# STUDY OF THREAD MILLING HOBS 

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The criteria set for the accuracy in machine production are increasingly rigorous. For the production of threaded machine components, engineering practice knows numerous methods of which milling is one of the most frequently used processes. While the productivity achieved by thread milling hobs is satisfactory, the accuracy is affected by many factors, that will be dealt with here from theoretical aspects.

The cutting edges of the thread milling hob consist of grooves running either parallel or at an angle to the axis. Tools with inclined grooves ensure more even and uniform running and better characteristics than tools with straight grooves. According to their intended use, thread milling hobs are produced in two variants: with bore, and with shank.

The life of the cutting edge and the accuracy of the thread strongly depend on the diameter of the tool.

Fig. 1 shows that the starting angle $\xi$ increases with increasing tool diameter, to cause an increasing distortion of the profile. When machining threads in internal surfaces, then - assuming identical dimensions - the starting angle and the profile distortion will increase (Fig. 2).


Fig. 1. Length of engaging arc between tool and workpiece in external thread cutting

The diameter of the tool can be determined as a function of the arbour strength. For the selection of the outside diameter for machining both external and internal threads, standards and recommendations are available.


Fig. 2. Length of engaging are in internal thread cutting

The length of the thread cutter $l_{1}=L+(2-3) h$. No thread cutter o optional length can be used since with increasing tool length the cutting force also increases and the accuracy criteria for the tool become more rigorous.

The advisable number of cutting edges in the thread cutter depends on the manufacturing conditions. Denser toothing ensures more even run and better surface finish, while teeth spaced farther apart are more favourable for relief work and allows for more regrindings.

In relieving thread cutters, the rate of radial relief (Fig. 3):

$$
\begin{equation*}
K=\frac{D_{s} \pi}{z} \operatorname{tg} \alpha_{1} \tag{1}
\end{equation*}
$$

With ground tools the degree of relief is greater, to facilitate the withdrawal of the wheel.

The angles of the thread milling hob vary along the axis. In the section normal to the axis the relief angle $\alpha_{x}$ pertaining to an optional point $P$ of the cutting edge can be determined on the basis of Fig. 3.

$$
\operatorname{tg} \alpha_{x}=\frac{K \cdot z}{2 R_{x} \pi}
$$

On the basis of Eqs 1 and 2:

$$
\begin{equation*}
\frac{\operatorname{tg} \alpha_{x}}{\operatorname{tg} \alpha_{1}}=\frac{D_{s}}{2 R_{x}} ; \quad \alpha_{x}=\operatorname{arctg}\left[\frac{D_{s}}{2 R_{x}} \operatorname{tg} \alpha_{x}\right] \tag{3}
\end{equation*}
$$

The angle $\alpha_{x}$ in Fig. 3 can be calculated from the triangle AOP:

$$
\begin{equation*}
\alpha_{\mathrm{ex}}=\arcsin \left[\frac{D_{s}}{2 R_{x}} \cdot \sin \gamma\right]-\gamma \tag{4}
\end{equation*}
$$



Fig. 3. Position of the thread milling hob and the workpiece during the machining process

The acting angles are approximately

$$
\begin{aligned}
& x_{m x}=\alpha_{x}-\alpha_{e x} \\
& \gamma_{m x}=\gamma_{x}+\alpha_{e x}
\end{aligned}
$$

For milling, the determination of the angles $x_{N}$ and $\gamma_{N}$ is of greatest importance. On the basis of Fig. 3 we may write that

$$
\begin{gather*}
\frac{\tan \alpha_{N}}{\tan \alpha_{1}}=\frac{\bar{B} \bar{C}}{\bar{A} \bar{B}}  \tag{5}\\
\alpha_{N}=\operatorname{arctg}\left(\operatorname{tg} \alpha_{1} \sin \frac{\varepsilon}{2}\right)
\end{gather*}
$$

