HOW TO DRAW A TANGENT TO ANY POINT OF THE CENTER POINT CURVE

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The center point curve is a stand-by of the interpolative design method of mechanisms [e.g. 1—5, 8, 9]. The application of computer shows marked progress in this field. To determine the center point curve computers giving the results not by tabulating but by drawing the curve wanted are well applicable. Such computers will not generally be used in Hungary for a long time yet. This is why it is reasonable today to consider accuracy of the well known drawing methods. To infinitely densify the points determined by drawing does not absolutely minimize errors. It would be more useful to determine tangents for some given points, or for points drawn at a due accuracy. The aim of this paper is to give the designers a method for drawing a tangent to any point of the center point curve.

The type of the center point curve with a double point is to be derived by means of a mechanism (Fig. 1) well known from the literature [2]. Joint L proceeds along the middle line v and the bar fixed to the bar LQ at an angle β goes always through the focus F. (It can be found in the literature how to draw the center point curve, the middle line and the focus [e.g. 2].) Point Q draws a center point curve with a double point in this case.

The fundamental idea of the tangent drawing comes from the generalization of this principle.

In this connection it is useful to recall one drawing method of the center point curve.

It is well known that the center point curve can also be drawn as consecutive intersections of the coordinated members in a pencil of radii and a series of non-concentrical circles [e.g. 2, 5]. The central line of the series of circles is the middle line of the center point curve and the holding point of the pencil of radii is the focus of the center point curve. Any line of the pencil of radii intersects the middle line in a point being the center of a circle crossing the line mentioned above in two points of the center point curve. The type of the series of circles is characteristic to the type of the center point curve. A one-parted center point curve without double point is determined by an elliptical series of circles; a one-parted center point curve with a double point by a parabolical series of circles; and a biparted center point curve by a hyperbolical series of circles.

The type of the series of circles can be determined by means of the middle line, the focus and any two points of the center point curve (Fig. 2). On Fig. 2a, b, c the focus (F), the middle line (v) and two poles (O_{12} and O_{32}) of the center point curve are given. In all of these three cases the middle line is crossed by the line FO_{12} at point O_1 and by line FO_{23} at point O_2. Points O_1 and

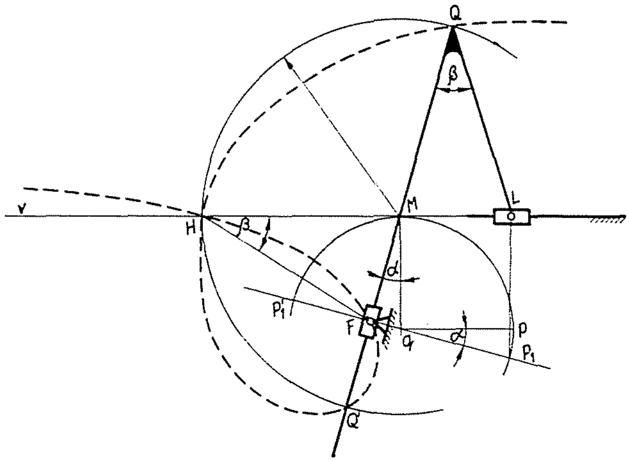

Fig. 1

O_2 are the centers of the circles k_1 and k_2, resp. Circles k_1 and k_2 pass through point O_{12} and point O_{23}, resp. Circles k_1, k_2 and their power line are members of a series of circles with v as a central line.

On Fig. 2a there is an elliptical series of circles for k_1 is not crossed by k_2, therefore the power line h is to be drawn yet. As h is the central line to the conjugated series of circles so circles k_1 and k_2 crossed by any circle k_3 the common chords of k_1, k_3 and k_2, k_3 are crossed in the power line h perpendicular to line v. The point e is cut out from the circle k_2 by the Thales circle drawn at distance $\overline{HO_2}$. Distance $\overline{He}$ is the radius of the circle k crossing the middle line v at point circles A_0 and B_0 of the elliptical series of circles. Every member of the elliptical series of circles crosses the circle k perpendicularly.

