DESIGN OF BRIDGE STRUCTURES COMPRISING HINGED MAIN GIRDERS

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1. Introduction

1.1 *Purpose of the paper*

The rapid evolution in road traffic imperatively calls for the development of an up-to-date highway network requiring a great number of engineering structures.

Owing to the modernization of bridge construction the prefabrication gets ever greater significance. The great number of short-span bridges requires mass prefabrication on the basis of standard designs. In this respect several problems (manufacturing, transportation, field assembly) have to be dealt with, in close connection to each other.

Several methods of assembling main girders in site are known [1]. Connecting methods following the commonly used, traditional design made more difficult the fabrication of main girders and required much field work. To eliminate these inconveniences, in the Soviet Union [2], in the United States of America [3], in Czechoslovakia [4] and in the German Democratic Republic [5] a system of connecting the main girder units has been adopted where the connecting elements are not exposed to bending moment. Significant advantages of the bridge structures assembled in this way are that the very same girders may be applied for bridges of whatever skew arrangement in plan little and only simple field work is required, and in applying main girders or appropriate torsional stiffness, the system ensures an advantageous load distribution. A general design method for such structures also applicable to skew bridges is not referred to in the literature to the author's knowledge.

Structural designs of the bridges built so far are not uniform, thus, it is impossible to work out a method of design holding true for all forms. In this paper, by making allowance for an easy to handle, mathematical solution, the existing structural designs have been kept in view.

In the following, the design method of skew bridge structures comprising hinged main girders will be presented.

The field of application of the mathematical procedure chosen for the solution of the problem covers a wide range of engineering calculations. This is why first the general method is reported, and only later the solution to the problem.

To the belief of the author, this calculation procedure is likely to ease the solution of all problems related to the strength of materials, the theory of stability and the vibrations at the desired accuracy with the help of a digital computer of low capacity, even those of surface structures hither to escaping analysis.

Notations

In this paper the following symbols are used which, in the text, are not explained or their meaning cannot distincly be understood from the figures.

E modulus of elasticity;
G shear modulus;
I_c torsional rigidity;
I_x, I_y flexural moments of inertia about axes x and y, respectively;
I_t warping rigidity.

2. Method of analysis of surface structures with the help of integral equations

For treating the problems mathematically, the results of the theory of integral equations have been utilized [6] [7]. Application of integral equations has several advantages: the possibility of numerically solving the problem may be predicted, the boundary conditions may easily be satisfied, and procedures of identical structure may be developed for solutions to different problems. Their disadvantages which, in the author's opinion, prevented their general practical acceptance, can be eliminated by using a computer of even low capacity: size of the set of equations to be solved is reduced, convergent series of functions may easily be computed at the desired accuracy.

2.1 *Basic principle of the procedure*

The surface to be analysed, which can be either a slab, a disc or a shell, is cut up by vertical planes into subregions (Fig. 1). The analysis is carried out in respect of these subregions. The unknown internal forces acting on the cross section are applied as external forces on each of the subregions, at their edges. The incidental intermediate constraints also are substituted by external forces.

In the case of continuously distributed internal forces, the force functions, i.e., the moment functions, and in the case of a concentrated force its value are considered as unknown (Fig. 2a). The displacement resulting from all these forces (the given external load and unknown junction forces) acting on a subregion may be established. At the locations of the cut-through connections as many deformation conditions may be established as there are unknown forces. The relationship between deflections and forces may be deduced with the help of the influence functions for all the missing connections. The conditions of connections may be expressed by the integral equations mentioned above.

The displacement influence functions may be established by virtue of the existence theorem of the Green's functions [6] and these will be the kernels of the integral equations. These influence relationships need only be known for a few simple basic cases; with their help also complex problems may be solved. By solving the set of integral equations the desired result may be obtained from the conditions of connection at the missing junctions.

Thus, for example, in analysing the slab structure in Fig. 1, according to the theory of strength of materials, not to be detailed here, it is sufficient to apply the usual solution of the simply supported slab shown in Fig. 2b; the result may be found at the desired accuracy by solving a set of equations comprising 11 unknown values.

On the basis of the results of the theory of the set of integral equations, in respect of the procedure, the following statements may be made.

