CRANK GEAR AND MASS BALANCE OF RADIAL ENGINES WITH EVEN NUMBER OF CYLINDERS

By

GY. DEZSÉNYI

Department of Gas-Engines and Automobiles, Technical University, Budapest

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Presented by Prof. Dr. A. JUREK

A very important characteristic for the operation or functioning of internal combustion engines is the balance of the inertia forces. This usually improves with the increase of the number of cylinders and it grows worse with the decrease of the connecting rod length. With the increase of the ratio $\lambda = r/l$ inertia force harmonics of higher order get a highly increased role.

The resultant effect of the respective inertia force components may be different for multi-cylinder engines. With certain arrangement of crank and cylinders they get added up, in other cases they decrease each other's effect, sometimes the value of the resultant may be zero.

It may be considered relatively favourable, when the effect of the resultant force can be eliminated by the inertia force of a balancing mass making simple motion for example circulair motion.

Multi-cylinder radial engines usually have such advantages.

Radial engines with four-stroke cycle operation have been built mainly for aircrafts, as the cooling conditions of cylinders radially situated in the same plane are extremely favourable.

Two-stroke cycle operation is rather unusual with radial engines, even though it has been tried with aircrafts as well (Deutz). From 1947 the Nordberg Manufacturing Co. (USA) have built eleven cylinder two-stroke cycle stationary radial engines for power plants.

Main data of the engine:

Bore:	355 mm (14″)	$N = 1700$ HP (1250 KW)
Stroke:	406 mm (16″)	$n = 400$ rpm

$V_h = 40$ litres — Performance of the scavenging pump:

$V_H = 440$ litres — $N_ö = 110$ HP

Hence the shaft horsepower of the engine is:

$$N_e = 1590 \text{ HP } (1170 \text{ KW})$$

The engine operates as a gas engine with compression or spark ignition, but it is constructed for operation with dual fuel also.

By special construction of the crank gear of the engine, the great disadvantage caused for radial engines in their mass balancing by the orbit of pins of articulated rods deviating from circular motion, could be eliminated [1, 2, 3].

With the Nordberg crank gear fitted with externally toothed epicyclic gear there is no master connecting rod but the pin-middle orbit of the 11

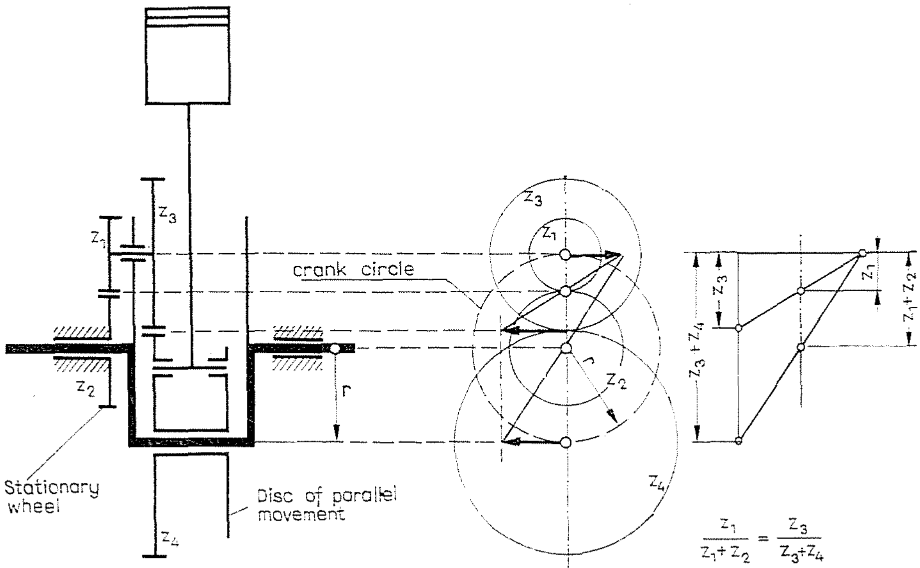

Fig. 1. Nordberg two-stroke cycle 11 cylinder radial engine. $N_e = 1700$ HP; n = 400/min.; $V_H = 400$ lit.; S/D = 406/355 mm

articulated rods, linked to the central disc is circular, because the disc moves in a path parallel with itself. This is determined on the basis of Fig. 1 by the given ratios of tooth numbers of gear wheels [4, 5].

The resultant primary force effect is of constant magnitude:

$$\sum_{1}^{11} F_{\mathrm{I}}^{k} = \frac{Z}{2} F_{\mathrm{I}} = \frac{11}{2} F_{\mathrm{I}}$$

$$F_{\mathrm{I}} = m_h \cdot R \cdot \omega^2$$

Hence it is possible that this force which is always radial is balanced together with the similarly radially rotating force by the centrifugal force of the balance weight placed on the extension of the crankweb.

From the secondary unbalanced forces no resultant force arises.

Recently the Nordberg firm changed over to the application of engines with 12, that is, even number of cylinders. With essentially the same cylinder dimension they increased the performance to 2100 KW [6].

This, likewise two-stroke cycle radial engine with cylinders lying in a horizontal plane drives a generator or a pump.

