DETERMINATION ACCURACY OF RELEASED HEAT BY CYLINDRICAL TENSILE SPECIMENS AT THE POINT OF CONTRACTION

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1. Introduction

In determining the failure of plastic metals, the amount of mechanical (external) work is to be known as a function of strain. Also the part of residual mechanical energy in the metal, transformed into heat is to be known. According to the first principal law of thermodynamics, the energy proportions are expressed by the common relationship:

$$W_M = W_S + W_T \tag{1}$$

where:

W_M mechanical (external) work
W_S residual mechanical energy in the metal
W_T residual mechanical energy transformed into heat.

According to (1), a certain external work can be constant, though the ratio of the part transformed into heat to that retained in the metal may vary between wide limits. The internal energy variation can be calculated if the other two energy members are known.

Tests have been done to determine the part of external work transformed into heat on the basis of thermoelectric differences. The thermo-couple was prepared from 0.08 mm Ø copper-constantan wires. The wires were welded on to the surface of the tensile specimen, thus the two branches of the thermocouple were connected through the specimen material.

To eliminate thermal conduction problems, an adiabatic change of state had to be maintained. Also, the change of state ought to be kept at a constant rate during deformation. These requirements could be met by tension carried out in short steps, and by selecting suitably refined stages. Deformation in small steps provides for slight caloric effect. Practically, the test deformation stages are to be determined so that the caloric effect can be recorded at a reliable accuracy.

High purity copper was used for the measurements. The basic material was twice remelted in a 10^{-6} Hgmm vacuum, by electron beam melting. The resulting 60 mm ∅ copper bar was cold rolled to 30 mm Ø. After rolling, the surface was treated by cutting and then holded for 3 hours in protective argon atmosphere at 600°C. The heat-treated bars were processed into tensile specimens 7 mm ∅, with a 20 mm long cylindric shaft and a threaded clamping head. Deformation of these specimens consisted in applying stepwise tensile stresses and the released heat was measured as outlined above.

2. Checking the accuracy of surface temperature gauging

2.1. *Estimating the character of thermal processes*

During stepwise deformation of cylindrical tensile specimens and at failure, the surface temperature values were measured at the point of contraction, after each tension, as a function of time.

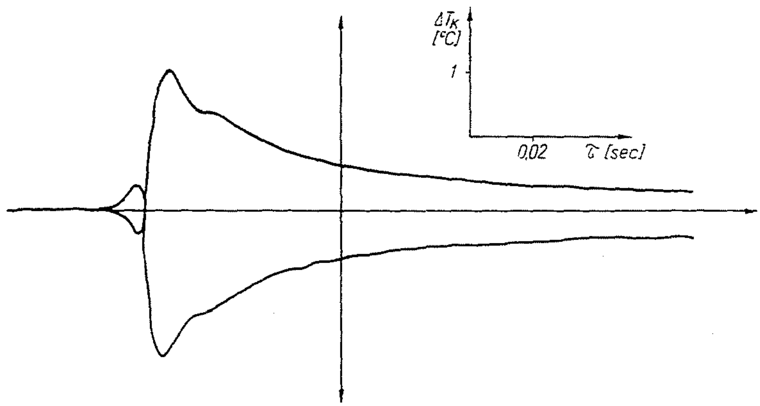

Fig. 1. Temperature variation at room temperature vs. time, in broken copper

Figure 1 (No. 335) is an excellent example for the character of a series of diagrams. The temperature determination after each deformation lasted $\tau_0 = 0.2$ sec. The vertical intercepts of the diagram are proportional to the values of ΔT_k, where

$$\Delta T_k = T_F - T_f$$

T_F specimen surface temperature at the point of contraction,
T_f ambient temperature: 20°C.

The cooling of specimen as a thermal process is to be regarded as unstationary. While the specimen is cooling, its temperature decreases, and thus heat is transferred to the ambient medium.

An object of our experiment was to follow the accuracy of temperature measurement by approximate calculations.

It was assumed that that part of external work was transformed into heat which was used to heat the deformed cross-section of the specimen to an evenly distributed initial temperature at the moment of deformation. Thus the initial condition was:

at the moment of $\tau = 0$, $T = f(\bar{r}) =$ constant.

A third-grade boundary condition was assumed, namely that the surface heat flux density was a function of the surface temperature. Accordingly, the heat conducted from inside the specimen to the surface passes from the surface to the atmosphere by heat transfer, in conformity with the Newtonian law of cooling.

As our assumed diagrams, similar to that in Fig. 1, are characteristic of the temperature field resulting from the spatial thermal conduction and the surface heat transfer, our calculations were based on this general case.

