CALCULATION OF BOBBIN-COP BODY LOADS

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In wire-works the wire coming down from the drawbench will be rolled up on a bobbin. The wire is transported or exposed to further heat treatment in such a rolled-up state.

When bobbins are opened the wire is often seen to be torn. Factors producing tear include obviously also the inner system of forces the coil body. The increase of forces at rolling up may result in imminent tear.

A more accurate strength design of bobbins regniries the knowledge of loads imposed by the wire.

The tested bobbin-cop consists of three parts, i.e. a cylinder and two annular end it–further on bobbin roll and sides.

Our investigations are restricted to thin, linearly elastic and perfectly flexible wire of circular cross-section. Let us suppose adjacent turns not to slide neither on each other, not on the bobbin-cop body. The contacting surfaces are perfectly smooth.

Load on a bobbin roll

Let us suppose that the radial displacement of the roll along the circumference is constant.

The tension developing in the elastic closed loop stretched to the roll is

$$F = AE\frac{l - l_0}{l_0}, \tag{1}$$

where $l = 2(R - u)\pi$.

During coil winding the wire is stretched by force F_0, in case of $u = 0$, $F = F_0$. From this condition l_0 can be determined. Since $1 \gg \frac{u}{R}$, the tension in the loop is

$$F(u) = F_0 - u\frac{AE}{R}.$$

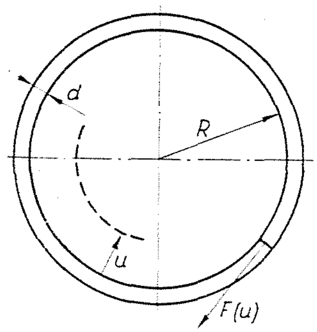

Fig. 1. Stretching force developing in the loop

This tension is balanced by the system of forces of intensity

$$f = \frac{F}{R}$$

distributed along the circle, for which an equivalent system of forces of intensity p, distributed on a roll shell of length d and radius R can be substituted

$$p(u) = \frac{F_0}{R} - u\frac{AE}{R^2 d}. \tag{2}$$

Eq. (2) is the load imposed by the first coil turn line, producing u_1. For the load, resulting from the second coil turn-line, the displacement u_1 will be taken into consideration just as the winding on an already increased radius $R + d$. The jacket of the bobbin roll is affected by the sum of the loads due to both coil turn lines.

$$p(u) = \frac{F_0}{Rd} - u\frac{AE}{R^2 d} + \frac{F_0}{(R+d)\,d} - (u - u_1)\frac{AE}{(R+d)^2 d}\,.$$

This load results in a displacement u_2. Continuing the train of thought, the load after winding up n coil turn lines amounts to

$$p(u) = \frac{F_0}{d}\sum_{i=0}^{n-1}\frac{1}{R+id} - u\frac{AE}{d}\sum_{i=0}^{n-1}\frac{1}{(R+id)^2} + \frac{AE}{d}\sum_{i=0}^{n-1}\frac{u_i}{(R+id)^2}, \tag{3}$$

$$u_0 = 0\,.$$

For wires of small diameter d this relationship simplifies into

$$\sum_{i=0}^{n-1} \frac{1}{R+id} = \frac{1}{d} \sum_{i=0}^{n-1} \frac{d}{R+id} .$$

With the notations in Fig. 2,

$id = b$ is the coordinate along the thickness of the bobbin,

$(i+1)d - id = \Delta b$ is the difference of the coordinate b.

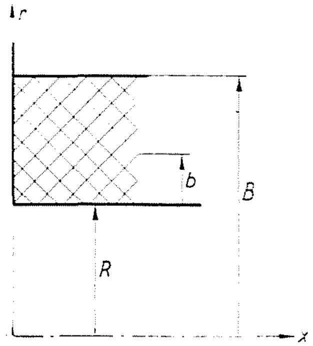

Fig. 2. Dimensions of the bobbin

For d small referred to $B - R$

$$\frac{1}{d} \sum_{i=0}^{n-1} \frac{d}{R+id} \approx \frac{1}{d} \int_{b=0}^{B-R} \frac{1}{R+b} \, db = \frac{1}{d} \ln \frac{B}{R} . \tag{4}$$

Similarly

$$\frac{1}{d} \sum_{i=0}^{n-1} \frac{d}{(R+id)^2} \approx \frac{1}{d} \int_{b=0}^{B-R} \frac{1}{(R+b)^2} \, db = \frac{1}{d} \left(\frac{1}{R} - \frac{1}{B} \right) . \tag{5}$$

Considering these all, the expression (3) will be of the form

$$p(u) = \frac{F_0}{d^2} \ln \frac{B}{R} - u \frac{AE}{d^2} \left(\frac{1}{R} - \frac{1}{B} \right) + \frac{AE}{d} \sum_{i=0}^{n-1} \frac{u_i}{(R+id)^2} , \tag{6}$$

$$u_0 = 0 .$$

To determine the inner system of forces on the jacket the deformation of the bobbin roll has to be iterated n-times according to the relationship (6).