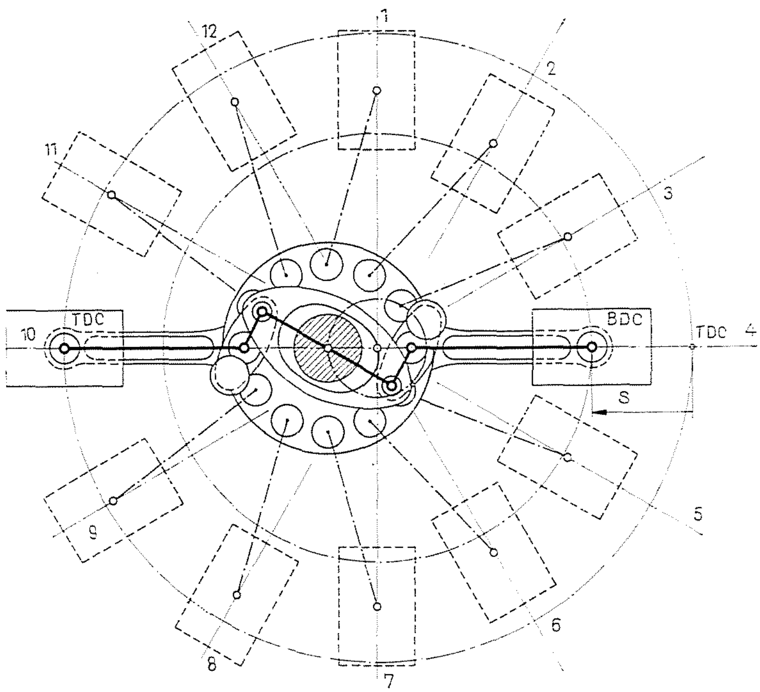

Fig. 2. Crank gear of the Nordberg two-stroke cycle 12 cylinder radial engine

Among the various modifications of the recent type, with increased number of cylinders, the most important one is the control of the motion of the central disc which serves the linking of the 12 connecting rods and is fixed on the crankpin of the engine.

The expensive differential pinion control is left out and the position of the central disc is determined by the laws of "quadrinomial antiparallelogram" motion known from the theory of mechanisms. The aim of further investigation is to find out how far and under what geometrical conditions, this solution approximates most the circular motion of the centre of the connecting rod pins — what is fulfilled at the driving gear of 11 cylinder engine — and by this very good mass balance conditions of the engine.

The structural arrangement of the crank gear is shown in Fig. 2.

The operation of the system can be followed from the simplified drawings of Fig. 3 wherein the notation also is given.

Into the connecting rod heads, linked with two pistons $d—d'$ moving opposite to each other, the bolt pins are rigidly fixed, but they can turn inside the disc k. The webs $r—r'$ are rigidly linked to these bolt pins, on the extension of which there are the balance weights $e—e'$, which compensate the mass forces derived from the motion of the link c, between the webs.

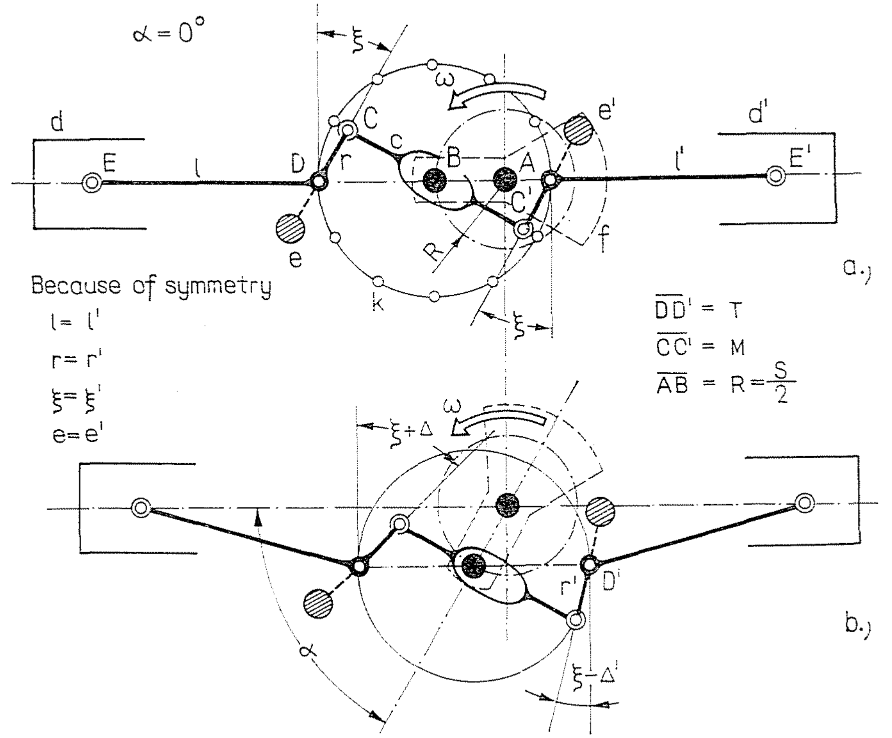

Fig. 3. Diagram of the crank gear of the Nordberg two-stroke cycle 12 cylinder radial engine. *a)* driving pistons in dead centre position, *b)* general position
Notation: A centre of rotation of the crankshaft; B centre of crank pin; C bolt pin; D casing of connecting rod head in disc k; E piston bolt pin; l driving connecting rod; r moving arm; c linking member; d piston; e oscillating balance weight; f balance weight serving for the balance of rotating and primary forces; k steering disc (disc with parallel movement)

In Fig. 3/a in inner dead centre position of the left side piston, the webs are situated symmetrically as a mirror image. As the crank turns to an α angle, the condition shown in Fig. 3/b arises. From the central position the connecting rods move together with the webs connected to them and with the link. Under given geometrical conditions a given position of disc k — which serves also for encasing the connecting rods — belongs to this.

