SOME PROBLEMS OF THE ACCURACY SYNTHESIS FOR MEASURING INSTRUMENT MECHANISMS

By

Z. KAPOSVÁRI

Department of Precision Mechanics and Optics, Technical University Budapest

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The accuracy of measuring instrument mechanisms can be improved, on the one hand, by reducing production technological deficiencies, by improving operational and functional circumstances, on the other hand, by reducing the errors involved in the so-called theoretical mechanisms which are free of production and operational deficiencies. Since the reduction of production defects requires a more precise production process, possibly an expensive machinery, we have to make efforts to elaborate and to apply methods likely of help in determining the mechanism with the minimum theoretical error.

The scope of the accuracy synthesis for measuring instrument mechanisms (mechanical systems) includes

kinematic accuracy synthesis and

geometric accuracy synthesis.

The kinematic accuracy synthesis is wanted to select the elements and assemblies to be used and to determine their most advantageous arrangement.

The geometric accuracy synthesis has to offer a rational method for determining optimum parameters of the mechanism likely to minimize the error of the theoretical mechanism (to the permitted value).

The problems of geometric accuracy synthesis are mostly solved by using one of the analytical methods of

interpolation,

method of least squares, or

equally "optimum" approximation.

The principle of the *interpolation method* is to approximate the theoretical function $F(x)$ by the polynomial $P(x)$ and to determine optimum parameters from the system of equations written for the equalities of ordinates $F(x) = P(x)$ (Fig. 1).

In the *method of least squares* the sum of squares of deviations from the theoretical function $F(x)$ is minimum (Fig. 2)

$$\Delta_k^2 = \frac{\int\limits_{x_0}^{x_m} [P(x) - F(x)]^2\, dx}{x_m - x_0}.$$

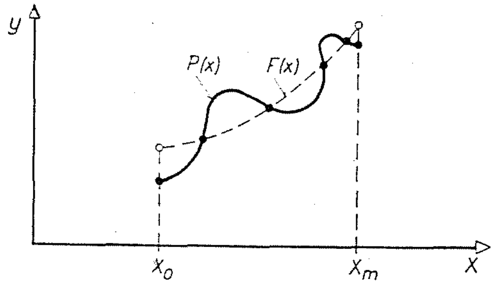

Fig. 1

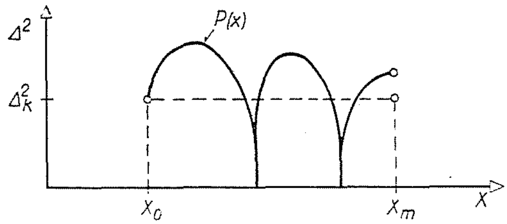

Fig. 2

This condition is satisfied by the Legendre polynomial:

$$P_k(x) = \frac{1}{2^k \cdot k!} \frac{d^k}{dx^k} (x^2 - 1)^k .$$

In accuracy calculations it is usual to employ the not normalized form of Legendre polynomials.

$$P_0(x) = 0$$

$$P_1(x) = x$$

$$P_2(x) = x^2 - \frac{1}{3}$$

$$P_3(x) = x^3 - \frac{3}{5} x$$

$$P_4(x) = x^4 - \frac{6}{7} x^2 + \frac{3}{35}$$

$$P_5(x) = x^5 - \frac{10}{9} x^3 + \frac{5}{21} x \ldots$$

The essential of the method of *equally "optimum" approximation* is that the value of the maximum deviation of the approximation polynomial $P(x)$ from the theoretical (specified) function $F(x)$ is minimum, i.e.:

$$\Delta_{\max} = \max |P(x) - F(x)| = \min.$$

This approximation is called equally close because the deviation of $P(x)$ from $F(x)$ is in the range $\pm E$ (Fig. 3).