The series of circles in Fig. 2b is parabolical for circle k_1 is touched by circle k_2. The power line h passes through this very point and is perpendicular to middle line v. Every member of the parabolical series of circles passes through the intersection point of middle line v and power line h.

The series of circles in Fig. 2c is hyperbolical for circle k_1 is crossed by the circle k_2 at points A and B. The power line passes through points A and B. Every member of the hyperbolical series of circles passes through

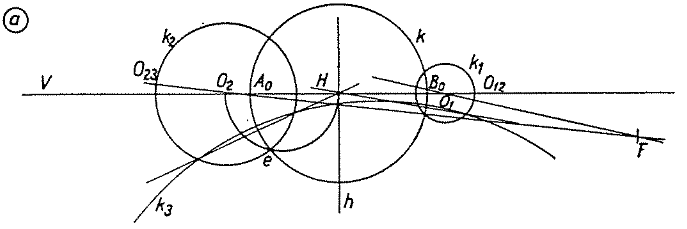

Fig. 2a

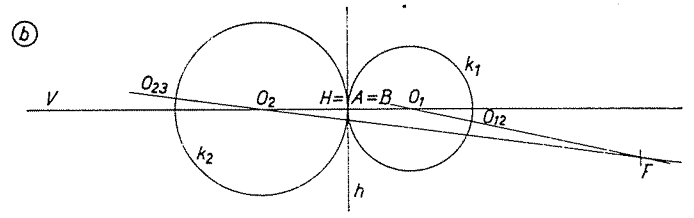

Fig. 2b

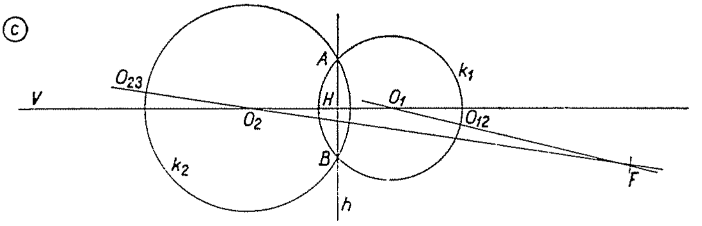

Fig. 2c

points A and B. In all the three cases (Fig. 2a, b, c) point H is the intersection point of lines v and h. On Fig. 2b point H coincides with the double point.

The characteristics of the elliptical (Fig. 3a), parabolical (Fig. 3b) and hyperbolical (Fig. 3c) series of circles are summarized as:

$$\overline{FH} = \text{constant} \qquad QML \sphericalangle = HMF \sphericalangle$$

$$\overline{HM} = \overline{QM} = \overline{Q'M}; \qquad \overline{ML} = \overline{MF}$$

From the characteristics listed above and got from the drawing it follows:

$$\overline{HF} = \overline{QL} = \text{constant}; \qquad MHF \sphericalangle = MQL \sphericalangle$$

As these conditions are enough for the mechanism in Fig. 1, theoretically it is possible to design a mechanism similar to that in all three cases (Fig. 3a, b, c) (Fig. 4). But while the point $Q = 1$ (Fig. 4b) draws directly the center point curve with a double point, in cases Fig. 4a and Fig. 4c points *1* and *1'*