In case of no heat source, the well-known common Fourier differential equation of thermal conduction is the following:

$$\frac{\partial T}{\partial \tau} = \frac{\lambda}{c\varrho}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2} + \frac{\partial^2 T}{\partial x^2}\right].$$

The temperature distribution in the contraction cross-section of the tensile specimen as a function of time, could be determined by several methods.

The mathematical solution of the problem has been discussed by CARSLAW and JAEGER [1].

For the solution, the method of finite differences elaborated by SCHMIDT [2] for cylinders may be used. The time-dependent temperature field formation in the specimen can be estimated by geometrical plotting.

Development of the unstationary temperature field can also be determined in hydraulic model tests, this method is discussed by FORBERT [3].

A fourth method applied for determining the temperature distribution in the studied cross-section, namely, the solutions by GRIGULL [4] was published as nomograms, $\vartheta_c = f(Fo, Bi)$ and $\vartheta_\omega = f(Fo, Bi)$, where

$$\vartheta_c = \frac{T_f - T_{\text{medium}}}{T_f - T_{\text{original}}},$$

and

$$\vartheta_\omega = \frac{T_f - T_F}{T_f - T_{\text{original}}},$$

with notations in Fig. 2.

Directly, the nomograms only result in the solution of a non-stationary heat conduction case for a cylinder of infinite length. At the same time, however, they are also characteristic of temperature field values as a function of temperature, three-dimensional heat conduction and surface heat transfer, found in tests on contraction cross-section, as a function of time. To take into consideration the three-dimensional heat conduction in calculation, the tensile test piece had to be regarded as a short cylinder. Thus, calculations will deliver the developing temperature field as a solution of the general case.

It is a well-known fact that, as smaller the length of a cylindrical body, as bigger the surface to volume ratio of the body, and higher the rate of heat propagation. According to MIHEEV [5], short cylinders can be tested as

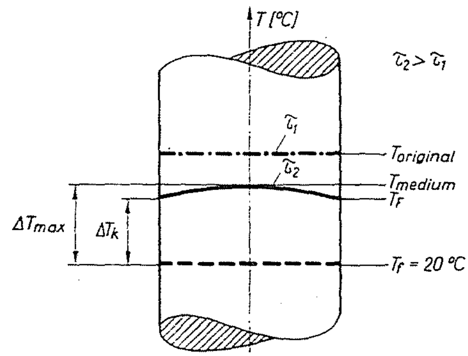

Fig. 2. Schematic diagram for notating temperature variations in the tensile specimen

bodies resulting from the intersection of a cylinder and a perpendicular plate of the thickness equal to the length of the cylinder.

In the above case, the relative surface temperature at the cylinder mid-length equals the product of the relative surface temperature ϑ_ω of the infinite length cylinder by the relative temperature ϑ_c at the middle of the infinite plate. Similarly, the relative temperature at the cylinder mid-axis equals the product ϑ_c for the infinite cylinder, by ϑ_c, for the infinite plate.

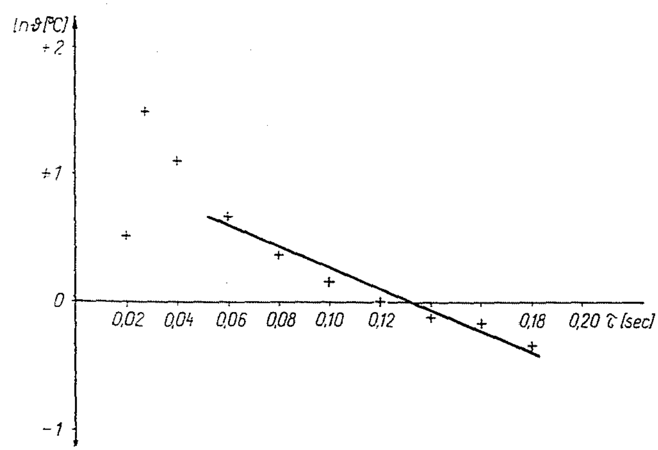

Fig. 3. Variation of ln ϑ vs. time

As there are no thermal sources in the tensile specimen, and both the ambient temperature, T_f, and the surface heat transfer factors (α) are approximately constant in time, three cooling stages can be discerned:

1. the stage of inordinate process,
2. the regular stage,
3. the stage of thermal balance.

As indicated in diagrams $\ln \vartheta = f(\tau)$, plotted on basis of the individual $\Delta T_k = f(\tau)$ diagrams, for each case of tension, the regular stage developed some 0.04—0.05 sec after tension, where function $\ln \vartheta = f(\tau)$ is well described by rectilinear negative direction tangent [5]. In this case $\vartheta = T - T_f$, where $T_f = 20°C$, and in our case $T = T_F$ (Fig. 3, No. 335).