The kernel of the integral equations is real, degenerated, symmetric, and will be positive definite owing to their physical content for any possible form of distribution of the force function.

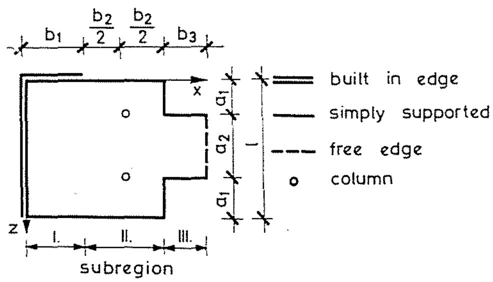

Fig. 1

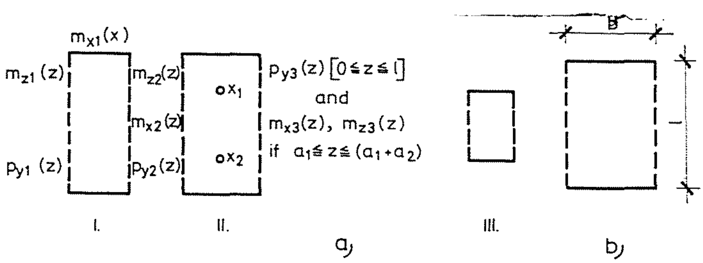

Fig. 2

Due to these properties of the kernel, by virtue of Fredholm's theorem of alternatives [6] [7], the set of equations has a definite unique solution, provided the parameter of the equation is not an eigenvalue and the function of all external forces is not identically equal to zero. The integral equation has at least one eigenvalue, this and the further ones are real values, and the associated eigenfunctions are orthogonal. In accordance with the Hilbert-Schmidt theorem, the solution may be expanded into series with respect to the eigenfunction which results in an absolute and monotonously convergent function.

In accordance with the foregoing it can be stated that there exist numerical solutions to the problems exposed to be achieved in the way as follows.

In the analyses according to the theory of strength of materials, the eigenfunctions of the various integral equations should be defined, and by replacing the result in the form of series according to the eigenfunctions into the set of integral equations, the yet unknown coefficients may be computed from a set of linear equations by identifying the functions established on the basis of the condition of osculation, thus, yielding the unknown junction forces.

In analyses according to the theory of stability and vibration, the set of integral equations may be reduced to a single integral equation, and the eigenvalues of this equation deliver the values of critical force and natural frequency. It should be noted that for problems of the theory of vibration, after multiplying by the mass distribution function, the kernel may be symmetrized.

2.2 *Advantages of the procedure*

The advantages described in the preceeding paragraph may be summed up as follows.

Surfaces of any form and with any support may be assembled from a few types of "elementary regions". Thus, the actual investigations must affect identical predetermined elementary types, thereby the way of solution is predetermined as well.

The various problems of the theory of strength of materials, stability and vibration may be treated according to the same concept and in the same form.

The originally two-dimensional problem has been reduced to a linear one.

The boundary conditions may readily be followed by making use of the elementary regions.

The possible interior and exterior supports do not necessitate the establishment of new kernels.

In the solution any desired accuracy may be achieved, independently of the number of elementary regions, except inclined edges.

A small set of linear equations is needed to the solution to the problem of an advantageous structure and this easy to solve.

The convergent series may quickly be computed with the help of a digital computer, and the accuracy requirements may be given in advance.

No large storage capacity is required from a computer, and several problems may even be solved without a computer, due to the advantageous structure of the small set of linear equations.

3. Analysis of hinged bridge structures

3.1 *Basic Assumptions*

With respect to the static model serving for basis for the computations in deriving the procedure, the following assumptions have been made:

1. The structure consists of simple beams of constant, simply symmetrical, but not necessarily identical, cross sections.
2. The beams are connected by ideal in-plane hinges, their axes being normal to the plane of the cross sections.
3. The hinges continuously connect the main girders throughout their length.
4. The beams are only subjected to forces causing elastic deflections.
5. Deflections due to shearing force may be neglected in comparison with those due to the bending moment.
6. The restraint of the ends of beams against torsion does not prevent the beam ends from bending rotation.

3.2 *Setting of the problem*

The calculation procedure has been worked out for a skew bridge structure of obliquity α (Fig. 3).