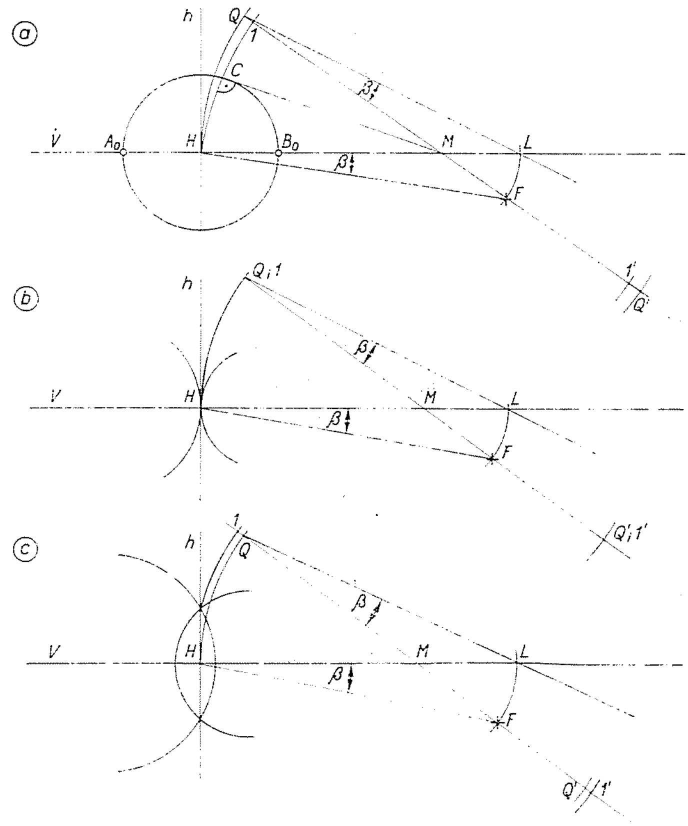

Fig. 3

of the one-parted center point curve without double point and the biparted center point curve are to be drawn by the help of the known method as used in Fig. 4.

In the case of Fig. 4a points Q and Q' are always farther and in the case of Fig. 4c nearer to the point M than points *1* and *1'* of the center point curve. As distance $\overline{Q1} = \overline{Q'1'}$ is variable the center point curve can really be drawn by this mechanism only in the case of $\overline{Q1} = \overline{Q'1'} = 0$ when the center point curve is one-parted with a double point. But as points *1* and *1'* are always on line QM and are always to be considered as ones belonging to member LQM of the fictitious mechanism so this theoretical mechanism can well be used for drawing tangents. For instance, to draw a tangent to point *1* it is necessary to determine pole P_1 in the intersection point of the lines passing through point F and perpendicular to line FM, and passing through L, and perpendicular to line ML, respectively. Tangent t_1 is perpendicular to pole radius $P_1 1$. Similarly tangent t_1' is perpendicular to pole radius $P_1' 1'$.