This condition is satisfied by the Tchebysheff polynomial:

$$T_n(x) = \cos(n \text{ arc } \cos x).$$

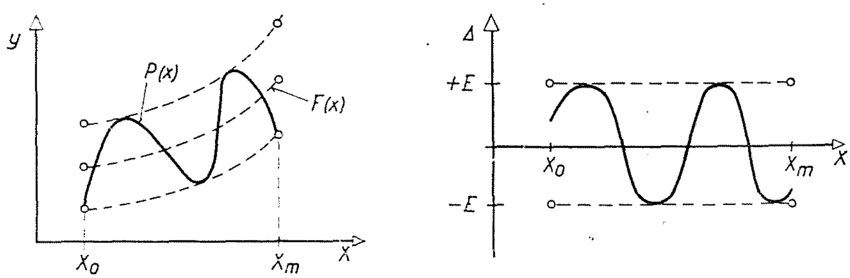

Fig. 3

It is advisable to employ the not normalized Tchebysheff polynomials in accuracy calculations.

$$T_0(x) = 1$$

$$T_1(x) = x$$

$$T_2(x) = x^2 - \frac{1}{2}$$

$$T_3(x) = x^3 - \frac{3}{4}x$$

$$T_4(x) = x^4 - x^2 + \frac{1}{8}$$

$$T_5(x) = x^5 - \frac{5}{4}x^3 + \frac{5}{16}x \ldots$$

The selection between the mentioned analytical methods depends on the problem to be handled.

Careful selection of the interpolation method should be pointed out, else the approximation polynomial $P(x)$ may cause rough deviations between the interpolation points.

In designing the mechanisms of high precision measuring instruments the use of the equally "optimum" approximation method can be recommended since it leads to the minimum of deviation between the approximative and the theoretical polynomial $P(x)$ and $F(x)$, respectively.

In applying the method of equally "optimum" approximation, the algorithm of solving the accuracy synthesis problem is the following.

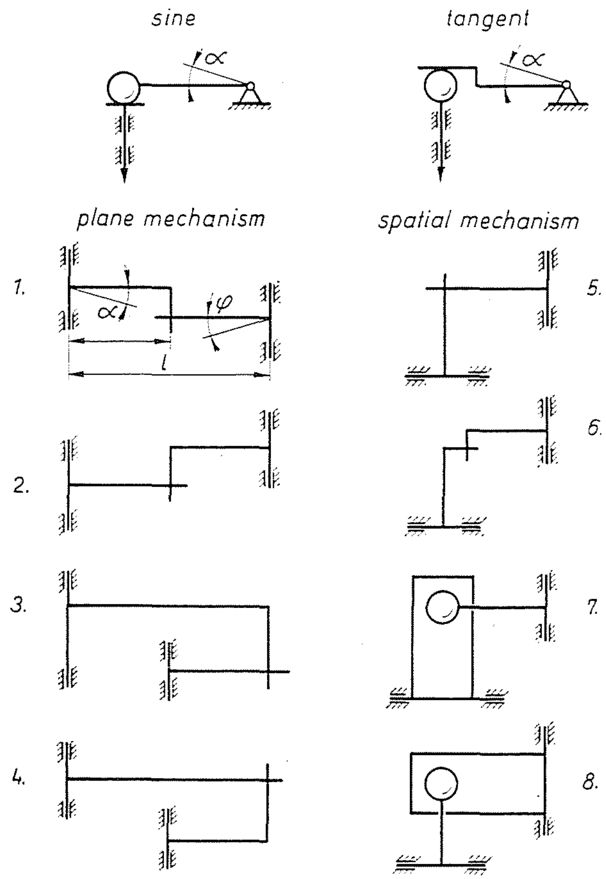

Fig. 4

1. Writing the error function of the tested theoretical mechanism:

$$\Delta(x) = P(x) - F(x).$$

2. Expanding the e rror function in power series up to the power satisfying accuracy requirements (practically the fifth or seventh power).

3. Transformation into the range $(-1, +1)$.

4. Equalizing the coefficients of the variable to the corresponding coefficients of the Tchebysheff polynomials.

5. Determination of optimum parameters (for the specified requirement) from condition.

It should be noted that this algorithm is also valid to the sense, for the Legendre polynomial with the least square deviation.