From Fig. 3, $\gamma_{N}$ may be calculated in the same way as $x_{N}$ :

$$
\begin{equation*}
\gamma_{N}=\operatorname{arctg}\left(\operatorname{ctg} \gamma \sin \frac{\varepsilon}{2}\right) \tag{6}
\end{equation*}
$$

The value of $\alpha_{N x}$ at an optional point, in the section normal to the cutting edge:

$$
\alpha_{N x}=\operatorname{arctg}\left(\operatorname{tg} \alpha_{x} \sin \frac{\varepsilon}{2}\right)
$$

and, in consideration of Eq. (3):

$$
\begin{equation*}
\alpha_{N x}=\operatorname{arctg}\left(\frac{D}{2 R_{x}} \operatorname{tg} \alpha_{1} \sin \frac{\varepsilon}{2}\right) . \tag{7}
\end{equation*}
$$

From relationship (7), it will be obvious that the angle $\alpha_{N x}$ is smaller than $\alpha_{1}$. For milling it is important that the minimum of the angle $\alpha_{N x}$ should be 3 deg. or above.

When determining the dimensions of the chip chute, the following factors should be borne in mind:
the volume occupied by the chips;
the space requirement for the runout of the relieving tool and the grinding wheel;
the required mechanical strength of the teeth.
The shape of the chute influences the run-off of the chips. The rate of run-off can be improved by rounding off the tooth root.

## The tool profile

Assuming zero rake angle the profile of the cutting edges of the milling hob will be equal to the thread to be cut. When the tool arbour and the workpiece mandrel are parallel, the annular grooves of the thread milling hob enclose the helix angle with the thread to be cut. If, on the other hand, the axis of the thread milling hob encloses an angle with the axis of the workpiece, the helix surface will be located over a hyperboloid surface.

With the axes arranged parallel, a suitably corrected tool profile may prevent the distortion of the thread, even if a thread milling hob is used. The correct dimensioning of the tool with modified section requires the analysis of the thread profile formation. Since the profile may be more distorted when internal threads are cut, we shall confine our investigations to internal threads.

Taking a workpiece of a theoretical $V$ thread profile and cutting it by a plane normal to its axis, the intersection line of helix and plane is a symmetric curve (Fig. 4).

As known, with a closed helical surface in the section perpendicular to the axis, an Archimedean spiral is obtained.

Fig. 5 shows the helix arisen during the machining process and its construction. Construction is done by dividing the tool rotation and the axial
feed to an optional but identical number of divisions. The figure indicates that the cut made with a thread milling hob is wider than the tool profile. The figure verifies furthermore that even with a perfectly sharp pointed nose the groove is rounded off after machining.

This construction can also be derived mathematically, helping to dimension the corrected section. The analysis of the evolution of the thread profile


Fig. 4. Cross-section of the workpiece


Fig. 5. Construction of helix in the section normal to the axis, in internal thread cutting
confirmed that the thread radius $Q$ pertaining to an optional point of the thread profile was not equal to the sum of the axial distance $e$ and the corresponding cutting radius $R$, i.e $(\varrho \neq e+R)$, since in the section normal to the axis the point of engagement between the helix and the hob does not fall into the straight line connecting the centre of the workpiece with the centre of the cutter.

To determine the data illustrated in Fig. 6, it is reasonable to select a polar co-ordinate system, with the origin coincident with the axis of the


Fig. 6. Belative positions of the tool and the workpiece
thread, and the polar axis passing across the point of the Archimedean spiral next to the pole, obtained by cutting the thread of given dimensions by a plane.

