EDDY-DIFFUSION COEFFICIENT, AS THE RATE OF LIQUID-MIXING IN CASE OF COMBINED PLATE CONSTRUCTIONS

By

K. MOLNÁR

Department of Chemical Machineries and Agricultural Industries,
Technical University, Budapest

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Presented by Prof. Dr. S. SZENTGYÖRGYI

Introduction

A considerable part of the processes in the chemical industry is a separation procedure. Such kinds of processes are e.g.: absorption of gas, desorption of gas, different processes of distillation, and so on. A common requirement of all these operations is the close contact of the gas (vapour) phase and the liquid phase, to create a diffusion mass transfer between the phases with the aim of changing their concentration.

Contact of gas (vapour) and liquid phase is realized mainly in column constructions. Among the types of column apparatus the most familiar are the bubble-cap plates and packed columns.

A lot of novel plate constructions appeared recently. The different dynamic plates and the so-called combined plates are well known, too. The most wide-spread is, for example, the so-called valve plate.

Combined plate constructions are characterized by the fact, that a part of the liquid entering the plate does not move off by falling over the outlet weir, but it is weeping through the plate. This phenomenon can well be observed when using a valve plate. Supposing normal operating parameters, the quantity of the weeping liquid may be even 30 to 40 per cent of the liquid entering the plate [1].

In column constructions there is a mass transfer between the phases. In such cases the procedure is determined by the rules of molecular diffusion as well as by the eddy-diffusion. In most common cases a resistance is developed by both phases. Thus knowledge of the liquid mixing is by all means necessary for the examination of the process, and the rate of the mixing is characterized by the diffusion coefficient.

The number of plates effecting the demanded selection, and the number of those to be incorporated in the column can be determined in knowledge of the eddy-diffusion coefficient for the sake of choosing the construction and operation parameters.

The phenomenon of weeping on the combined plate constructions has its effect on the mixing relations developing on the plate. So we have to elaborate a model, likely of help in determining the relations of liquid mixing of these types.

Liquid mixing on (static) bubble-cap plates

It was assumed earlier that the liquid was completely mixed on the plate. Consequently the liquid concentration was everywhere identical with that of the liquid leaving the plate. Murphree plate efficiency and point efficiency are equal in this case. Real conditions of the cross-flow plates are, however
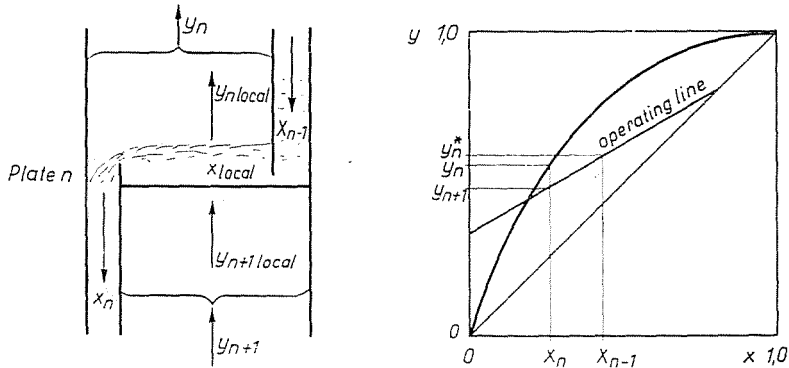

Fig. 1

not the kinds to allow a perfect mixing of the liquid on the plate. A concentration gradient is developing, determining the relation between the plate efficiency and the point efficiency. This relation depends upon the gradient of concentration or upon the rate of liquid mixing.

Fig. 1 shows the nth plate of a bubble-cap column in case of distillation.

Definition of the Murphree plate efficiency and point efficiency, expressed by the difference of concentration on the vapour side, on the basis of Fig. 1. is the following:

$$E_{MV} = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}} , \qquad (1)$$

$$E_{OG} = \frac{y_{n\,local} - y_{n+1}}{y_{n\,local}^* - y_{n+1}} . \qquad (2)$$

For $x_{n\,local} = x_n$, i.e. when the liquid on the plate is completely mixed, the plate efficiency coincides with the point efficiency ($E_{MV} = E_{OG}$).