A general arrangement of the antiparallelogram system can be seen in Fig. 4.

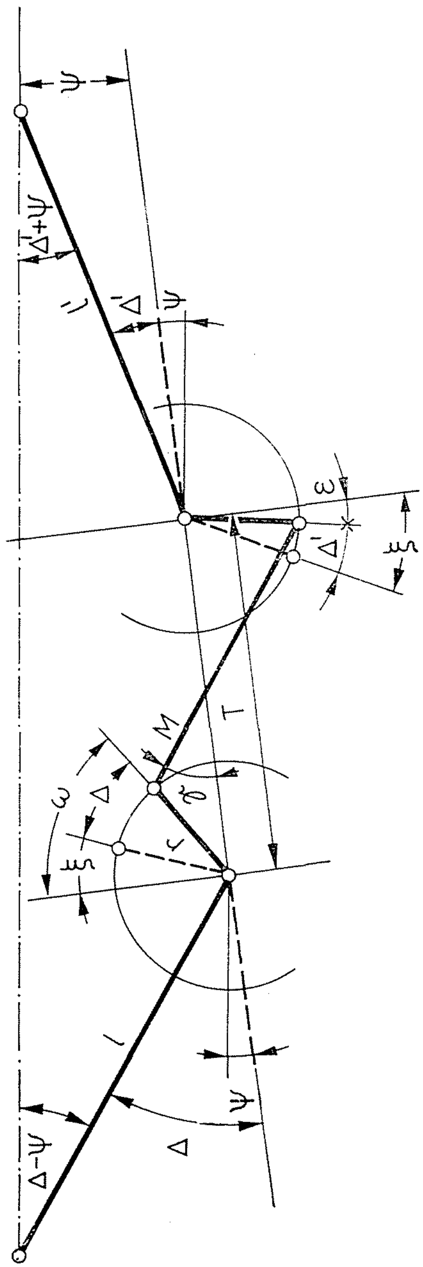

Fig. 4. Characteristic data of the crank gear in general position

On the basis of the notation the aim is to determine the change of the angle ψ and by this the position of the driving disc, changing the characteristic dimensions. As a matter of fact, parameters cannot be changed arbitrarily except within the structural possibilities of internal combustion engines. And this will be used in our further calculations. As a first step the relationship of the angles Δ and Δ' must be determined. (In case of exact parallelity $\Delta = \Delta'$ as for example with the 11 cylinder Nordberg engine.)

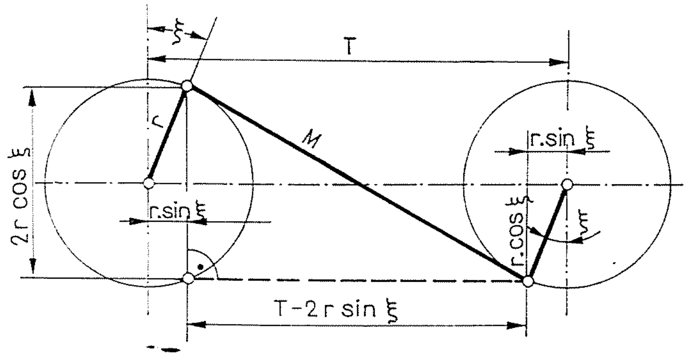

Fig. 5. Dimensions of the antiparallelogram mechanism

Taking the notation of Fig. 4 as basis we can write:

$$r \cdot \sin \omega + M \cdot \cos \gamma + r \cdot \sin \varepsilon = T \text{ and}$$
$$r \cdot \cos \omega + r \cdot \cos \varepsilon = M \cdot \sin \gamma \tag{1}$$

Squaring and arranging Eq. (1) and introducing the characteristic ratios

$$\frac{T}{r} = C \quad \text{and} \quad \frac{M}{r} = C^x$$

the basic equation:

$$\frac{C^x}{2} + C \cdot \sin \omega + C \cdot \sin \varepsilon - \cos \omega \cdot \cos \varepsilon - \sin \omega \cdot \sin \varepsilon - 1 = \frac{C^{x^2}}{2} \tag{2}$$

In case of an antiparallelogram structure symmetric as a mirror image between C and C^x the following relationship can be stated:

$$C^{x^2} = C^2 - 4C \sin \xi + 4 \tag{3}$$

Substituting the latter in Eq. (2) the basic equation can be transformed into:

$$C \cdot \sin \omega + C \cdot \sin \varepsilon - \cos \omega \cdot \cos \varepsilon - \sin \omega \cdot \sin \varepsilon - 1 = 2C \cdot \sin \xi - 2 \quad (4)$$

on the basis of Fig. 4.