The investigation has been carried out in respect to the primary system obtained by omitting the hinges. Unknowns are the uniformly distributed vertical and horizontal forces $p(z)$ and $h(z)$, respectively, as well as the vertical and horizontal displacement functions of the hinge axis $y(x)$ and $u(z)$, respectively. The unknown forces are marked with a subscript consisting of two characters, the first of which designates the serial number of the hinge, and the second, that of the beam, that is, the reference system. If the omission of the second character in the subscript referring to the beam, does not involve misunderstanding, it may be omitted. Distribution of the hinge forces is assumed over length L, according to Fig. 3.

Let us denote the Green's function associated with the differential equation of the bent beam in the case of vertical bending by $G_y(z, \zeta)$; in the case of horizontal bending by $G_x(z, \zeta)$; and that belonging to the differential equation of torsion, by $G_\varphi(z, \zeta)$. This last one was established for the dimensionless unit torsional moment.

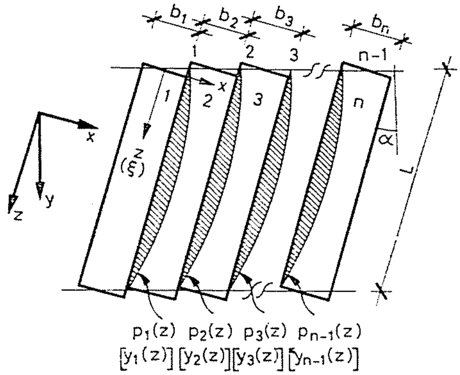

Fig. 3

Under the effect of forces acting in the positive direction, as shown in Fig. 4, the vertical and horizontal displacements of the i-th row of hinges may be established with the help of Green's functions. It may be pointed out that along the axis of the row of hinges only the forces acting on the two adjacent beams cause relative displacements. Shift of the axis of the row of hinges may be written in the form of integral equations with the help of displacement influence functions derived from the Green's functions. After reduction of these equations and introduction of the notation as follows

$$\begin{aligned} K_{1i}(z, \zeta) &= G_{yi}(z, \zeta) + \frac{b^2}{4} G_{\varphi i}(z, \zeta) \\ K_{2i}(z, \zeta) &= G_{yi}(z, \zeta) - \frac{b^2}{4} G_{\varphi i}(z, \zeta) \\ K_{3i}(z, \zeta) &= G_{xi}(z, \zeta) + t^2 G_{\varphi i}(z, \zeta) \\ K_{4i}(z, \zeta) &= \frac{bt}{2} G_{\varphi i}(z, \zeta) \end{aligned} \tag{1}$$

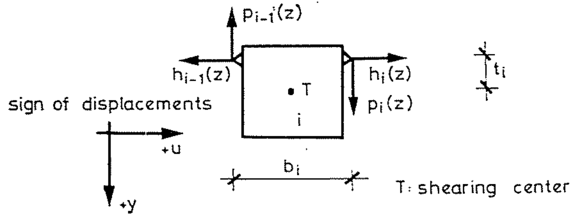

Fig. 4

the function of the vertical displacement at the left-hand side beam becomes

$$y_{ii}(z) = \int_0^L K_{1i}(z,\zeta)\, p_{i,i}(\zeta)\, d\zeta - \int_0^L K_{2i}(z,\zeta)\, p_{i-1,i}(\zeta)\, d\zeta + \\ + \int_0^L K_{4i}(z,\zeta)\,[h_{i,i}(\zeta) - h_{i-1,i}(\zeta)]\, d\zeta + y_{0i,i}(z) \tag{2}$$

where $y_0(z)$ is the displacement function due to the external load. The function of the horizontal displacement at the left-hand side beam will be

$$u_{ii}(z) = \int_0^L K_{4i}(z,\zeta)\,[p_{i,i}(\zeta) + p_{i-1,i}(\zeta)]\, d\zeta + \\ + \int_0^L K_{3i}(z.\zeta)\,[h_{i,i}(\zeta) - h_{i-1,i}(\zeta)]\, d\zeta + u_{0i,i}(z) \tag{3}$$

where $u_0(z)$ is the displacement function due to the external load.