The other limiting case is where the liquid does not mix altogether. This is the so-called plug flow of liquid, demonstrated by LEWIS [2]:

$$\frac{E_{MV}}{E_{OG}} = \frac{e^{E_{OG}\lambda} - 1}{E_{OG}\lambda} ,$$

where

$$\lambda = \frac{mG_M}{L_M} .$$

In reality, the liquid mixing is between these two limiting cases.

We know two general models describing the liquid mixing. KIRSCHBAUM [3], and later on GAUTREAUX and O'CONNEL [4] proposed the so-called "mixed pool" model. Their presumption was that from the inlet to the outlet weir the plate can be divided into n parts, where liquid mixing can be considered as perfect. Accordingly the mixing was interpreted as the sequence of perfectly
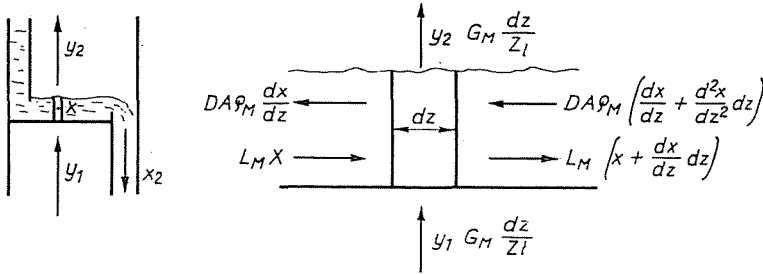
Fig. 2

mixed liquid pools. On the basis of this supposition a connection was deducted on the behaviour of the actual plate:

$$\frac{E_{MV}}{E_{OG}} = \frac{\left(1 + \frac{\lambda E_{OG}}{n}\right)^n - 1}{\lambda E_{OG}}.$$

Difficulty of the application of this method lies in the determination of the number n of cells.

Other model is the so-called eddy-diffusion model, used first by Wehner and Wilhelm. It was used in the standard works [5, 6], too. On the basis of this eddy-diffusion model the rate of mixing of the diffusing component on the plate is supposed to be proportional to the concentration gradient of the component.

Fig. 2 shows an elementary part of a static plate with the application of the eddy-diffusion model. The mass balance of the more volatile component concerning the elementary part and introducing the dimensionless location co-ordinate w yields the differential equation:

$$\frac{1}{Pe} \frac{d^2 x}{dw^2} - \frac{dx}{dw} - \lambda E_{OG} (x - x^*) = 0, \tag{3}$$

where

$$Pe = \frac{vZ_l}{D} \qquad \text{is the Peclet-number}$$

and

$$w = \frac{z}{Z_l}$$ is the dimensionless location co-ordinate.

WEHNER and WILHELM [5] solved the differential equation assuming the following boundary conditions:

$$w = 1 \qquad (z = Z_l),$$

$$x = x_2 \quad \text{and} \frac{dx}{dw} = 0.$$

Solution:

$$\frac{x - x^*}{x_2 - x^*} = \frac{e^{(\eta + Pe)(w-1)}}{1 + \dfrac{\eta + Pe}{\eta}} + \frac{e^{\eta(1-w)}}{1 + \dfrac{\eta}{\eta + Pe}}, \tag{4}$$

where

$$\eta = \frac{Pe}{2} \cdot \left[\sqrt{1 + \frac{4E_{OG}}{Pe}} - 1 \right].$$

On the basis of Equ. (4) the relationship of the plate efficiency and the point efficiency is

$$\frac{E_{MV}}{E_{OG}} = \frac{1 - e^{-(\eta + Pe)}}{(\eta + Pe)\left(1 + \dfrac{\eta + Pe}{\eta}\right)} + \frac{e^{\eta} - 1}{\eta\left(1 + \dfrac{\eta}{\eta + Pe}\right)}. \tag{5}$$

From Equ. (5) it is apparent that the relationship of the plate efficiency and the point efficiency can be determined from knowledge of the rate of mixing (D or Pe).