$$\omega = \xi + \Delta \quad \text{and} \quad \varepsilon = \xi - \Delta'$$

Let us introduce the following simplifying notation:

$$\begin{aligned} \sin \Delta = A \quad &\text{and} \quad \sin \xi = S \\ \cos \Delta = B \quad &\text{and} \quad \cos \xi = K \end{aligned}$$

By these Eq. (4) can be changed to:

$$\sin \Delta'(A - CK) + \cos \Delta'(CS - B) + CSB + CKA - 2CS + 1 = 0 \quad (5)$$

Using a simple trigonometric relation it can be transformed into a quadratic equation:

$$\begin{aligned} \sin^2 \Delta'[(A - CK)^2 + (CS - B)^2] + \sin \Delta'[2Z(A - \\ - CK)] + Z^2 - (CS - B)^2 = 0 \end{aligned} \quad (6)$$

$$Z = CSB + CKA - 2CS + 1$$

The application of the following new nomenclature may mean further simplification:

$$(A - CK) = f \text{ and } (CS - B) = e$$

With them the value of $\sin \Delta'$ is:

$$\sin \Delta'_{1,2} = \frac{-zf \pm e\sqrt{f^2 + e^2 - z^2}}{e^2 + f^2} . \quad (7)$$

From Eq. (5) $\cos \Delta'$ can also be obtained

$$\cos \Delta'_{1,2} = \frac{-zf \pm f\ \sqrt{f^2 + e^2 - z^2}}{e^2 + f^2} . \quad (8)$$

It can be seen from the relations that the value of Δ' in the antiparallelogram mechanism as a function of Δ is influenced by the intermediate variables

$$\frac{T}{r} \text{ and } \frac{M}{r}$$

as well as the value of ξ. Changing C practically has no importance, for the mechanism is used in a given radial engine, and because of the relative restriction of geometrical dimensions. (For example the ratio crankthrow — connecting rod, positioning of the central disc, etc.) Therefore we shall calculate with the constant value of C taken as 4.

Henceforth Eq. (7) will be used and the relation $\Delta' = f(\Delta)$ will be calculated in tabular form for various values of ξ.

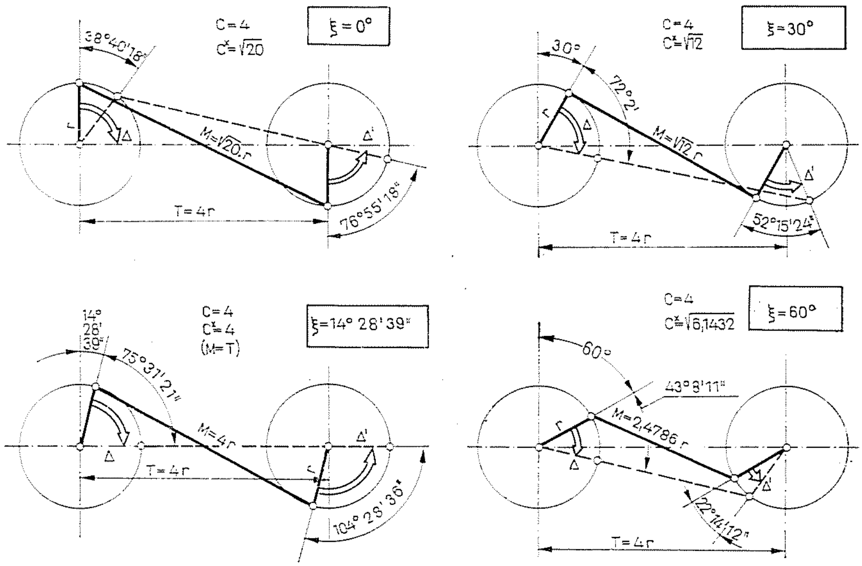

Fig. 6/a. Extreme positions of the mechanism with the starting position $\xi = 0°$
Fig. 6/b. Extreme positions of the mechanism with the starting position $\xi = 14°\ 28'\ 39''$
Fig. 6/c. Extreme positions of the mechanism with the starting position $\xi = 30°$
Fig. 6/d. Extreme positions of the mechanism with the starting position $\xi = 60°$

1) Let us take the positions $\xi = 0$ as the starting point. The independent variable Δ is examined up to the extreme position shown in Fig. 6/a (38°10'18").

The substituting values are:

$$C = \frac{T}{r} = 4 \qquad e = 4.0 - B = -B$$

$$C^x = \frac{M}{r} = \sqrt{20} \qquad f = A - 4.1 = A - 4$$

$$S = \sin \xi = 0 \qquad z = 4.1 \cdot A + 1 = 4A + 1$$

$$K = \cos \xi = 1$$

Of course, the calculations can be made for negative Δ values as well, it means a deflection of the connecting rod in opposite direction with respect to the centre line of the cylinder to $\Delta_{\max} = 76°\ 55'\ 18''$.