2.2. *Time-dependent variations in temperature field in the contraction cross-section of tensile specimen*

To determine the variations in the temperature field, the symbols according to Fig. 2 were introduced and the values ϑ_c and ϑ_ω referring to the tensile specimen — as a short cylinder — were determined after each tension.

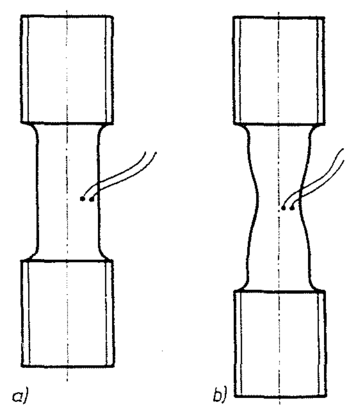

Fig. 4. Shape of the tensile specimen up to the limit of uniform true strain *(a)* and following the beginning of contraction *(b)*

In our calculations a cylinder length of $l = 0.02$ m was considered up to the limit of uniform true strain $\lambda_0 = 0.31$ while beyond this limit, approximating the deformed volume by a cylinder, the length of the cylinders was considered to equal the prevailing contraction diameter. (The actual shape of the tensile specimen is as in Fig. 4.)

Then, the functions $\vartheta_c = f(Fo, Bi)$ and $\vartheta_\omega = f(Fo, Bi)$ were solved by means of the relative Grigull-diagrams.

The solution requires knowledge of Bi and Fo with

$$Bi = \frac{\alpha R}{\lambda_{\text{specimen}}} \quad \text{and} \quad Fo = \frac{a\tau}{R^2}$$

where R radius of the actual contraction cross-section. (For No. 335, $R = 0.00176$ m.) For the tested copper specimen $\lambda = 340$ kcal/m, h, °C, $a = 0.412$ m²/h, while the α value is to be calculated as follows:

The specimen heat content is transferred to the atmosphere by free flow heat transfer after each tension. For determining the heat transfer factor, the observation by MIHEEV was involved, namely that the Nu-number can be produced in every case of this type of heat transfer, as a function of the product of the Gr-number by the Pr-number:

$$Nu = C(Gr \cdot Pr)^n$$

where the C and n values can be established in relation to the $Gr \cdot Pr$ product [5]. The α value can be established in knowledge of the thermal characteristics of the applied medium:

$$T_{\text{ref.}} = \frac{T_f + T_F}{2} \cong 20\ °\text{C}$$

$$Gr = \frac{\beta \cdot g \cdot \Delta T \cdot 1^3}{\nu^3} = \frac{3.43 \cdot 10^{-3} \cdot 9.81 \cdot 5 \cdot 8 \cdot 10^{-6}}{(15.1 \cdot 10^{-6})^2} = 5 \cdot 10^3 .$$

For air: $Pr = 0.71$.

With the above data

$$Gr \cdot Pr = 3.5 \cdot 10^3 = 0.35 \cdot 10^4.$$

As

$$5.10^2 < Gr \cdot Pr < 2 \cdot 10^7, \text{ thus } c = 0.54 \text{ and } n = 0.25.$$

On basis of the above

$$Nu = 0.54\,(0.35 \cdot 10^4)^{1/4} = 4.12$$

$$Nu = \frac{\alpha \cdot 1}{\lambda\ \text{air}}$$

thus

$$\alpha = \frac{Nu \cdot \lambda\ \text{air}}{1} = \frac{4.12 \cdot 0.0257}{2 \cdot 10^{-3}} = 52\ \text{J/m}^2,\ \text{s},\ °\text{C}, = 44.4\ \text{kcal/m}^2,\ \text{h},\ °\text{C}.$$

Therefore

$$Bi = \frac{44.4 \cdot 0.00176}{340} = 0.000227 \text{ and thus } \frac{1}{Bi} = 4400 .$$

The Fo-number for a cylinder of infinite length was calculated assuming $a = 0.412$ m²/h constant while $d = 2R$ values were measured after each tension, and the results compiled in Table 1.