The rate of mixing is characterized by the eddy-diffusion coefficient, which is not a constant of the mass, but it is depending on the peculiarities of the flow, in contradiction to the molecular diffusion, which is caused by the BROWNIAN movement, so it can be characterized by a single mass constant, i.e. the molecular diffusion coefficient.

Since the eddy-diffusion coefficient depends on the features of the flow, so it depends on the plate construction, on the shape of the cap and on other construction data, too, influencing the formation of the flow.

In the quoted standard works [5, 6] the scientists determined the empirical relationship suitable for reckoning with of the eddy-diffusion coefficient in bubble-cap plate columns. This relationship is proposed by other summarizing works [7, 8], too for use under given validity conditions. It is the following:

$$D^{0.5} = 0.00378 + 0.0171 u_G + 0.00102 L + 0.0001758 h_w. \tag{6}$$

Method of determining the eddy-diffusion coefficient in case of combined plate constructions

1. *Analytic method*

As mentioned in the Introduction, weeping of the liquid appears in case of combined plate constructions. So the eddy-diffusion model has to be modified accordingly.

Equ. (6), determined on the basis of the model in Fig. 2, is not suitable for combined weeping plate constructions.

Taking also the weeping into consideration, mixing parametres are determined by analysing the concentration profile of the stationary condition. The most wide-spread method of determining the mixing parametres is that by examining the concentration profile of the indicator, taking no part in the mass transfer process, and continuously fed into the apparatus on a certain place. This method was used first of all by GILLILAND and MASON [9] to the examination of the hydro-dynamic characteristics of the fluidized state. The above method was used to the analysis of bubble-cap plate columns, too [5, 6]. The above-mentioned Equ. (6) has been set up on its basis. The method is suitable to analyse combined plate constructions, too, but the quantity of the wept liquid is to be taken into consideration in this case.

The weeping occurs e.g. in case of a valve-plate through the chimney openings of the valve-caps. If there is not a too high liquid gradient apparent on the plate, the same quantity of liquid can be assumed to be wept off each chimney opening. At the same time, having a nearly indentical number of caps per cap lines, the liquid weeping by cap lines can be assumed to be constant. Assume, further, the liquid weeping from the inlet to the outlet weir to be continuous, and on the basis of the foregoing, to be steady, accordingly on a given position z of the plate the actually flowing liquid quantity can be determined by the function:

$$L_M = L_{Mbe} - \frac{S_M}{Z_l} z. \tag{7}$$

Fig. 3 shows a plate with a weeping, and the eddy-diffusion model and injection of indicator is adopted.

The liquid is assumed to be perfectly mixed in an elementary section. Differential mass balance of the solved component:

$$DA\varrho_M \frac{d^2 x}{dz^2} dz + L_M x - (L_M - dL_M)\left(x + \frac{dx}{dz} dz\right) -$$
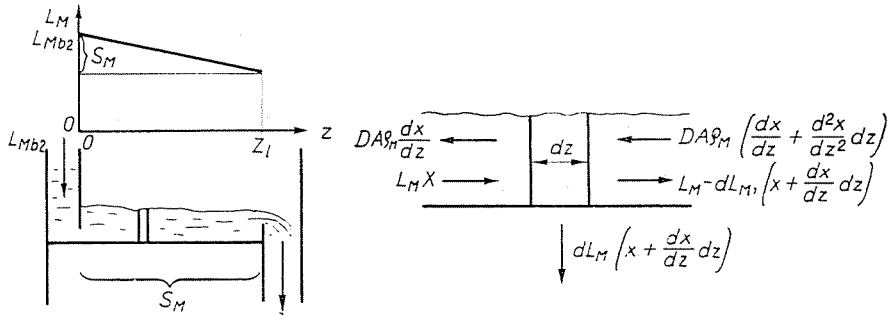$$- dL_M\left(x + \frac{dx}{dz} dz\right) = 0. \tag{8}$$