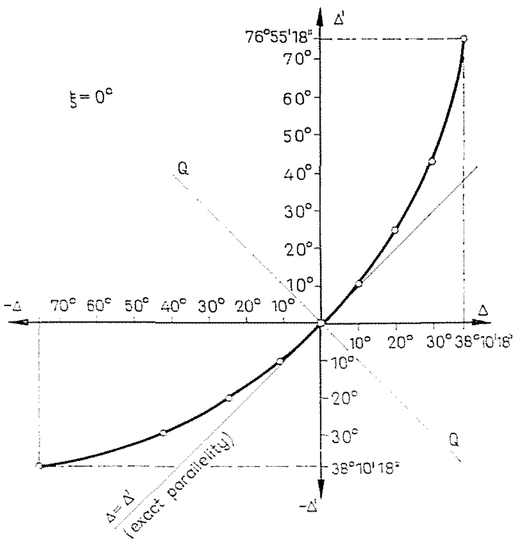

Fig. 7. Displacement conditions of moving arms ($\xi = 0°$)

2) $\xi = 14° 28' 39''$ (Under this condition $C = C^x$ what means simplification of calculations.) Extreme positions can be seen in Fig. 6/b.

The substituting values are

$$
\begin{aligned}
C &= 4 & e &= 1 - B \\
C^x &= 4 & f &= A - \sqrt{15} \\
S &= 1/4 & z &= B + \sqrt{15}\,A - 1 \\
K &= \sqrt{15}/4
\end{aligned}
$$

While representing the results, the symmetry of the construction may be considered, on account of which for example Fig. 10 is symmetric to the axis $Q - Q$. Therefore in the following Δ' will be shown only with change of $+\Delta'$ (Fig. 8).

3) $\xi = 30°$ (Fig. 6/c)

In this position:

$$
\begin{aligned}
C &= 4 & e &= 2 - B \\
C^x &= \sqrt{12} & f &= A - \sqrt{12} \\
S &= 1/2 & z &= 2B + \sqrt{12}\,A - 3 \\
K &= \sqrt{3}/2
\end{aligned}
$$

Extreme positions:

$$\Delta = 72^\circ\, 2'$$
$$\Delta' = 52^\circ\, 15'\, 24''$$

Deviation from the line $\Delta = \Delta'$ is shown in Fig. 9.

4) $\xi = 60^\circ$ (Fig. 6/d.)

$$\Delta = 43^\circ\, 8'\, 11''; \qquad \Delta' = 22^\circ\, 14'\, 12''$$

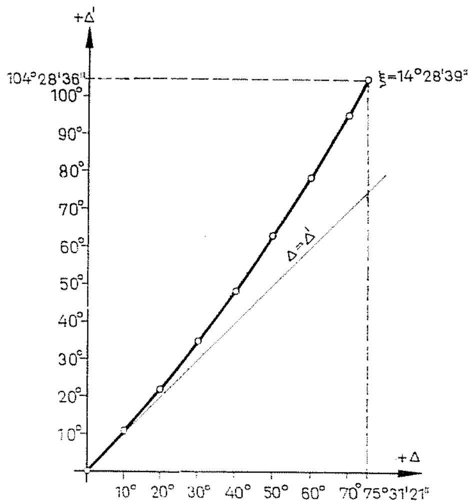

Fig. 8. Displacement of moving arms ($M = T$)

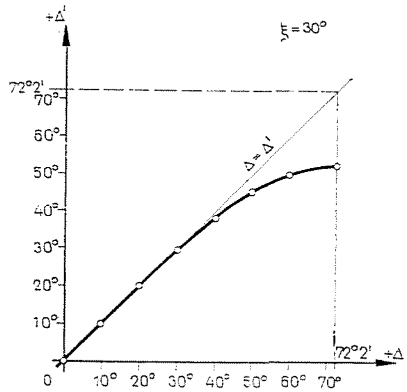

Fig. 9. Displacement of moving arms ($\xi = 30^\circ$)

Here already the values of the extreme positions of the mechanism are considerably decreasing.

$$C = 4 \qquad S = \sqrt{3}/2 \qquad e = 2\sqrt{3} - B$$
$$C^x = \sqrt{6.1432} \qquad K = 1/2 \qquad f = A - 2$$
$$z = 2\sqrt{3} + 2A - 4\sqrt{3} + 1$$

Results can be seen in Fig. 10.

The role of the position of the moving arm becomes especially clear if the results of the calculations made for four various cases are drawn together. (See Fig. 11.)

$\xi = 0$. In this case the extreme condition arises quickly and the value of $\Delta' - \Delta$ is significant already at the beginning.

$\xi = 14^\circ\, 28'\, 39''$ is an interesting geometrical combination, as $M = T$ and hence after $\Delta = 75^\circ\, 31'\, 21''$ movements take place according to the laws of the symmetrical parallelogram (see the straight line $\overline{N}\overline{M}$ in the upper part of Fig. 11).

$\xi = 30^\circ$ seems to be the most favourable case.

First the diagram clings to the straight line $\Delta - \Delta'$ and even later the deviation increases only moderately. The case $\xi = 60^\circ$ is the least favourable from all, in every respect.

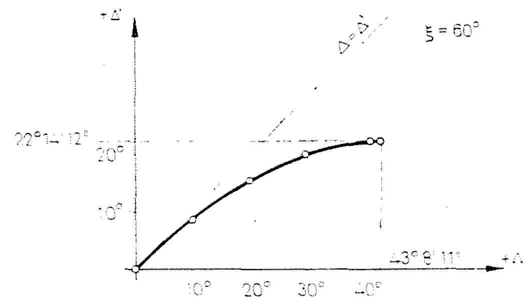

Fig. 10. Displacement of moving arms ($\xi = 60^\circ$)

It can also be seen from the combined diagram that in case of any ξ in the position $\Delta = 0$ $\Delta' = 0$ and from the cases shown $\xi = 14^\circ\, 28'\, 39''$ at $\Delta = 90^\circ$ $\Delta = \Delta'$.

