Hence, a considerable damping is likely to develop in the torus of the thin shell. Increase of damping is apparent from the narrowing zone of secondary stresses for lesser wall thicknesses, and from the membrane stresses prevailing in a great part of the crown. There is a third local maximum meridionally, on the outer crown surface, again a tension.

Circumferential stress maxima occur on the outer surface, in the knuckle. Resultant stresses are compressions, and mostly secondary membrane, rather

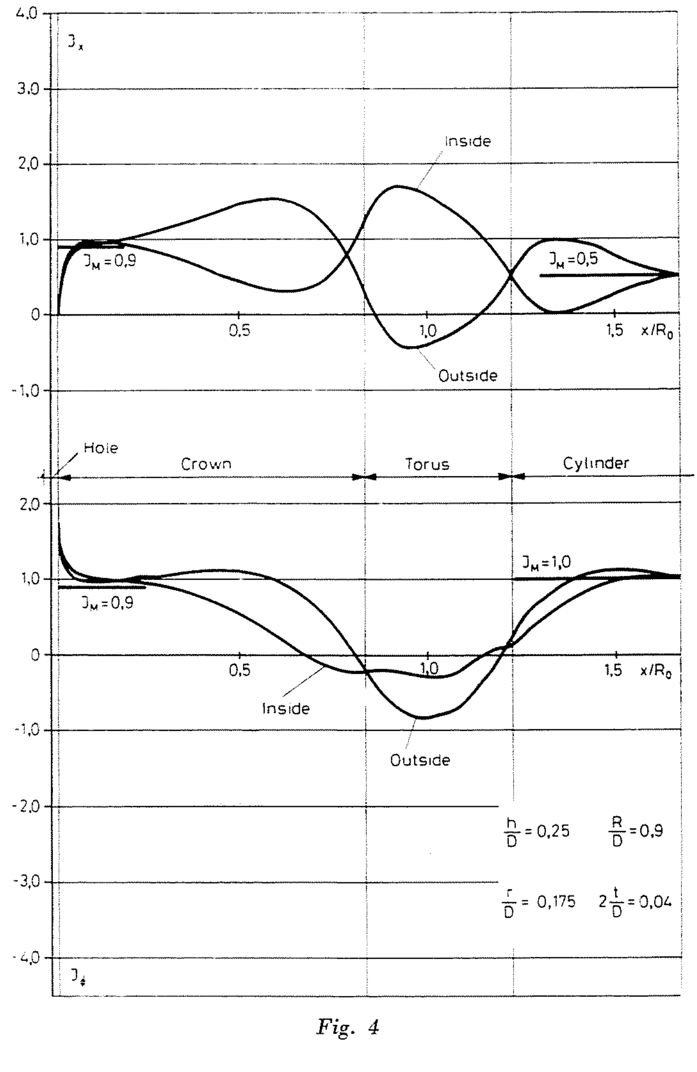

Fig. 4

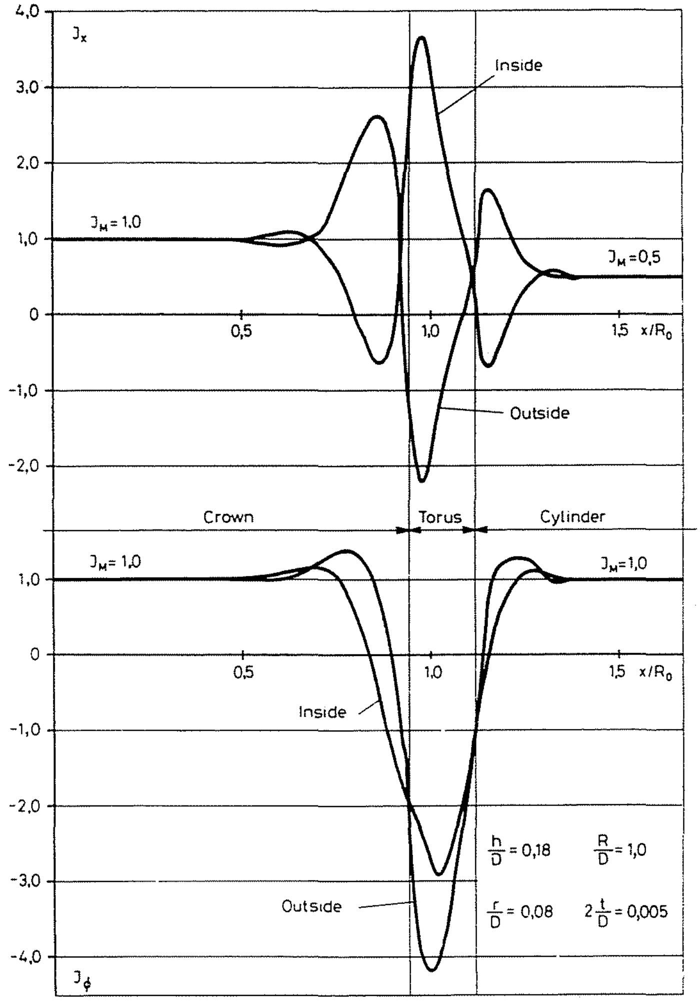

Fig. 5

than bending, stresses. In very thin shallow vessel heads type b) with $2\,t/D <$ < 0.01 the absolute value of this stress exceeds even the meridional maximum (Fig. 2).