Let us select the most frequently used four plane and four spatial mechanisms of measuring instruments. The selection shown in Fig. 4 is arbitrary though advisable since these mechanisms are frequently applied e.g. in dial gauge type length measuring instruments [3]. In the design of measuring instrument mechanisms, we have to choose among many arrangement possibilities.

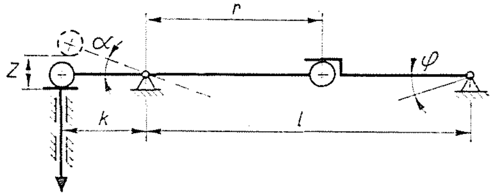

Fig. 5

Inserting a sine or tangent input before the 8 mechanisms shown in Fig. 4 produce 16 mechanisms.

By combining the 8 mechanisms in every possible variant we obtain

$$V_8^{2,i} = 64$$

various mechanisms. Inserting a sine or tangent mechanisms before each 128 arrangements are to be examined.

Obviously the infinity of kinematic variaties obtained by further increasing the number of mechanisms shown in Fig. 4 converting angular displacement to angular displacement would be inhibitive for the selection of the most favourable mechanism.

As an example, let us insert a sine mechanism before the mechanism No. 1 in Fig. 4 (see Fig. 5). The transfer function of the mechanism is

$$\varphi = \operatorname{arctg}\left(\frac{r \cdot z}{k\left(l - r\sqrt{1 - \frac{z^2}{k^2}}\right)}\right). \tag{1}$$

Coupling another mechanism, e.g. that No. 3 behind the mechanism shown in Fig. 5, the transfer function of the resulting mechanism (Fig. 6) is:

$$\beta = \operatorname{arctg}\left[\frac{r_1 \cdot \sin\left(\operatorname{arctg}\left(\frac{r \cdot z}{k\left(l - r\sqrt{1 - \frac{z^2}{k^2}}\right)}\right)\right)}{r_1 \cdot \cos\left(\operatorname{arctg}\left[\frac{r \cdot z}{k\left(l - r\sqrt{1 - \frac{z^2}{k^2}}\right)}\right]\right) - l_1}\right]. \tag{2}$$

Beyond the difficulties arising in testing great many complicated transfer functions, another problem is that of designing for specified error functions.

Measuring instruments may be subject to various accuracy specifications. Let us examine some common error functions.

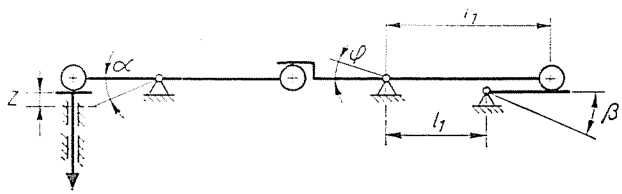

Fig. 6

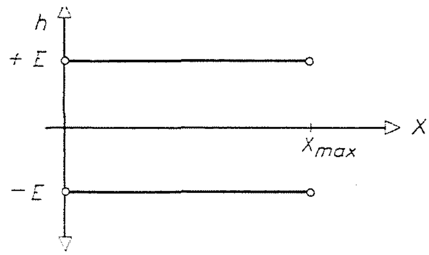

Fig. 7

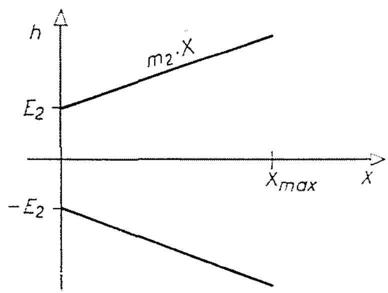

Fig. 8

In the case of pressure and electric meters the class of accuracy of the instrument is specified in percentage error related to full scale deviation, the error equation of the instrument (Fig. 7) being

$$h = \pm E \tag{3}$$

where E is the specified error limit.