Fig. 3

Reducing and substituting the value of L_M determined by Equ. (7):

$$D\frac{d^2x}{dz^2} - \frac{1}{A\varrho_M}\left(L_{Mbe} - \frac{S_M}{Z_l}z\right)\frac{dx}{dz} = 0. \qquad (9)$$

Introducing a dimensionless location co-ordinate:

$$w = \frac{z}{Z_l}$$

$$\frac{D}{Z_l^2}\frac{d^2x}{dw^2} - \frac{1}{A\varrho_M Z_l}(L_{Mbe} - S_M w)\frac{dx}{dw} = 0. \qquad (10)$$

Since the velocity of the instantaneous liquid flow can be expressed as:

$$v = \frac{L_M}{A\varrho_M} = \frac{L}{Z_c Z_w},$$

hence

$$\frac{S_M}{A\varrho_M} = \frac{S}{Z_c Z_w}.$$

so the differential equation (10) is the following:

$$\frac{d^2x}{dw^2} - \frac{1}{D}\frac{Z_l}{Z_c Z_w}(L - Sw)\frac{dx}{dw} = 0. \qquad (11)$$

Introducing notations:

$$\frac{Z_l}{Z_c Z_w} = K$$

$$\frac{KS}{D} = a$$

$$\frac{KL}{D} = b.$$

The quantities K, a and b are constant under given construction and stationary operation conditions, regardless of the place.

Substituting into (11):

$$\frac{d^2 x}{dw^2} + (aw - b)\frac{dx}{dw} = 0.$$ (12)

Boundary conditions:

$$w = 0 \qquad x = x_0,$$
$$w = 1 \qquad x = x_g.$$

General solution of differential equation (12):

$$x = C_1 + C_2 \int\limits_{0}^{w} e^{-\frac{a}{2} w^2 + bw} \, dw \cdot$$ (13)

Eliminating the integration constants:

$$\frac{x - x_0}{x_g - x_0} = \frac{\int\limits_{0}^{w} e^{-\frac{a}{2} w^2 + bw} \, dw}{\int\limits_{0}^{1} e^{-\frac{a}{2} w^2 + bw} \, dw} .$$ (14)

Forming a full square from the exponent of the quantity behind the integral and introducing a new variable:

$$u = w\sqrt{a} - \frac{b}{\sqrt{a}} ,$$

becomes:

$$\frac{x - x_0}{x_g - x_0} = \frac{\int\limits_{-\frac{b}{\sqrt{a}}}^{\left(w\sqrt{a} - \frac{b}{\sqrt{a}}\right)} e^{-\frac{u^2}{2}} \, du}{\int\limits_{-\frac{b}{\sqrt{a}}}^{\left(\sqrt{a} - \frac{b}{\sqrt{a}}\right)} e^{-\frac{u^2}{2}} \, du} .$$ (15)

Since

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{x} e^{-\frac{t^2}{2}} \, dt ,$$

the so-called error integral Equ. (15) can be expressed as:

$$\frac{x - x_0}{x_g - x_0} = \frac{\Phi\left(w\sqrt{a} - \frac{b}{\sqrt{a}}\right) - \Phi\left(-\frac{b}{\sqrt{a}}\right)}{\Phi\left(\sqrt{a} - \frac{b}{\sqrt{a}}\right) - \Phi\left(-\frac{b}{\sqrt{a}}\right)} .$$

Since

$$\Phi(-x) = 1 - \Phi(x),$$

and the quantity of the liquid wept is always less than that entering the plate, hence

$$\frac{b}{\sqrt{a}} > \sqrt{a} \geq w\sqrt{a}.$$

Thus

$$\frac{x - x_0}{x_g - x_0} = \frac{\Phi\left(\dfrac{b}{\sqrt{a}}\right) - \Phi\left(\dfrac{b}{\sqrt{a}} - w\sqrt{a}\right)}{\Phi\left(\dfrac{b}{\sqrt{a}}\right) - \Phi\left(\dfrac{b}{\sqrt{a}} - \sqrt{a}\right)}. \tag{16}$$