Functions (2) and (3) may also be written as calculated for the right-hand-side beam

$$y_{i,i+1}(z) = - \int_0^L K_{1i+1}(z,\zeta)\, p_{i,i+1}(\zeta)\, d\zeta + \\ + \int_0^L K_{2i+1}(z,\zeta)\, p_{i+1,i+1}(\zeta)\, d\zeta + \\ + \int_0^L K_{4i+1}(z,\zeta)\,[h_{i,i+1}(\zeta) - h_{i+1,i+1}(\zeta)]\, d\zeta + \\ + y_{0i,i+1}(z) \tag{4}$$

and

$$u_{i,i+1}(z) = \int_0^L K_{4i+1}(z,\zeta)\,[p_{i,i+1}(\zeta) + p_{i+1,i+1}(\zeta)]\, d\zeta + \\ + \int_0^L K_{3i+1}(z,\zeta)\,[h_{i+1,i+1}(\zeta) - h_{i,i+1}(\zeta)]\, d\zeta + \\ + u_{0i,i+1}(z) \tag{5}$$

By writing down the above equations for the functions $y_{i,j}(z)$ and $u_{i,j}(z)$ where $i = 1, 2, \ldots, n-1$ and $j = 1, 2, \ldots, n$; in the range $0 \leq j-i \leq 1$, and taking into consideration that $p_0(z) = h_0(z) = p_n(z) = h_n(z) = 0$, the set of integral equations expressing the equilibrium and compatibility of the structure is obtained. Solving this set of equations yields functions $p_i(z)$ and $h_i(z)$.

On the basis of what has been said in paragraph 2, solution of the integral equations by the convergent series of the eigenfunctions may be obtained for any form of the kernel. In order to speed up the convergence of the result functions (1) have to be expanded into Fourier's series.

The Green's functions of bending are given by the trigonometric series of the deflection diagram constructed for the case of a load $P = 1$ acting at ζ

$$G_y(z, \zeta) = \sum_{m=1}^{m=\infty} \frac{2L^3}{m^4\pi^4 EJ_x} \sin\frac{m\pi}{L} z \sin\frac{m\pi}{L}\zeta. \tag{6}$$

The function $G_x(z)$, only differs from (6) in the value of the flexural rigidity: EJ_y in lieu of EJ_x.

In analysing the torsion one distinguishes pure and warping torsion [9]. The differential equations relating to the warping torsion of beams of closed cross-section are obtained in the very same form as that of the beams of open cross-section [10]. The pure torsion may be applied as an approximation, if the following inequality holds [10]:

$$J_t \ll \frac{G}{E}\left(\frac{L}{\pi}\right)^2 \cdot J_c. \tag{7}$$

Denoting the flexural rigidity by T_c:

$$T_c = GJ_c \quad \text{or} \quad T_c = \frac{EJ_t(m^2\pi^2 + k^2L^2)}{L^2}, \tag{7a}$$

where

$$k^2 = \frac{GJ_c}{EJ_t}. \tag{7b}$$

Inequality (7) being satisfied, from the two values the former one should be chosen. This, in the further discussion both the warping and the pure cases of torsion are taken into account:

$$G_\varphi(z, \zeta) = \sum_{m=1}^{m=\infty} \frac{2L}{m^2\pi^2 T_c} \sin\frac{m\pi}{L} z \sin\frac{m\pi}{L}\zeta. \tag{8}$$

Being aware of the Green's functions, on the basis of (1) the kernels are given.

3.3 *Solution to the problem*

In order to solve the set of integral equations, first, a solution to one equation should be found. The eigenvalues are simple to find, due to the orthogonal functions of the degenerated kernel. Their values have, in the analysis of the bridge structure, no more significance than their utilization for the determination of the minimum torsional stiffness where the participation of all of the main girders of the bridge in the development of the stress pattern is ensured. The value may be calculated from the relationship

$$T_c = EJ_x \left(\frac{b\pi}{2L} \right)^2 . \tag{9}$$

If the value of the torsional stiffness of each of the main girders is the same as given by (9) then, only the loaded beam and the adjacent two beams are engaged in the development of the stress distribution. In order to ensure the participation in load bearing of all of the girders, the value of the stiffness against torsion must be higher than that calculated from (9). The eigenfunctions will be obtained in the form

$$\varphi_m(z) = \sin \frac{m\pi}{L} z . \tag{9a}$$

Thus, the solution is given by the functions

$$p_i(z) = \sum_{m=1}^{m=\infty} p_i(m) \sin \frac{m\pi}{L} z \tag{10}$$

and

$$h_i(z) = \sum_{m=1}^{m=\infty} h_i(m) \sin \frac{m\pi}{L} z . \tag{11}$$

The still unknown coefficients $h(m)$ and $p_i(m)$ may be found as follows.