Systematic errors of instruments are frequently specified — on account of inaccuracies in the mechanism, as a function of deviation (Fig. 8):

$$h = \pm m_1 \cdot x \tag{4}$$

where x is the input signal (e.g. displacement, angular displacement, etc.).

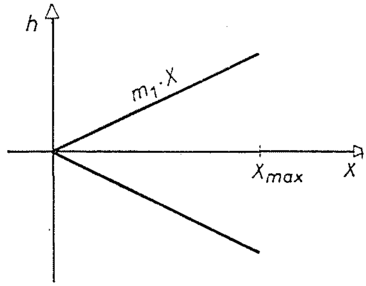

Fig. 9

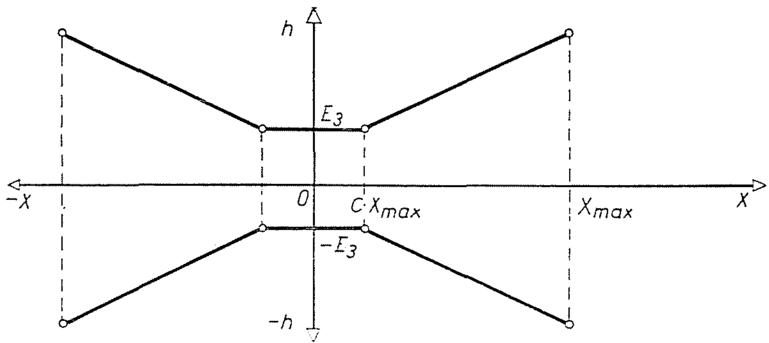

Fig. 10

If the measuring instrument has a zero error, the error equation is (Fig. 9):

$$h = \pm(E_2 + m_2 \cdot x)\,. \tag{5}$$

In the case of certain force and torque measuring instruments the error equation is given in the form (Fig. 10):

$$h = \pm \begin{cases} E_3 \text{ for } x \leqq c \cdot x_{\max} \\ m_3 \cdot x \text{ for } c \cdot x_{\max} < x \leqq x_{\max}\,. \end{cases}$$

This means that up to e.g. $c = 1/5$ of the upper limit x_{max} of the measuring range the error is E_3, constant above this limit the error is proportional to the deviation.

If the measuring range is symmetrical about the zero point the error functions are of the character shown in Fig. 10.

Increasing requirements for measuring instruments may involve error functions differing from the above.

On the basis of the aforesaid the described analytical methods of accuracy synthesis may involve the following difficulties:

a) Great many relatively complicated mechanisms and functions are possible. In our case the mechanism best suited to specifications should be selected from 16 mechanisms characterized by equations type (1) and from 128 mechanisms with equations type (2). This number may further increase, of course, by widening the range of mechanisms shown in Fig. 4.

b) Functions type (1) or (2) should be expanded in Taylor series up to the fifth or seventh (in some cases even higher) power of the variables.

c) Tchebysheff polynomials only permit the determination of optimum parameters resulting in minimum error. No determination of optimum parameters for other modifications (of engineering accuracy) is possible.

d) Attempts were made to determine parameters satisfying the proportional error function by using a suitable weighting function with the Tchebysheff polynomials [3].

No deductive calculation method determine optimum parameters satisfying a specified, arbitrary error function, likely to facilitate the work of designers is available, a timely problem.

Thus, the described analytical methods for accuracy synthesis are tedious, time and labour consuming on account of the complexity and high number of functions, inadequate for a rational design for prescribed error functions.

Thus, a method, likely to permit rational determination of optimum parameters so as to satisfy the specified error functions, has to be elaborated.

Our measuring instruments are required to have a possibly wide measuring range, involving a possibly minimum measurement error (specified value). In most practical cases these two conditions result in contradictory design requirements.

Hence in the course of the accuracy synthesis of instrument mechanisms we have to establish in which measuring range (up to which input signal) the mechanism error is lower than the specified error, or what are the parameters for which the error of the theoretical mechanism is at a minimum.

For the above problem a computer method has been developed. The algorithm of the solution is the following.

