INVESTIGATION OF LINEAR FORCE EQUATIONS WHEN CUTTING STEEL

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1. Introduction

Cutting force is known to depend on several conditions, mainly on cutting speed and cutting geometry. Fig. 1 shows the effect of cutting geometry on cutting force, assuming idealized conditions of orthogonal cutting, where

1. Work material (W, work piece, test piece)
2. Tool (T)
3. Ambient medium (M)
4. Cutting speed (v [m/min])
5. Cutting force (F_R [kp])
6. Angle Φ determining shear plane.

A chip of width b_d [mm] contacts the tool rake face over a length L [mm], thus the tool face of b_dL [mm^2] is under a cutting force F_R [kp] due to an average of non-uniform load values. The symbols of cutting force components are shown in Fig. 2, in the coordinate system of tool, machine and chip. Removing undeformed chip of thickness h [mm] at speed v [m/min], the chip W_d really produced will be of thickness h_d.

The following known equations express the relationship between main cutting force and undeformed chip cross section:

$$F' = C_{F_h} h^p b \tag{1}$$

$$F'' = C_F hb \tag{2}$$

$$F''' = A^0 A_0 + B^0 \tag{3}$$

The attempt to predict cutting force on the analogy of other force calculations, from strength and area, resulted in a new concept: "specific cutting force". In accordance with the three equation types it will be

$$k_s' = \frac{C_{F_h}}{h^{1-p}} \qquad \text{and} \qquad F' = k_s' A_0 \tag{4}$$

$$k_s'' = C_F \qquad \text{and} \quad F'' = k_s'' A_0 \tag{5}$$

$$k_s''' = A^0 + \frac{B^0}{A_0} \quad \text{and} \quad F''' = k_s''' A_0 \tag{6}$$

By introducing k_s the three equation types (1, 2, 3) will have the same form offering comparison between different cutting methods. The precalculated k_s values are available either in tables or graphically expressed, consider-

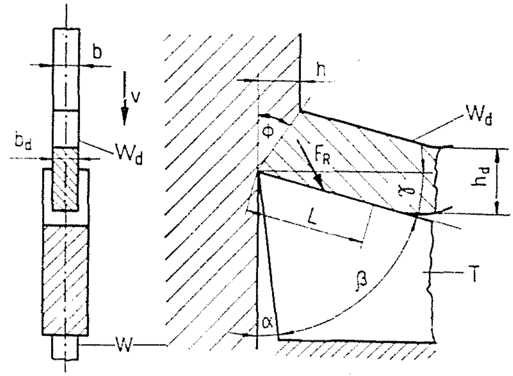

Fig. 1. Elements of cutting process

ably facilitating any further calculation. The use of specific cutting force presents the advantage to simplify cutting force prediction: the known data (b, h, $bh = A_0$) have to be multiplied with the appropriate tabulated k_s values.

The practical advantages need completion by some theoretical considerations.

The specific cutting force on a surface unit has a physical meaning only if the area concerned and the cutting force are coherent. Remind that the area is under force effect and the relationship between area and force can be expressed numerically.

The shear plane A_Φ as well as the tool rake face contacting the chip A_γ are the main factors influencing force acting during the cutting process while the nominal cross section A_0 does not intervene. The value of A_0 can be expressed exactly and the resulting ratio is a good reference, but A_0 is of no practical use in finding the real cutting force.

It is evident from the linear force equation $F = A^0 A_0 + B^0$ that the increase of the chip cross section does not affect the component B^0. Namely B^0 has no role in generating the mean shear strength. If the function $k_s''' = A^0 + \frac{B^0}{A_0}$ contained $A_0 \rightarrow 0$, the resulting k_s values would be extremely high and had no physical meaning. Analyses made about $A_0 = 0$ raise the problem of the lower limit of validity for the equation.