Meridional maximum stress concentration is seen to little depend on wall thickness, but the circumferential stress concentration is steeply ascending with decreasing wall thickness—especially for the shallower head type b).

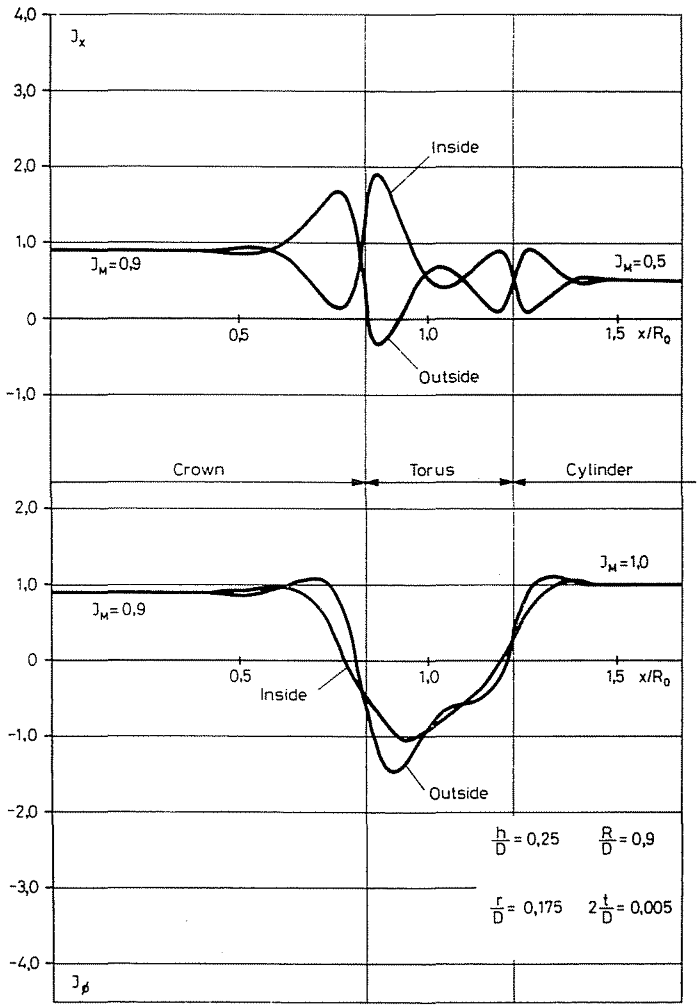

Fig. 6

Conclusions

Standard design shape factors and stress concentration factors have been compiled in Table 1.

The structurally needed vessel head thickness specified in the standard:

$$t' = \frac{pD}{4\,\dfrac{\sigma_F}{z}\,v}\,y.$$

The shape factor value being referred to the meridional, rather than on the circumferential, stress in the standard, for the sake of comparison, a division by 2 has been applied in column 2 of Table I. The shape factor value is seen to be much lower than the stress concentration factor, since, however, most of the latter are local stresses and less dangerous than membrane stresses, the comparison is more realistic by examining the first-yield point.

Table I

Type	Shape factor y	Shape factor $1.5y$	S.C.F. Min.	S.C.F. Max.	S.C.F. Hole
1	2	3	4	5	6
(a)	2.0/2 = 1.0	1.5	1.62	1.85	1.8
(b)	3.3/2 = 1.65	2.475	3.1	(–) 4.2	2.0

In determining the permissible stress, a safety factor of 1.5 being specified for the yield point, column 3 contains shape factor values multiplied by 1.5. From columns 3, 4 and 5 a local plastic deformation is seen to develop in heads designed according to the standard. Even the negative value in column 5 warns of loss of stability, referred to also in the literature [6].

Summary

Based on the linear theory of thin shells, an ALGOL program has been developed, applying the method of finite differences, likely to suit determination of elastic stresses in pressure vessels of axisymmetric shell surface. Calculated stress curves are presented for two torispherical vessel head types of different wall thicknesses specified by Hungarian Standards:

(a) $h/D = 0.25, \quad R/D = 0.9, \quad r/D = 0.175,$

(b) $h/D = 0.18, \quad R/D = 1.0, \quad r/D = 0.08.$

Notations

(a), (b)		bottom types
D	[cm]	cylinder diameter
h	[cm]	head height
I_x		meridional stress intensity
$I\varphi$		circumferential stress intensity
I_M		membrane stress intensity
p	[kp/cm²]	pressure
R	[cm]	crown radius
$R_0 = \frac{D}{2}$	[cm]	cylinder radius
r	[cm]	knuckle radius
S.C.F.		stress concentration factor
t	[cm]	vessel head thickness
v		weld efficiency factor

x	[cm]	arc length from the crown centre
y		shape factor
z		yield point safety factor
σ_F	[kp/cm^2]	yield point

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Dr. Pál REUSS, H-1521 Budapest