By this the points O, M, and N as well as the area enclosed by the lines connecting them acquire special significance because between $\xi = 14^\circ\, 28'\, 39''$ $(T = M)$ and $\xi = 30^\circ$ $(T = \sqrt{4/3} \cdot M)$ at any ξ we confine a value (apart from $\Delta = 0$) where exact parallelity is realized, that is, $\Delta = \Delta'$.

Considering the actual geometrical conditions of the crank gear, let us see which value fulfils the condition $\Delta = \Delta'$ when probably $\Delta = 15^\circ$ is the maximum.

The calculation is made for four intermediate values taken.

From the values $\Delta' - \Delta$ plotted against ξ we get a straight line the intersection of which with the axis ξ means the condition $\Delta' - \Delta = 0$. Using the property of similar triangles with the notation of Fig. 12, and expressing the result in seconds:

$$y = \frac{514800}{254} \approx 2027''$$

corresponds: $33'\, 47''$, that is

$$x = 26'\, 13''$$

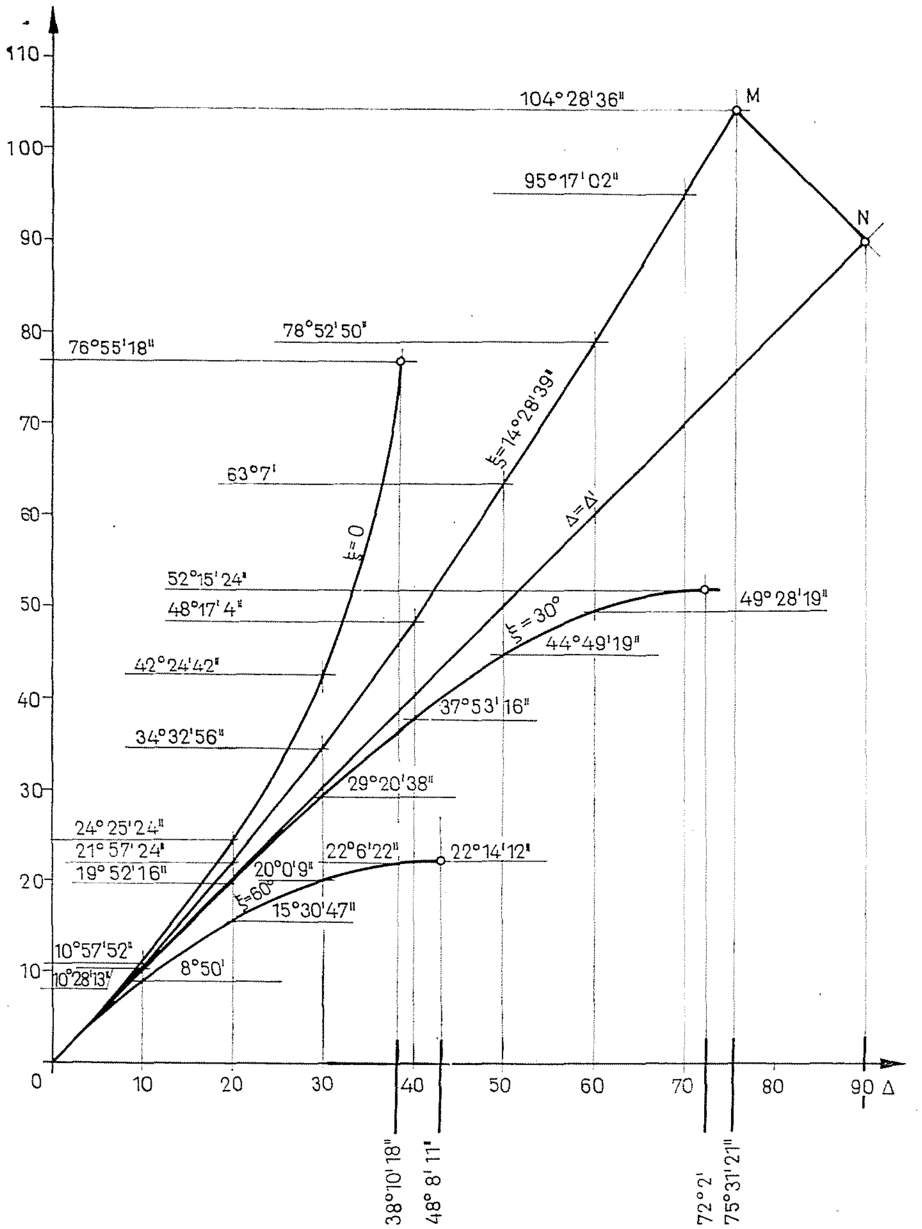

Fig. 11. Representation of displacement conditions of antiparallelogram mechanism fitted with moving arms, of different basic positions, in a combined diagram

The required value of ξ at a deflection of $\Delta = 15°$

$$\Delta' = 15°; \qquad \xi = 29° \, 26' \, 13''$$

The result becomes very clear if $\Delta' - \Delta$ is represented with an accuracy of seconds.