Sometimes a less accurate, simpler expression may be satisfactory.

The fractional displacements u_i satisfy the inequalities

$$u_i \geq 0\,, \quad u_i > u_{i-1} \qquad i = 1, \ldots, n-1\,.$$

In the last term of (6) the displacements u_i can be substituted by the greatest value u_{n-1}. Considering Eq. (5)

$$p(u) < \frac{F_0}{d^2} \ln \frac{B}{R} + (u_{n-1} - u) \frac{AE}{d^2} \left(\frac{1}{R} - \frac{1}{B} \right).$$

Since $u_{n-1} - u \approx 0$, the load on the jacket of the bobbin roll can be determined by the simpler approximate relationship

$$p = \frac{F_0}{d^2} \ln \frac{B}{R}\,. \tag{7}$$

Lateral load

The sides are loaded in the direction of axis x. In determining the axial load the approximation $u = 0$ involves some error, namely the u value is smaller near the roll ends. Also the effect of the axial elongation of the bobbin roll will be omitted.

The superimposed coil turn lines are of opposite pitch. In winding up of the last turn of the coil turn line the wire rises from the roll shell radius r to $r + d$, as imposed by the planes of the last turn and of the side.

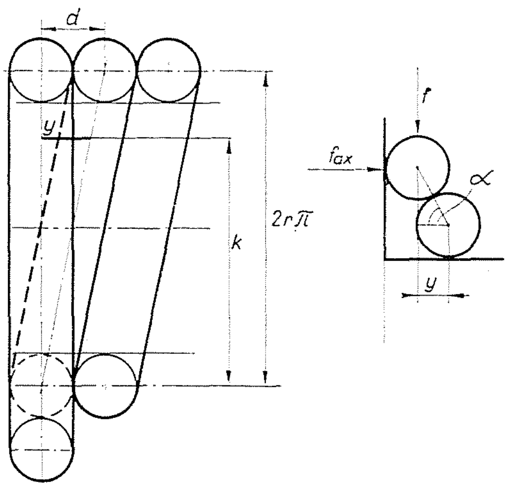

Fig. 3. Lateral load

The system of forces of intensity f_{ax} affecting the side distributed along a spiral — can be calculated from the radial load f. (f is the load due to wire lines over the roll shell of radius r.) According to Fig. 3

$$f_{ax} = \frac{fk}{2\, r\pi \sqrt{1 - \left(\frac{k}{2\, r\pi}\right)^2}} .$$

Since d is small, the spiral can be substituted by the circle of radius r. Let us determine the system of forces of intensity p_{ax} — distributed on the shell with the resultant equal to that of f_{ax}

$$p_{ax} = \frac{1}{2\, r\pi d} \int_{k=0}^{2r\pi} f_{ax}\, dk = \frac{f}{d} .$$

With the above consideration from Eq. (2)

$$p_{ax} = [p\,(u,\, B) - p\,(u,\, r)]_{u=0} .$$

Substituting (6)

$$p_{ax} = \frac{F_0}{d^2} \ln \frac{B}{r} . \qquad (8)$$

Summary

The paper deals with a possible method to determine the system of forces arising between the coil body — which consists of thin, circular wire — and the surface of bobbin will be presented.

Loads are calculated from the geometry and the deformations.

The calculation is simplified by applying approximations giving, however, greater values than the true ones.

The knowledge of the load permits the strength design of the bobbin — cop body and the analysis of the factors of imminent tear.

Symbols

E	[kp/cm²]	modulus of elasticity of the material of wire
d	[cm]	diameter of wire
A	[cm²]	cross-sectional area of the wire
F_0	[kp]	stretching force
x, r, b, k	[cm]	coordinates
$2R$	[cm]	diameter of bobbin roll
$2B$	[cm]	diameter of coil body
p	[kp/cm²]	pressure on the surface of bobbin roll
p_{ax}	[kp/cm²]	lateral pressure on the bobbin-cop body
u	[cm]	radial displacement of bobbin roll
n		number of coil turns

References

KOZÁK, I.: Szilárdságtan. Tankönyvkiadó Budapest, 1967.
NEAL, B. G.: Behaviour of filament yarn wound on a flanged cylinder. Strain Analysis. London, 1967. III.
VICENTINI: Calculation of the elastic deformation of textile beams during yarn winding. Strain Analysis. London, 1969. I.

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