The results (10) and (11) are substituted into the set of Eqs (2) to (5) and integrated.

In carrying out the substitution, it has to be taken into consideration that each of the girders defines its own system of coordinates. The unknown forces are always to be sought in the system of coordinates of the beam of the very same serial number. Introducing the notation

$$\begin{aligned}
k_{1i}(m) &= \frac{2L^3}{m^4\pi^4 EJ_{xi}} + \frac{b_i^2 L}{2m^2\pi^2 T_{ci}} \\
k_{2i}(m) &= \frac{2L^3}{m^4\pi^4 EJ_{xi}} - \frac{b_i^2 L}{2m^2\pi^2 T_{ci}} \\
k_{3i}(m) &= \frac{2L^3}{m^4\pi^4 EJ_{yi}} + \frac{2t_i^2 L}{m^2\pi^2 T_{ci}} \\
k_{4i}(m) &= \frac{b_i t_i L}{m^2\pi^2 T_{ci}}
\end{aligned} \tag{11a}$$

further

$$d_i = 0{,}5 \cdot b_i \operatorname{tg} \alpha$$

$$c_i = \frac{b_i + b_{i+1}}{2} \operatorname{tg} \alpha \tag{11b}$$

we can write down the vertical deflection function (2) to the i-th beam

$$y_{i,i}(z) = \frac{L}{2} \sum_{m=1}^{\infty} \Big\{ -k_{2,i}(m)\, p_{i-1}(m) \cos \frac{m\pi}{L} c_{i-1} + k_{1i}(m)\, p_i(m) + k_{4i}(m) \Big[h_i(m) - h_{i-1}(m) \cos \frac{m\pi}{L} c_{i-1} \Big] \Big\} \cdot \sin \frac{m\pi}{L} z + y_{0,i,i}(z) . \tag{12}$$

The function of the horizontal displacement for the i-th beam may be obtained with the help of (3)

$$u_{i,i}(z) = \frac{L}{2} \sum_{m=1}^{\infty} \Big\{ k_{4i}(m) \Big[p_i(m) + p_{i-1}(m) \cos \frac{m\pi}{L} c_{i-1} \Big] + k_{3i}(m) \Big[h_i(m) - h_{i-1}(m) \cos \frac{m\pi}{L} c_{i-1} \Big] \Big\} \sin \frac{m\pi}{L} z + u_{0i,i}(z) . \tag{13}$$

Both of these displacement functions may also be calculated for the $(i + 1)$ th beam from Eq. (4):

$$y_{i,i+1}(z) = \frac{L}{2} \sum_{m=1}^{\infty} \Big\{ - k_{1i+1}(m)\, p_i(m) \cos \frac{m\pi}{L} c_i + k_{2i+1}(m)\, p_{i+1}(m) + k_{4,i+1}(m) \Big[h_i(m) \cos \frac{m\pi}{L} c_i - h_{i+1}(m) \Big] \Big\} \sin \frac{m\pi}{L} z + y_{0,i,i+1}(z) \tag{14}$$

and from Eq. (5)

$$u_{i,i+1}(z) = \frac{L}{2} \sum_{m=1}^{\infty} k_{4,i+1}(m) \Big[p_i(m) \cos \frac{m\pi}{L} c_i + p_{i+1}(m) \Big] + k_{3,i+1}(m) \Big[h_{i+1}(m) - h_i(m) \cos \frac{m\pi}{L} c_i \Big] \Big\} \sin \frac{m\pi}{L} z + u_{0,i,i+1}(z). \tag{15}$$