Cutting is now discussed as a plane process, therefore results may simply be compared by referring the forces obtained from $\left(F_1 = F\frac{b_1}{b}\right)$ [kp] to the cross section $b_1h = A_1$ [mm²] for unit length of edge. Without knowing Φ, the first step will deliver the linear equations

$$F_1 = AA_1 + B \text{ for the main cutting force,}$$
$$F_{N1} = A_N A_1 + B_N \text{ for the normal force.}$$

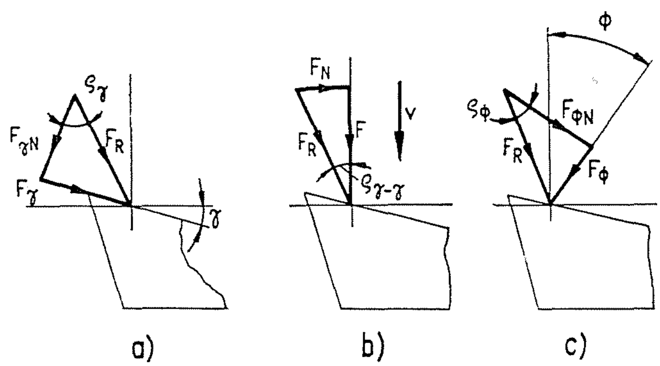

Fig. 2. Components of cutting force

For about similar cutting operations in nearly identical conditions of temperature, strain and strain rate, the value of Φ will be unaffected or nearly so, consequently the relationship between the nominal area A_1 and the shear plane $A_{\Phi 1}$, i.e. $A_{\Phi 1} = A_1/\sin \Phi$ will remain constant. Any change in Φ will impair the linear approximation. Essentially, Φ is decisive for the intervals of linear approximation.

2. Calculation of mean shear stress

The linear equation $F_1 = AA_1 + B$ is expressing the main cutting force. AA_1 is seen to be the only term which increases proportionally to A_1 and for unchanged also to $A_{\Phi 1}$. Be A_1' the lowest limit of validity of the linear equation within the range of measured data it may be written (Fig. 3):

$$A(A_1 - A_1') = \bar{\tau}_\Phi \frac{A_1 - A_1'}{\sin \Phi} \cdot \frac{\cos \psi}{\cos(\Phi + \psi)} \tag{7}$$