Fig. 13 shows well that in case $\xi > 30°$ till $M = T$ by adjusting the moving arms as a function of the magnitude of the deflection Δ, the error $\Delta' - \Delta$ can be decreased to minimum.

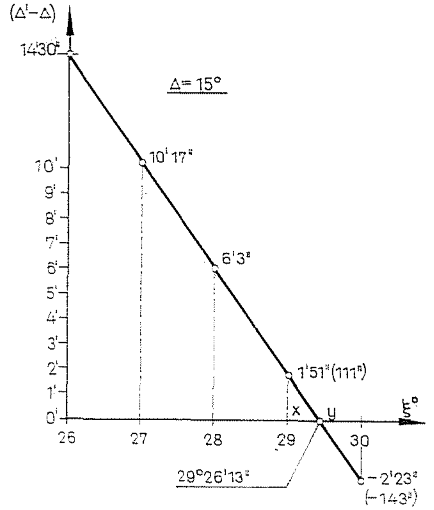

Fig. 12. Finding out the equal displacement position of the two moving arms

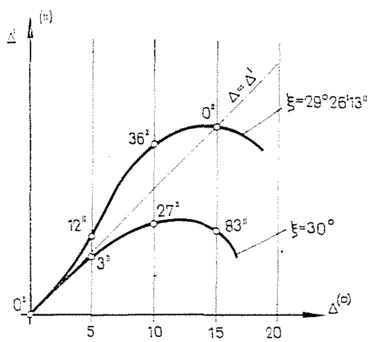

Fig. 13. Representation of $\Delta - \Delta'$ in seconds

If the geometrical arrangement of the construction is like in the case described in this paper, that is the displacement of the moving drive arm from its basic position does not reach $\Delta = 15°$, choosing $\xi = 30°$ seems to be the most favourable, that is to say, when the centre line of the link is perpendicular to the moving arms. In case of the Nordberg engine the deflection of connecting rods from the centre line of cylinders — with exactly parallelly moving disc —

$$\text{tg}\, \Delta_{\max} = \frac{203}{900} = 0.2255$$

$$\text{arc tg}\, 0.2255 \cong 12° \, 45''$$

(length of the connecting rod: 1 = 900 mm)

With such relatively small angular deflection $\xi = 30°$ is the most suitable in our example.

Further on the question is how the relation of the other geometric dimensions affects the relation $\Delta = f(\Delta')$ (Fig. 5). Only the value of C can influence the result.

By increasing it the error decreases, for if

$$C = \infty; \qquad \xi = 0°; \qquad C = C^x$$

and the angle of the moving arm and the link is $\pi/2$. Then the displacement of the moving arm would be the same, that is

$$\Delta = \Delta'$$

The distance (T) of the casing of the connecting rod heads in practice cannot be raised considerably over the value given in the example, because

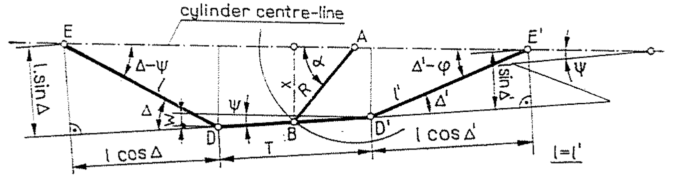

Fig. 14. Showing the position of the central disc represented by $D - D'_3$ in a general position of the crank mechanism (for finding out the displacement angle ψ)

the outer dimensions are already large on account of the large number of cylinders of the engine.

The maximum outer dimension at present (on the basis of Fig. 2) is the maximum distance of the lower edges of cylinders from each other.

After making sure $\Delta = f(\Delta')$ turning off ψ of the central disc with respect to the centre line of cylinder, can be determined with the notations of Fig. 14 made according to Fig. 4.

The following equations can be written:

$$\operatorname{tg} \psi = \frac{l(\sin \Delta - \sin \Delta')}{l(\cos \Delta + \cos \Delta') + T} \tag{9}$$

$$x = \frac{l \sin(\Delta - \psi) + l \sin(\Delta' + \psi)}{2} . \tag{10}$$

Obtaining $\cos \Delta'$ from equation (5).

Writing C^{xx} as $\frac{1}{r}$

$$\operatorname{tg} \psi = \frac{C^{xx}(A - \sin \Delta')}{C + C^{xx}\left[B - \dfrac{\sin \Delta'(A - CK) + Z}{CS - B}\right]} . \tag{11}$$

The latter expression is suitable for finding out the relation $\psi = f(\Delta)$ for given or assumed values.

In accordance with the equations already given:

$$\xi = 30^\circ \qquad CS = 2 \qquad \text{and}$$

$$C = 4 \qquad C^x = \frac{l}{r} = \frac{900}{180} = 5$$

$$CK = \sqrt{12} \qquad (1 = 900 \text{ mm, assumed value})$$

Doing the tabular calculation degree by degree from $\Delta = 0$ to $15°$ the result is given in Table I.