Carrying out the transformation of coordinates required, the functions at the one side (12) and (14), and at the other one (13) and (14) may be set equal, since for the value of the displacement functions the very same result will be obtained by calculating either from the right-hand or the left-hand side beam. After arranging the obtained equation, it is multiplied by the function $\sin (m\pi/L)z$, and integrated in the range $z = 0$ to L; then the following equation will be obtained for the unknown coefficients where in the left-hand-side the terms of the expressions $y_{0,i,i+1}(m)$ and $y_{0,i,i}(m)$ denote the Fourier's coefficients of the displacement functions calculated from the given load

$$\begin{aligned}
&- k_{2i}(m) \cos\frac{m\pi}{L} c_{i-1} \cos\frac{m\pi}{L} d_i p_{i-1}(m) + \left[k_{1i}(m) \cos\frac{m\pi}{L} d_i + \right.\\
&\left. + k_{1,i+1}(m) \cos\frac{m\pi}{L} c_i \cos\frac{m\pi}{L} d_{i+1}\right] p_i(m) - \\
&- k_{2i+1}(m) \cos\frac{m\pi}{L} c_i p_{i+1}(m) - k_{4i}(m) \cos\frac{m\pi}{L} c_{i-1} \cdot \\
&\cdot \cos\frac{m\pi}{L} d_i h_{i-1}(m) + \left[k_{4i}(m) \cos\frac{m\pi}{L} d_i - k_{4,i+1}(m) \cdot \right.\\
&\left. \cdot \cos\frac{m\pi}{L} c_i \cos\frac{m\pi}{L} d_{i+1}\right] h_i(m) + k_{4,i+1}(m) \cos\frac{m\pi}{L} d_{i+1} \\
&\cdot d_i h_{i+1}(m) = \frac{2}{L}\left[y_{0,i,i+1}(m) \cos\frac{m\pi}{L} d_{i+1} - \right.\\
&\left. - y_{0,i,i}(m) \cos\frac{m\pi}{L} d_i\right].
\end{aligned} \tag{16}$$

From the equality of the functions expressing the horizontal connection of the beams, we have

$$\begin{aligned}
&k_{4i}(m) \cos\frac{m\pi}{L} c_{i-1} \cos\frac{m\pi}{L} d_i p_{i-1}(m) + \left[k_{4,i}(m) \cos\frac{m\pi}{L} d_i + \right.\\
&\left. + k_{4,i+1}(m) \cos\frac{m\pi}{L} c_i \cos\frac{m\pi}{L} d_{i+1}\right] p_i(m) - k_{4,i+1}(m) \cdot \\
&\cdot \cos\frac{m\pi}{L} d_{i+1} p_{i+1}(m) - k_{3i}(m) \cos\frac{m\pi}{L} c_{i-1} \cos\frac{m\pi}{L} d_i \cdot \\
&\cdot h_{i-1}(m) + \left[k_{3i}(m) \cos\frac{m\pi}{L} d_i + k_{3i+1}(m) \cos\frac{m\pi}{L} c_i \cdot \right.\\
&\left. \cdot \cos\frac{m\pi}{L} d_{i+1}\right] h_i(m) - k_{3i+1}(m) \cos\frac{m\pi}{L} d_{i+1} = \\
&= \frac{2}{L}\left[u_{0,i,i+1}(m) \cos\frac{m\pi}{L} d_{i+1} - u_{0i,i}(m) \cos\frac{m\pi}{L} d_i\right]
\end{aligned} \tag{17}$$

wherein the term $u_{0,i,j}(m)$ in the right-hand side of the equation denotes the Fourier's coefficients of the displacement functions associated with the external load.

By applying Eqs (16) and (17) to the cases $i = 1, 2, \ldots, n-1$, the unknown coefficients may be calculated from the set of equations for the values $m = 1, 2, 3, \ldots$, whereby the problem has been solved.

On the basis of investigations performed with respect to main girders rigid to torsion [10], the horizontal hinge forces — for the accuracy needed in practice — may be ignored. Thus, substituting the values $h_i(z) = 0$ ($i = 1, 2, \ldots, n-1$) into the results obtained so far, and omitting Eq. (17) expressing the horizontal connection, significant simplification may be achieved.