The equation of the thread profile in the polar co-ordinate system:

$$
\begin{equation*}
\varrho_{v}+r+k \widehat{q} \tag{8}
\end{equation*}
$$

The value of the coefficient $K$ can be determined from the condition that while the polar angle increases from 0 to 180 deg , the length of the leading radius varies by the height $t$ of the theoretical sharp $V$ thread. Accordingly, if $\varphi=\pi$ and $\varrho_{\pi}=r+i$, then $r+t=r+k$, whence

$$
\begin{equation*}
k=\frac{t}{\pi} \tag{9}
\end{equation*}
$$

On the basis of Fig. 6:

$$
\begin{equation*}
t=\frac{h}{2 \operatorname{tg} \frac{\psi}{2}} \tag{10}
\end{equation*}
$$

Combining Eqs (9) and (10):

$$
\begin{equation*}
k=\frac{h}{2 \pi \operatorname{tg} \frac{\psi}{2}} . \tag{11}
\end{equation*}
$$

The value of $r$ from Eq. (8) is

$$
r=\frac{D_{2}}{2}-\frac{t}{2} .
$$

Substituting into Eq. (10):

$$
\begin{equation*}
r=\frac{1}{2} / D_{2}-\frac{h}{2 \operatorname{tg} \frac{\psi}{2}} . \tag{12}
\end{equation*}
$$

The cutting radius is obtained from the triangle $O_{1} C Q$ of Fig. 6.

$$
\begin{equation*}
R^{2}=\left(\varrho_{\varphi}-e \cos \delta\right)^{2}+(e \sin \delta)^{2} \tag{13}
\end{equation*}
$$

As shown by the figure, the angle $\delta$ in the equation is as follows:

$$
\begin{equation*}
\delta=v-\varphi \tag{14}
\end{equation*}
$$

There are three unknown quantities in Eqs (13) and (14): $R, \delta$ and $\varphi$. The missing equation can be determined from the premise according to which in the point of engagement both curves in the section have a common tangent. Thus, the directional tangent of the tangents is identical.

The directional tangent of the tangent to the helix is

$$
\begin{equation*}
\operatorname{tg} \omega=\frac{d \varrho_{\varphi}}{\varrho_{\varphi} d \varphi}=\frac{d(r+k \varphi)}{\varrho_{\varphi} d \varphi} \tag{15}
\end{equation*}
$$

The slope of the tangent can be expressed from Eq. (13) as well:

$$
\begin{gather*}
\varrho_{\varphi}=\sqrt{R^{2}-(e \sin \delta)^{2}+e \cos \delta,} \\
\operatorname{tg} \omega=\frac{d \varrho_{q}}{\varrho_{q} d \varphi}=\frac{\left.\left.d\left[\sqrt{R^{2}-e^{2} \sin ^{2}(v-\varphi( }+e \cos \right) v-\varphi\right)\right]}{\varrho_{\tau} d \varphi} . \tag{16}
\end{gather*}
$$

From the two equations of the directional tangent the following relationship is obtained:

$$
\frac{d(r+k \varphi)}{\varrho_{\varphi} d \varphi}=\frac{d\left[\sqrt{R^{2} e^{2} \sin ^{2}(v-\varphi)}+\cos (v-\varphi)\right]}{\varrho_{\varphi} d \varphi} .
$$

After simplification and derivation we arrive at

$$
\begin{equation*}
\frac{e^{2} \sin (\nu-\varphi) \cos (v-\varphi)}{\sqrt{R^{2}-e^{2} \sin ^{2}(\nu-\varphi)}}=k-e \sin (v-\varphi) . \tag{17}
\end{equation*}
$$

Substituting (13) into the above relationship:

$$
\frac{d^{2} \sin (v-\varphi) \cos (\nu-\varphi)}{\sqrt{\left[r+k \varphi-e \cos (v-\varphi)^{2}+e^{2} \sin ^{2}(v-\varphi)-e^{2} \sin ^{2}(v-\varphi)\right.}}=k-e \sin (\nu-\varphi) .
$$

Simplified and rearranged:

$$
\begin{equation*}
e \cos (v-\varphi)=(r+k \varphi)\left[1-\frac{e}{k} \sin (\nu-\varphi)\right] . \tag{18}
\end{equation*}
$$

Using the relationships, the cutting radius can be determined. Eq. (18) being, however, a transcendent function which cannot be arranged into explicit form, the calculation of the angle is rather cumbersome.