It is well known that where $x > 1$ and $0 < \Theta < 1$ is a positiv number, the error integral can be expressed by the following function [10]:

$$\Phi(x) = 1 - \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi x}\left(1 + \dfrac{\Theta}{x^2}\right)}. \tag{17}$$

Relationship (17) adopted to Equ. (16):

$$\frac{x - x_0}{x_g - x_0} = \frac{\dfrac{e^{-\frac{a}{2}w^2 + bw}}{\left(\dfrac{b}{\sqrt{a}} - w\sqrt{a}\right)\left(1 + \dfrac{\Theta}{\left(\dfrac{b}{\sqrt{a}} - w\sqrt{a}\right)^2}\right)} - \dfrac{1}{\dfrac{b}{\sqrt{a}}\left(1 + \dfrac{\Theta}{\dfrac{b^2}{a}}\right)}}{\dfrac{e^{-\frac{a}{2} + b}}{\left(\dfrac{b}{\sqrt{a}} - \sqrt{a}\right)\left(1 + \dfrac{\Theta}{\left(\dfrac{b}{\sqrt{a}} - \sqrt{a}\right)^2}\right)} - \dfrac{1}{\dfrac{b}{\sqrt{a}}\left(1 + \dfrac{\Theta}{\dfrac{b^2}{a}}\right)}}. \tag{18}$$

Since:

$$0 < \Theta < 1,$$
$$b > 1,$$
$$b > a,$$
$$a \geq 0,$$

and in the neighbourhood of the position $w = 1$ (injection place) the fraction

$$\frac{1}{\dfrac{b}{\sqrt{a}}\left(1 + \dfrac{\Theta}{\dfrac{b^2}{a}}\right)},$$

is negligible compared to the other members, and about $w = 1$ there is:

$$\frac{1 + \dfrac{\Theta}{\left(\dfrac{b}{\sqrt{a}} - \sqrt{a}\right)^2}}{1 + \dfrac{\Theta}{\left(\dfrac{b}{\sqrt{a}} - w\sqrt{a}\right)^2}} \approx 1.$$

Expression (18) after reductions becomes:

$$\frac{x - x_0}{x_g - x_0} \approx \frac{e^{-\frac{a}{2}w^2 + bw}}{e^{-\frac{a}{2} + b}} \frac{\left(\dfrac{b}{\sqrt{a}} - \sqrt{a}\right)}{\left(\dfrac{b}{\sqrt{a}} - w\sqrt{a}\right)} . \tag{19}$$

Since

$$b = \frac{b^*}{D} ,$$

and

$$a = \frac{a^*}{D}$$

thus

$$\frac{\dfrac{b}{\sqrt{a}} - \sqrt{a}}{\dfrac{b}{\sqrt{a}} - w\sqrt{a}} = \frac{b^* - a^*}{b^* - wa^*} .$$

Substituting into (19) and transposed:

$$\frac{b^* - wa^*}{b^* - a^*} \frac{x - x_0}{x_g - x_0} \approx e^{-\frac{a}{2}w^2 + bw + \frac{a}{2} - b} \tag{20}$$

Introducing notations:

$$\psi = \frac{b^* - wa^*}{b^* - a^*} ,$$

and

$$\eta = (1 - w)\left[b^* - \frac{a^*}{2}(1 + w)\right].$$

Substituting into Equ. (20) and taking the natural logarithm of both sides:

$$\ln\left(\psi \frac{x - x_0}{x_g - x_0}\right) = -\frac{1}{D}\eta. \tag{21}$$

On the basis of Equ. (21) it can be realized that the eddy-diffusion coefficient can be determined by injecting an indicator (provided the quantity

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$\ln [\psi(x - x_0)/(x_g - x_0)]$ is described vs. η, a straight-line, the slope of which is the inverse of the eddy-diffusion coefficient).