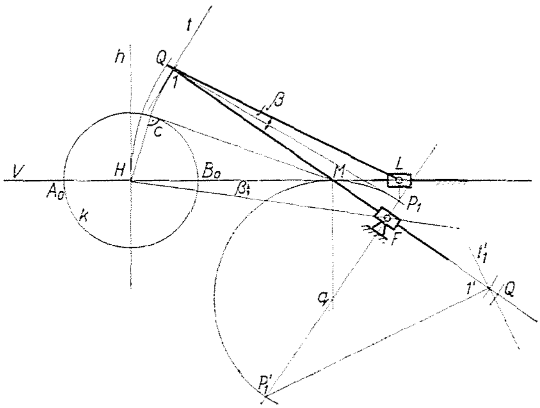

Fig. 4a

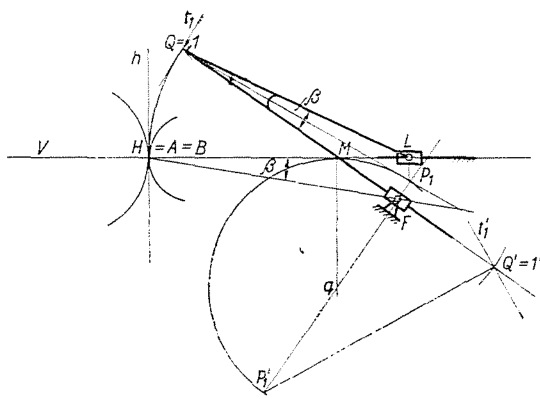

Fig. 4b

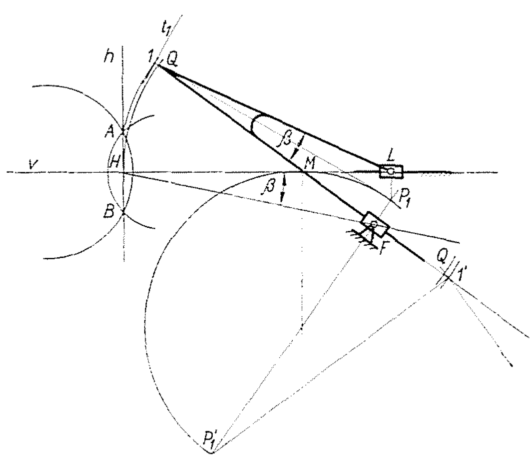

Fig. 4c

Poles P_1 and P_1' are simple to draw (Fig. 4) by learning from Fig. 1 that:

$$\overline{Mq} = \frac{\overline{FM}}{\cos\alpha} = \varrho\,; \quad \overline{qp} = \overline{ML} = \varrho\cos\alpha$$

So the intersection point q of the lines passing through points M and F and perpendicular to middle line v and FM, respectively, is the center point of a circle of radius $\varrho = \overline{Mq}$ and crossing line Fq at points P_1 and P_1'. The pole radii from P_1 and P_1' are perpendicular to the tangents.

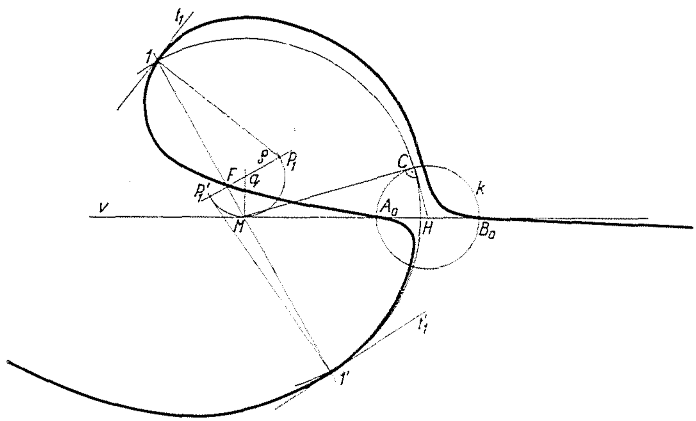

Fig. 5

At last, knowing the line v, point F and the series of circles any point of the center point curve with its tangent can be drawn by the following method:

1. In the case of an elliptical series of circles (Fig. 5): any line passing through point F crosses middle line v at point M. Point c is staked out on the circle k by its tangent passing through point M. Two points of the center point curve (*1* and *1'*) are determined on the line $\overline{FM}$ by intersecting it by a circle with center M and radius Mc ($\overline{M1} = \overline{M1'} = \overline{Mc}$). Line $\overline{Mq}$ is perpendicular to line v; line Fq is perpendicular to line FM. Moreover distance $qM = \varrho$ and points P_1 and P_1' are pointed out on line Fq crossed by a circle with center q and radius ϱ. Thus lines $P_1 1$ and $P_1' 1'$ are perpendicular to tangents t_1 and t_1'. Lines t_1 and t_1' are tangents to the center point curve at points *1* and *1'*.

2. In the case of a parabolical series of circles (Fig. 6): any line through point F crosses middle line v at point M. Points *1* and *1'* of the center point curve are determined on line $\overline{FM}$ crossed by a circle with center M and radius $\overline{MH}$. ($\overline{M1} = \overline{M1'} = \overline{MH}$). Line Mq is perpendicular to line v and distance $qM = \varrho$. Points P_1 and P_1' are determined on line Fq crossed by a circle with center q and radius ϱ. Then lines $P_1 1$ and $P_1' 1'$ are perpendicular to tangents