The obtained [10] result analysing the rapidity of convergence of the series of trigonometric functions, was the function of beam geometry. In the case of bridge structures of box-section main girders where the horizontal rigidity of the beams is significantly higher than the minimum value given by the relationship (9) determined from the eigenvalues of the integral equations, the convergence is very rapid. In investigating the deflections of the different beams, even if a single term is taken into consideration, the error, due to the neglect of the other terms, is lower than 2 to 4 per cent. Therefore the set of Eqs (16) yielding the final result of the problem, needs only be solved for the minor values of m which — considering the structure of the set of equations and negligibility of the horizontal hinge force — does not give hard work, nor requires the use of a computer. By making use of the approximate procedure to be described in the next paragraph, even this calculation work may be reduced.

The sets of Eqs (16) and (17), deliver the hinge forces and hereby, the problem is solved. In knowledge of the hinge forces, stress and deflection data may be evaluated.

3.4 *Calculation of the coefficients of trigonometric series by the iterative method*

In applying the procedure described in Chapter 2, the calculation of the coefficients of series according to the eigenfunctions is rather frequently needed for various values of m. This necessitates repeated solution to the sets of equations similar to that given by (16) and (17).

In the following, a relationship will be deduced for the determination series of the coefficients of trigonometric by the iterative method. This procedure yields exact results if

a) all terms of the equation defining the unknown coefficients of the harmonic series converge at the same rapidity;

b) the principle of superposition is valid.

Let us write the equation to be solved in the form

$$\mathbf{A}(m) \cdot x(m) = y(m) \tag{18}$$

where $y(m)$ is a known vector and $\mathbf{A}(m)$ a coefficient matrix complying with the conditions a) and b). The vector $x(m)$ has to be found. In the case of an equation satisfying the above two conditions, the coefficients may be determined by iteration decomposed into two factors 10 or even, without decomposition.

The vector $y(m)$ comprises the Fourier's coefficients of the displacement functions induced by the given external load, therefore the structure of coefficients which might be taken into account in the solution, has been considered.

On the basis of condition b) it is sufficient to find a solution to $y(m)$ to meet the following requirements: the elements of the known vector should be decomposed into two factors depending on m; the first of them will be constant and the second a trigonometric function. Two vectors may be formed from these elements whose logical multiplication gives the original vector. In the elements of the second vector only the identical trigonometric functions of identical quantities are involved, and in respect to the first vector, the following relation is true

$$y_1(m+1) = \beta(m) y_1(m) \tag{a}$$

wherein $\beta(m)$ is a scalar number.

On the basis of condition a) it may be written

$$\mathbf{A}(m+1) = c(m) \cdot \mathbf{A}(m) \tag{b}$$

wherein $c(m)$ is a scalar number.

With these notations, making use of the known relationships relating to the product and sum of the trigonometric functions — if the solution to the values $m = k$ and $m < k$ is known, then the following relationship is obtained for the value of the vector $x(k+1)$:

$$x(k+1) = \frac{2\beta(k)}{c(k)} \cos \frac{\pi}{L} a_0 \cdot x(k) - \frac{\beta(k) \cdot \beta(k-1)}{c(k)\, c(k-1)} x(k-1) \tag{16}$$

wherein a_0 is a constant depending on the type, distribution and location of the load.

From this relation either the total value of the vector x or an arbitrary number of its elements may be calculated to any value of m.

In the case of the bridge structure analysed, relationship (19) may be applied as an approximation, provided the torsional stiffness of the main girder is significantly greater than the value from Eq. (9). In this instance, no iterations are needed to the set of equations of favourable structure for achieving the desired accuracy.

Summary

Elaborateness of the analysis of bridge structures with hinged main girders called for a computerizable — though not necessarily computerized — method.

The possible applications and advantages of the set of integral equations in analysing surface structures are related to problems of the theory of strength of materials, stability and vibration. There is a variety of problems lending themselves to identical computer methods of solution at the desired accuracy.

In analysing a skew bridge structure comprising hinged main girders — to achieve practical accuracy — only a set of equations, simple in form, has to be solved, of a structure eliminating the need for a computer.

In a subsequent paper, the calculation of a bridge structure consisting of beams of usual form will be dealt with, together with the analysis of the given problem on the basis of another model.

References

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