Thus, measuring the indicator concentration at different spots w from the injection point against the liquid flow but around $w = 1$, then, since η and ψ can be measured, in knowledge of all constant magnitudes S, L, F, h_w, the turbulent diffusion coefficient can be determined, taking the constant weep into consideration.

2. Determination of the eddy-diffusion coefficient by a grapho-analytic method

If the change of concentration of the indicator is known, the eddy-diffusion coefficient can be determined at about position $w = 1$, on the basis of Equ. (13):

$$\frac{dx}{dw} = C_2 e^{-\frac{a}{2} w^2 + bw} . \tag{22}$$

Taking the natural logarithm of both sides:

$$\ln \frac{dx}{dw} = \ln C_2 + \left(-\frac{a}{2} w^2 + bw \right) ,$$

be

$$\ln C_2 = C,$$

and after

$$a = \frac{a^*}{D} .$$

and

$$b = \frac{b^*}{D} ,$$

thus

$$\ln \frac{dx}{dw} = \frac{1}{D} \left(-\frac{a^*}{2} w^2 + b^* w \right) + C . \tag{23}$$

Introduced notation:

$$b^* w - \frac{a^*}{2} w^2 = \xi ,$$

becomes (23):

$$\ln \frac{dx}{dw} = \frac{1}{D} \xi + C . \tag{24}$$

From Equ. (24) it appears that knowing the change of concentration of the indicator vs. dimensionless co-ordinate w the gradient of concentration change dx/dw can be graphically determined at arbitrary location w. Quantities

ln dx/dw vs., one gets a straight, with a slop $1/D$, i.e. the inverse of the eddy-diffusion coefficient.

This method can involve a considerable inaccuracy, since it is partly graphical, namely the gradient of the concentration change has to be determined by differentiating, and the concentration of the indicator is diminishing rapidly away from the place of injection.

Notations

h_w	[mm]	height of weir
m		slope of equilibrium line
n		number of cells
u	[m/sec]	linear gas (vapour) velocity
v	[m/sec]	velocity of phase flow
w		dimensionless place co-ordinate
x	$\left[\dfrac{\text{kmol dissolved mass}}{\text{kmol solution}}\right]$	concentration of liquid phase
x^*, y^*	$\left[\dfrac{\text{kmol dissolved mass}}{\text{kmol solution}}\right]$	equilibrium concentrations
y	$\left[\dfrac{\text{kmol dissolved mass}}{\text{kmol solution}}\right]$	concentration of gas (vapour) phase
z	[m]	co-ordinate
A	[m^2]	flow cross-section of liquid current
D	[m^2/sec]	eddy-diffusion coefficient
E_{MV}		Murphree plate efficiency
E_{OG}		Murphree point efficiency
G_M	[kmol/h]	molar gas rate
L	[m^3/sec, m^3/h]	quantity of liquid
L^*	[m^3/mh]	quantity of liquid referred to unit length of the weir
L_M	[kmol/h]	molar liquid flow
Pe		Peclet number
S	[m^3/sec]	quantity of liquid wept
S_M	[kmol/h]	molar quantity of liquid wept
Z_c	[mm, m]	height of clear liquid on the plate
Z_l	[mm, m]	distance of inlet to outlet weir
Z_w	[mm, m]	mean breadth of liquid flow on the plate
ϱ	[kg/m^3]	density
ϱ_M	[kmol/m^3]	molar density

Subscripts

be	entering
g	being at the injection grid
G	vapour phase
$local$	local value
M	molar
n	n^{th}
$n-1$	$(n-1)^{\text{th}}$
$n+1$	$(n+1)^{\text{th}}$
o	being at the inlet weir
w	weir

Summary

Plate columns are often used in chemical industry for separation. In order to determine the number of plates needed for a separation of given clearness the rate of mixing of the liquid on the plates has to be known. The paper presents methods for the experimental determination of the eddy-diffusion coefficient in the case of combined plate constructions, affected by a considerable amount of weeping.

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Dr. Károly MOLNÁR, 1502 Budapest P. O. B. 91. Hungary

* In Hungarian.