Table I

Δ	tg ψ	$\psi^{(11)}$
0	0.00000	0
1	0.00000	0
2	0.00000	0
3	0.00000	0
4	0.00000	0
5	0.00000	1
6	0.00001	2
7	0.00001	2.5
8	0.00002	4
9	0.00003	6.2
10	0.00004	9.7
11	0.00007	14.0
12	0.00010	20.0
13	0.00014	29.0
14	0.00019	39.0
15	0.00025	52.0

Representing the deviations of the central disc from exact parallelity in seconds it can be proved that the deflection is very small (see Fig. 15). As the deflection of connecting rods does not reach even 13°

$$\psi_{\max} < 29''$$

And this value cannot be considerably larger than that obtained, as the constructional dimensions or ratios determining the same do not change in the order of magnitude.

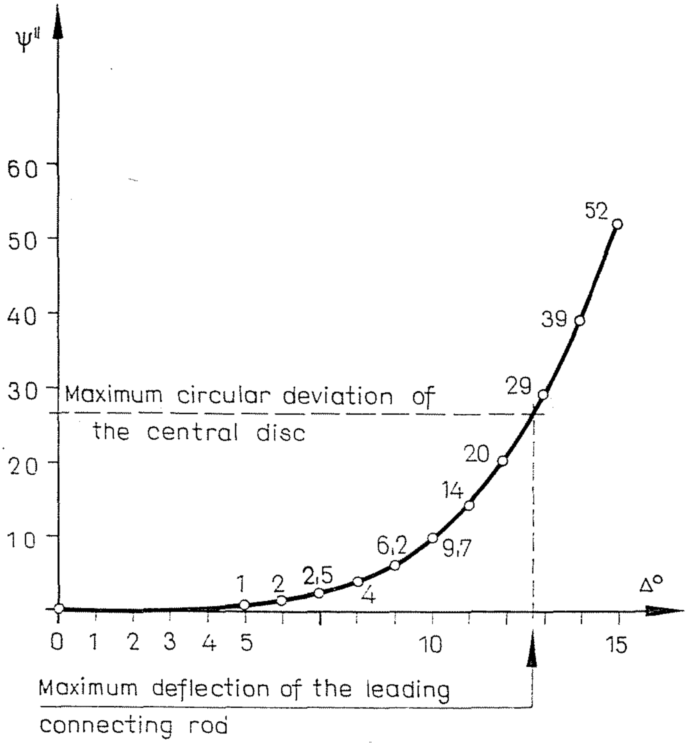

Fig. 15. Representation of the function $\psi = f(\Delta)$ based on tabular calculation

If we take the dimension $T = 720$ mm as basis for the engine of our example, in this case the deviation of the central disc from parallelity according to Fig. 14 is

$$W = T \cdot \operatorname{tg} \psi = 720 \cdot 0.00014 = 0.1 \text{ mm}.$$

The movement of point B is determined by the turning off of the crank. The distance of point B from the centre line of cylinders is:

$$x = R \cdot \cos \alpha \tag{12}$$

It can be calculated with equation (10) too, but because of the very small values of ψ it is of practical insignificance.

The deflection of points D and D' from the centre line EE' differs from X by $\pm W/2$. It is only 0.05 mm, therefore the orbit of the other (10) connecting rods hardly deviates from the exact circular orbit of point B. The turning off is 0.02% of the magnitude of the crankthrow.

Thus our investigation has proved the result that the new system crank gear of two-stroke cycle radial engines with even number of cylinders is practically equivalent to the driving gear of the more complicated engine, with a central disc driven by an epicyclic gear.

The resultant of the primary forces — similarly to that of the 11 cylinder engine — is always radial and of constant value, and that makes it possible to balance with rotating balance weight (Fig. 3, mark f).

$$\sum_{1}^{12} F_{\mathrm{I}}^{k} = \frac{z}{2} F_{\mathrm{I}} = 6F_{\mathrm{I}}.$$

The resultant of the secondary forces is zero.

The masses $e\ e'$ shown in Fig. 3/a balance the effect of forces of the link c, deriving from its relative deflection with respect to the central disc.

The antiparallelogram mechanism discussed here can be used with other machines too if deflections are not too big for practically "exact" parallel movement, in places where simultaneously big loads have to be transferred.

Summary

In this work the author has investigated the crank gear of the two-stroke cycle diesel engine with even number of cylinders. In the course of the investigation — which was done first of all in order to find out the mass balance conditions of the engine — he has kinetically analyzed the crank mechanism of the new system. Within the limitations of the geometrical conditions, he has chosen the most suitable condition and for this he has proved numerically that instead of the solution formerly applied with radial engines with odd number of cylinders, the construction discussed herein is simpler and practically equivalent to it.

References

1. Biezeno, u. Grammel: Technische Dynamik. J. Springer, Berlin, 1939.
2. Schrön, H.: Die Dinamik der Verbrennungskraftmaschine. Springer, Wien, 1947.
3. Kimmel: Die freien Massenkräfte des Sternmotors. Ing. Arch. **11**, 424 (1940).
4. Jurek, A.: Belsőégésű motorok. Tankönyvkiadó, Budapest 1961.
5. MTZ **11** 47—49 (1950).
6. MTZ **28** 509—510 (1967).

Dr. György Dezsényi, Budapest XI., Műegyetem rkp. 24., Hungary