Table 1

No.	$d(mm)$	λ_0	ΔT_k (°C)
:	:	:	:
:	:	:	:
335	3.53	1.3692	4.667
:	:	:	:
:	:	:	:

The τ values were taken uniformly as $\tau = 0.2$ sec, though the ΔT_k values pertaining to $\tau = 0.02$ sec were applied for determining functions $\vartheta_c = f(Fo, Bi)$ and $\vartheta_\omega = f(Fo, Bi)$. With this step, the certainty and reliability was increased, namely calculating with values $\tau = 0.02$ sec, the resulting Fo numbers would be so low that the differences of $\Delta T_{max} - \Delta T_k$ would be even less. Similarly, certainty and reliability was increased and thus the real error of temperature gauging was increased too, by using in the calculations the maximum $\frac{1}{Bi} = 100$ value still present in Grigull nomograms instead of the resulting real $\frac{1}{Bi}$ values.

The partial results for No. 335 and the final T_{medium} and $T_{initial}$ are indicated in Table 2.

Table 2

No.	ϑ_c ∞ cylinder	ϑ_ω ∞ cylinder	$\frac{1}{Bi}$ plate	Fo plate	ϑ_c plate
335	0.89	0.86	100	7.1	0.93

No.	ϑ_c finite cylinder	ϑ_ω finite cylinder	ΔT_k	T initial (original)	T medium
335	0.826	0.80	4.667	25.83	24.82

2.3. *Checking the accuracy of surface temperature gauging*

For checking the accuracy of temperature gauging, the formation of surface temperature gauging "error" as a function of the true strain was sought for. The true strain values, λ, can be calculated after each tension and, by definition, the "error" of temperature gauging is:

$$h = \frac{\Delta T_{\max} - \Delta T_k}{\Delta T_k} \cdot 100\,\% \text{ (see Fig. 2).}$$

Table 3 indicates the calculation results for our example, No. 335.

Diagram $h = f(\lambda_0)$ in Fig. 5 was plotted from data in Table 3. The value of h is seen to increase with the increase of plastic strain, hence the accuracy

Table 3

No.	T_{original}	T_{medium}	T_P	ΔT_k	$\Delta T_{\max}$	$\Delta T = \Delta T_{\max} - T_k$	h %
:	:	:	:	:	:	:	:
335	25.83	24.82	24.667	4.667	4.82	0.153	3.28
:	:	:	:	:	:	:	:

of surface temperature gauging decreases. Maximum value of $h = h_{\max} = 3.68\%$ was observed in case of $\lambda_0 = 1.64$.

Analysing the above results it has to be emphasized, however, that the real "error" and deviation, in the surface temperature and the specimen axis

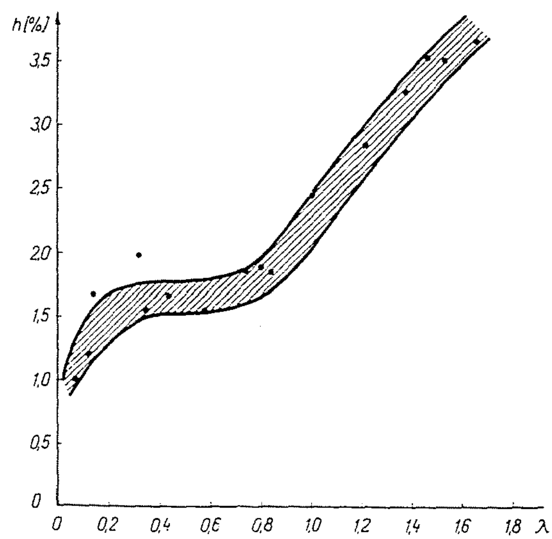

Fig. 5. Error in temperature gauging vs. true strain

temperature (in the contraction plane) can be but *less* than the values obtained by us.

As, due to the short time of strain application, the exact, analytical solution would have caused too many problems, a few approximations have been made which undoubtedly increased the real "error". The resulting values (Fig. 5) give a high-accuracy, reassuring answer as to at what certainty the surface temperature values could be relied on for the metal-physical processes in the specimen centreline.

Summary

One of the methods for testing the resistance to failure of metals is based on the changes in the energy conditions. According to the first main law of thermodynamics, a part of the external work (W_M) producing plastic strain increases the internal energy (W_S) of the metal, while the other part is transformed into heat (W_T). The energy retained in the metal is characteristic of the response of the metal to failure. The changes in the material struture can be examined, determining this energy-member, more exactly. The internal energy changes from the external work and the part transformed into heat can be determined by the relationship $W_M - W_T = W_S$. The reliability of this method depends on how accurately the heat released during strain can be determined. In our experiment we defined the errors inherent with thermoelectric difference thermometry in tensile tests, on high-purity copper specimens, i.e. at their point of contraction. Accordingly, the part of mechanical work W_T transformed into heat at the point of contraction can be determined at an accuracy of 3.5% even in the least favourable case.

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