There is a simpler method available to calculate the cutting radius. From the triangle $0 O_{1} Q$ of Fig. 6 we have:

$$
\begin{equation*}
R=e \frac{\sin \delta}{\sin \omega} \tag{19}
\end{equation*}
$$

From Eq. 15 the unknown angles $\delta$ and $\omega$ can be calculated:

$$
\operatorname{tg} \omega=m \frac{d \varrho_{\psi}}{\varrho_{\psi} d \varphi} \cdots \frac{d(r+k \varphi)}{\varrho_{\varphi} d} \frac{k}{\varrho_{\varphi}} .
$$

Substituting Eq. 8:

$$
\begin{equation*}
\operatorname{tg} \omega=\frac{k}{r+k \varphi} . \tag{20}
\end{equation*}
$$

The angle of shift in the point of engagement, as per Fig. 7:

$$
\begin{equation*}
\delta=\vartheta-\omega \tag{21}
\end{equation*}
$$

From the triangle $O O_{1} Q$ :

$$
\sin \vartheta=\frac{\varrho_{q}}{e} \sin \omega
$$

and, considering Eq. (8):

$$
\begin{equation*}
\sin \vartheta=\frac{r+k \varphi}{e} \sin \omega . \tag{22}
\end{equation*}
$$

The rate of feed $L$ pertaining to $v=360^{\circ}$ is equal to one pitch. For an optional $\nu$ angle we may write down the following relationship

$$
\frac{v}{2 \pi}=\frac{L}{h}
$$

whence

$$
\begin{equation*}
L=\frac{\widehat{v}}{2 \pi} h=\frac{v^{\circ}}{360^{\circ}} h . \tag{23}
\end{equation*}
$$

In the knowledge of the cutting radii pertaining to angles $\varphi$ and spacings $L$, the tool profile can be calculated without any difficulty.

In the section of the helix cut by a thread milling hob normal to the axis, no spiral curve will develop everywhere since part of it bounding the thread is a circular arc. For a central angle $2 \delta^{\prime}$ pertaining to the section with a permanent radius, the range of validity of the formulae is:

$$
0 \leqq \phi \leqq \pi-\delta^{\prime}
$$

Knowing the angles $\delta^{\prime}=\pi-\varphi^{\prime}$ the greatest cutting radius will be

$$
\begin{equation*}
R_{s}=0 \frac{\sin \delta^{\prime}}{\sin \omega^{\prime}} \tag{24}
\end{equation*}
$$

where

$$
\begin{equation*}
\omega^{\prime}=\operatorname{arctg} \frac{k}{r+k \varphi^{\prime}} . \tag{25}
\end{equation*}
$$



Fig. 7. Sections of the tool and the workpiece

From the triangle $K L O$ in Fig. 7, the thread radius can be expressed by the following equation:

$$
\begin{equation*}
\varrho_{v_{1}}=e \cos \left(\pi-v_{1}\right)+\sqrt{R_{s}^{2}-e^{2} \sin ^{2}\left(\pi-v_{1}\right)} . \tag{26}
\end{equation*}
$$

$L$ values pertaining to radii determined by this formula can be determined by Eq. (23).

The nose angle of the tool can be calculated on the basis of Fig. 7:

$$
\begin{equation*}
\operatorname{tg} \frac{\varepsilon}{2}=-\frac{L_{2}-L_{1}}{R_{2}-R_{1}} . \tag{27}
\end{equation*}
$$

The tool profile being narrower than the thread section, the centre lines of the sections shift in relation to one another by $A R$.