1. Write the transfer function φ of the theoretical mechanism.

2. Compose the error function of the theoretical mechanism,

$$Dy = \varphi - \varphi_e$$

where φ is the transfer function of the theoretical mechanism,
φ_e is the prescribed scale function.

3. Compare error function By with the specified error function (Figs 7, 8, 9 and 10).

A possible way of comparing error function Dy of the theoretical instrument mechanism with the specified error function is represented by a section of the flow chart in Fig. 11.

Error function *Dy* is determined for each theoretical mechanism, then at the first logical condition from among the error functions shown in Figs 7,

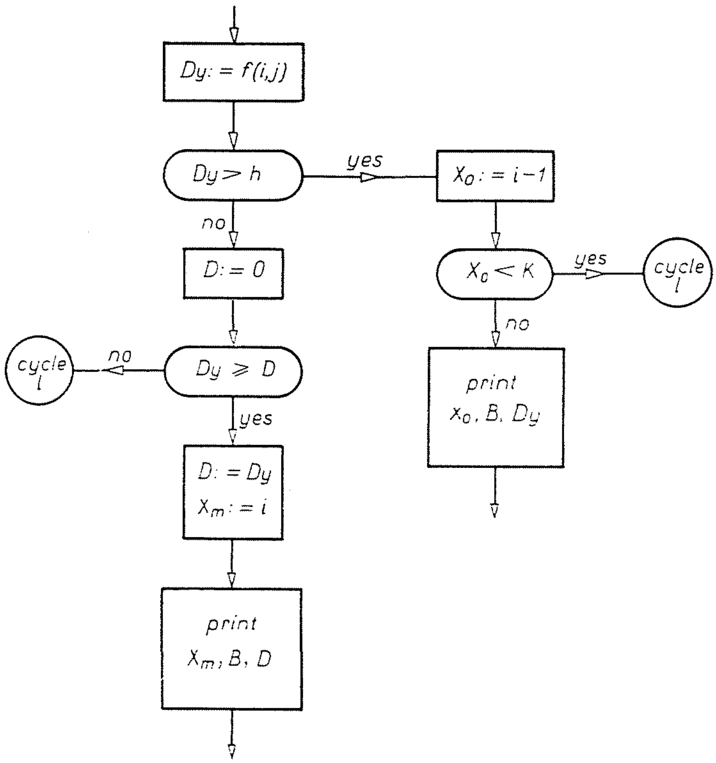

Fig. 11

8, 9 and 10 the desired error limit gets active in place of *h* in the right-hand side of the inequality.

If the error function exceeds the specified error limit within the measuring range, then the maximum input value x_0 (above limit *K*) where the error is lower than the specified value, that is, up to which the examined mechanism can be used, will be output.

If the first logical condition is frustrated, i.e. the error of the measuring instrument does not exceed the specified error limit within the measuring range, then the maximum error value *D* within the range will be found and printed out, together with the corresponding abscissa x_m and the optimum parameter value *B*.

Accordingly, the described computer method of accuracy synthesis permits the determination of the optimum parameters of measuring instrument mechanisms in a deductive way, namely from the specified (arbitrary) error function.

The described method was elaborated for solving problems arising in the accuracy synthesis of lever and rod type instrument mechanisms shown in Fig. 4.

Actual examinations aim at finding the validity conditions of the above method for the accuracy synthesis of other, mixed systems (e.g. mechanical-pneumatic, electromechanical, etc.).

Summary

The problems of geometric accuracy synthesis of measuring instrument mechanisms are mostly solved by using one of the analytical methods, interpolation, method of least squares, and equally "optimum" approximation.

These analytical methods are rather tedious, time and labour consuming accuracy synthesis on account of the complexity and high number of functions, permitting no rational design for prescribed error functions.

The presented algorithm lends itself to find the optimum parameters for various functional requirements such as signal transfer function and error restriction (error function).

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Dr. Zoltán KAPOSVÁRI, 1502 Budapest, P. O. B. 91. Hungary