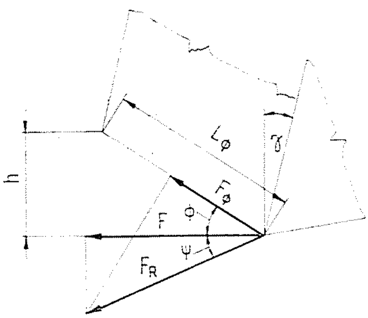

Fig. 3. Calculation of main cutting force

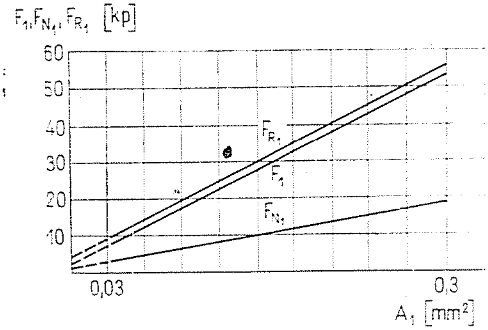

Fig. 4. Scalar values of main cutting force and its components

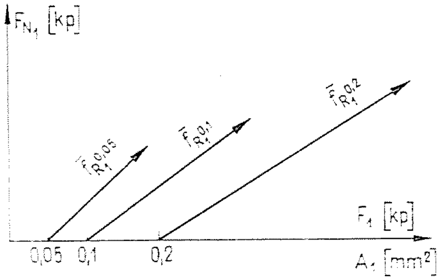

Fig. 5. Vectors of cutting force

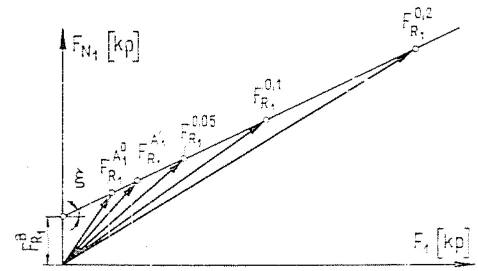

Fig. 6. Vectors shifted to the origin

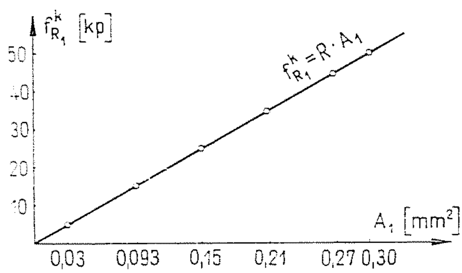

Fig. 7. Scalar values of difference vectors

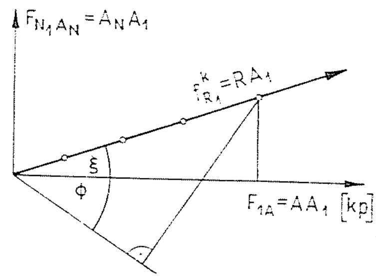

Fig. 8. Calculation of mean shear stress

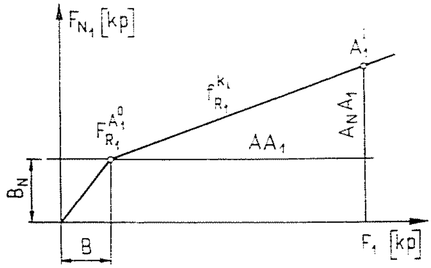

Fig. 9. Calculation of difference vectors

Introducing the expression $\Omega = \dfrac{\cos \psi}{\cos(\Phi + \psi) \sin \Phi}$

$$A = \bar{\tau}_\Phi \frac{\cos \psi}{\cos(\Phi + \psi) \sin \Phi} = \bar{\tau}_\Phi \Omega \tag{8}$$

The mean shear strength $\bar{\tau}_\Phi = \dfrac{A}{\Omega}$,

while

$$F_1 = \bar{\tau}_\Phi \Omega A_1 + B$$

In orthogonal cutting, the cutting force is determined from two measured components, mostly the main cutting force F[kp] and the normal force F_N[kp] perpendicular to the former. These are, naturally, the components most frequently discussed in literature. References on the resulting force F_R, its measure and direction, are scarcely found in literature. The fact that any change in direction of the resulting force has also to be considered, makes difficulties in linearization, except for components of a given direction.

To investigate this problem, Fig. 4 shows not only the curve describing the two components but also that of the resulting scalar value. Some cutting force vectors pertaining to the relevant A_1 values are also plotted (Fig. 5). Shifting the same vectors to the origin, the F_{N1} values are plotted against F_1 (Fig. 6). Each $F_1 - F_{N1}$ group determines a point $F_{R_1}^{A_1^i}$. These points fit a curve of slope ξ, intersecting the axis F_{N1} at point F_{R1}^B. In accordance with the linear equations of the components, the values B and B_N extrapolated to A_1^0 determine the vector $\bar{f}_{R_1}^{A_1^0}$. The difference vectors $\bar{f}_{R_1}^{A_1^i} - \bar{f}_{R_1}^{A_1^0} = f_{R_1}^{k_i}$ are proportional to the corresponding A_1^i values. Fig. 7 indicates the scalar values of the difference vectors by A_1, i.e. $f_{R_1}^k = RA_1$. Thus, the cutting force vector is obtained by adding the vector $\bar{f}_{R_1}^{A_1^0}$ to $f_{K_1}^{k_i}$, the difference vector RA_1 of slope ξ. The mean shear stress is obtained from the variation of the difference vector.

The curve of scalar values of the difference vector in the $AA_1 - A_N A_1$ system (Fig. 8) is completed with the shear plane illustrated in physically true direction. The component of RA_1 in the shear plane is

$$F_{\Phi 1} = RA_1 \cos(\Phi + \xi) \tag{9}$$

its reference plane being $A_1/\sin \Phi$
thus

$$\bar{\tau}_\Phi = R \cos(\Phi + \xi) \sin \Phi \tag{10}$$

where ξ is a constant.