From the triangle $A B C$ :

$$
\begin{align*}
R_{i:} & =R_{\lrcorner}+\left(\frac{h}{4}-L_{1}\right) \operatorname{ctg} \frac{\varepsilon}{2},  \tag{28}\\
\Delta R & =\frac{D_{2}}{2}-\left(R_{k}+e\right) . \tag{29}
\end{align*}
$$

The profile height being:

$$
\begin{equation*}
t_{1}=t_{m_{1}}+\Delta R \tag{30}
\end{equation*}
$$

where:

$$
\begin{align*}
t_{m 1_{\min }} & =\frac{3}{8} t=\frac{3}{8} \cdot \frac{h}{2} \operatorname{ctg} 30^{\circ}=0,325 h \\
t_{2} & =t_{m 2}  \tag{31}\\
t_{m 2} & =\frac{t}{4}=0,217 \mathrm{~h}
\end{align*}
$$

The cutting profile formulae refer to a tool with zero rake angle. To improve the conditions of cutting, it is advisable to evolve a non-zero rake angle.

According to Fig. 8, the height of the crest of the rake profile is

$$
\begin{equation*}
t_{1}^{\prime}=R_{k} \frac{\sin \tau}{\sin \gamma} \tag{32}
\end{equation*}
$$

The angle $\tau$ in (32) can be calculated from the triangle $A B O$, in the following way:

$$
\begin{equation*}
\tau=\arcsin \left|\frac{R_{k}+t_{1}}{R_{k}} \sin \gamma\right|-\gamma \tag{33}
\end{equation*}
$$



Fig. 8. Analysis of the profile elements of the tool with positive rake angle

The root height of the tool's rake profile, from the triangle $B C O$ will be:

$$
\begin{align*}
t_{2}^{\prime} & =\left(R_{k}-t_{2}\right) \frac{\sin \left(\tau_{2}-\tau_{1}\right)}{\sin \left(\gamma+\tau_{1}\right)}  \tag{34}\\
\tau_{2} & =\arcsin \left[\frac{R_{k}}{R_{k}-t_{2}} \sin \left(\gamma+\tau_{1}\right)\right]-\gamma . \tag{35}
\end{align*}
$$

The nose angle of the tool's rake profile:

$$
\varepsilon^{\prime}=2 \operatorname{arctg}\left\langle\frac{t_{1}}{t_{1}} \operatorname{tg} \frac{\varepsilon}{2}\right\rangle
$$

The height of the tool profile in the radial section:

$$
\begin{align*}
& t_{01}=A B=t_{1}-K_{1} ; \text { while } K_{1}=K z \frac{\tau_{1^{\circ}}}{360^{\circ}}  \tag{36}\\
& t_{01}=t_{1}-K z \frac{\tau_{1^{\circ}}}{360^{\circ}} \\
& t_{02}=t_{2}-K z \frac{\tau_{2} \tau_{1}}{360} . \tag{37}
\end{align*}
$$

The nose angle of the thread profile in the section under study:

$$
\begin{equation*}
\varepsilon_{0}=2 \operatorname{arctg}\left(\frac{t_{1}}{t_{01}} \operatorname{tg} \frac{\varepsilon}{2}\right) . \tag{38}
\end{equation*}
$$

The distance between the centre line of the profile and the cutting axis:

$$
\begin{equation*}
R_{O h}=R_{k}+K_{1} \tag{39}
\end{equation*}
$$

The above calculation method suits tools for machining internal threads, but it can be adapted also for the machining of external threads.

## Summary

The theoretical study of the process of cutting internal threads has shown that, although with the use of suitable designed tools the distortion of the thread profile can be prevented. the rounding-off of the thread root cannot be eliminated even through the correction of the cutting profile. The larger the tool diameter, the wider is the rounded-off profile section of the workpiece. In the production of simple joints it is seldom necessary to determine the cutter profile from point to point, since the distortion remains within the permissible tolerances. The variation of the thread section depends also on the length of the are along which the workpiece and the tool engage. When external threads are cut, the profile distortion is slighter than in internal thread cutting. While the dimensional changes due to re-grinding of the tool have practically no effect upon the geometric accuracy, the value of the rake angle must be strictly adherent to.

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