Fig. 9 idealizing the difference vectors clearly shows the length of the difference vectors to be

$$f^k_{R_1} = A_1\sqrt{A^2+A_N^2} = RA_1 \tag{11}$$

thus $R = \sqrt{A^2+A_N^2}$

furthermore $\xi = \text{arc tg}\,\dfrac{A_N}{A}$

According to the equation

$$\bar{\tau}_\Phi = R\cos(\Phi+\xi)\sin\Phi = \sqrt{A^2+A_N^2}\cos\left(\text{arc tg}\,\frac{A_N}{A}+\Phi\right)\sin\Phi \tag{12}$$

the factors determining the mean shear stress are the angle Φ of the shear plane and the constants of the components' equations.

3. Numerical determination of the mean shear stress on the basis of data by Thomsen, Lapsley and Grassi [1]

As stated by a number of authors [2, 3, 4] the relationship between undeformed chip thickness and cutting force components in the range $h < 0.3$ mm is a linear one. Let me quote some data from "Deformation work absorbed in metal cutting" by Thomsen, Lapsley and Grassi (Table I; $\gamma = 25°$). Working conditions: material Shelby tubing, $D = 6$ in, chip thickness 0.475 in, tool HSS, clearance angle 6°, $v = 90$ fpm, dry cutting.

Table I

Data by Thomsen, Lapley and Grassi [1]

$\Phi°$	$\psi°$	h (in)	F (lb)	F_N (lb)	A_1 (mm²)	F_1 (kp)	F_{N1} (kp)
20.9	25.0	0.0025	380	224	0.0635	14.29	8.42
21.5	27.0	0.0035	475	281	0.0889	17.86	10.57
24.0	26.0	0.0050	643	357	0.1270	24.18	13.42
20.1	26.0	0.0060	728	398	0.1524	27.37	14.96
22.4	27.5	0.0085	992	551	0.2159	37.30	20.72

In Table I the converted values h [mm], F_1 and F_{N1} [kp] are also given. The paper referred to contains values for $\gamma = 35°, 40°, 45°$, too.

The constants calculated from the equations

and $F_1 = AA_1 + B$

$F_{N1} = A_NA_1 + B_N$

as well as the standard deviation of the linear approximation are shown in Table II. The mean shear stress values and the data needed for their calculation are found in Table III.

Table II

Constants and standard deviation of linear force equations

γ°	A (kp/mm²)	B (kp)	$s_r^{F_1}$ %	A_N (kp/mm²)	B_N (kp)	s_r^{FN1} %
25	151.15	4.62	0.93	79.41	3.33	1.85
35	106.49	2.56	1.86	32.92	1.82	2.45
40	99.87	2.47	1.41	20.23	3.16	3.16
45	87.18	3.20	1.25	12.09	1.88	2.75

Table III

Values calculated from the constants of linear equations

γ°	Φ°	ξ°	$(\Phi + \xi)^\circ$	R (kp/mm²)	$\cos(\Phi+\xi)$	$\sin\Phi$	$\bar{\tau}_\Phi$ (kp/mm²)
25	21.8	27.7	49.5	170.0	0.65	0.37	41.0
35	31.8	15.7	47.5	112.0	0.68	0.53	40.7
40	36.6	11.3	47.9	102.0	0.67	0.58	39.5
45	40.7	7.9	48.6	88.0	0.66	0.65	38.0

The value of $(\Phi + \xi)$ is seen to be nearly constant $(48.5^\circ \pm 1^\circ)$. If angle γ increases by 20°, Φ will increase and ξ decrease to the same extent. The angle $(\Phi + \xi)$ will rotate by as much as the increase of angle γ. For a growth by 20° of angle γ, R will be smaller by 48%, $\bar{\tau}_\Phi$ by 7.5%.

Summary

Specific cutting forces calculated from known force equations have no physical meaning. Physically meaningful indices require consideration of coherent forces and areas of cut. Between their ranges of validity, linear force equations may deliver coherent force to area values. Their constants and the angle Φ of the shear plane yield the mean shear stress.

References

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