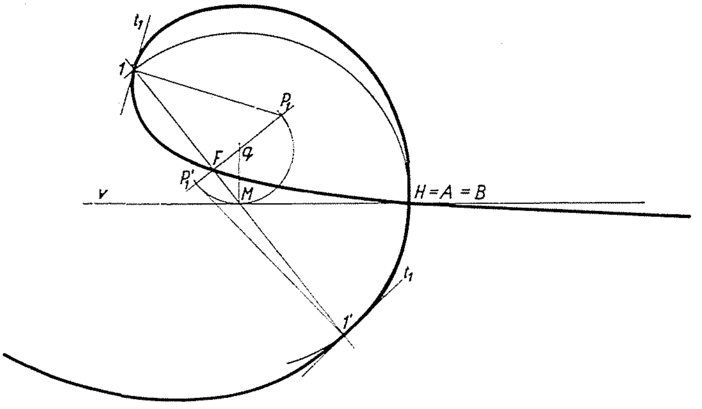

Fig. 6

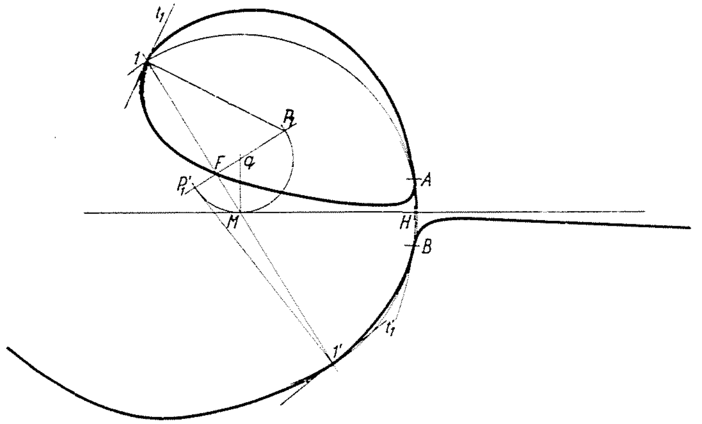

Fig. 7

t_1 and t_1', resp. Lines t_1 and t_1' are tangents to the center point curve at points *1* and *1'*, resp.

3. In the case of a hyperbolical series of circles (Fig. 7): any line through point F crosses line v at the point M. Points *1* and *1'* of the center point curve are determined on line FM crossed by a circle with center M and radius $\overline{MA} = \overline{MB}$ ($\overline{M1} = \overline{M1'} = \overline{MA} = \overline{MB}$). Line Mq is perpendicular to line v and distance $\overline{Mq} = \varrho$. Points P_1 and P_1' are pointed out on line Fq crossed by a circle with center q and radius $\overline{Mq}$. Then lines $P_1 1$ and $P_1' 1'$ are perpendicular to tangents t_1 and t_1', resp. Lines t_1 and t_1' are tangents to the center point curve at points *1* and *1'*, resp.

So the drawing for each type of series of circles is different only up to determining the points of the center point curve. Having these points it is possible to draw a tangent to any of them in the same way. The method is identical whether the center point curve is one-parted with or within a double point, or it is biparted.

Summary

Center point curve is a stand-by of the interpolative design method of mechanisms. Because of some viewpoints it seems to be useful to draw a tangent to any point of the center point curve. The tangent drawing method given by this paper is independent from the type of the center point curve.

References

1. ARTOBOLEVSKY, I. I., LEVITSKY, N. I., TCHERKUDINOV, S. A.: Sintes ploskih mekhanismov. Gosudarstvennoe Isdatelstvo Fisika Matematicheskoy Literaturi. Moscow, 1959.
2. BEYER, R.: Kinematische Getriebesynthese. Springer-Verlag. Berlin, 1953.
3. BURMESTER, L.: Lehrbuch der Kinematik. Verlag von Arthur Felix. Leipzig, 1888.
4. TCHERKUDINOV, S. A.: Sintes ploskih skarnirnorichashnih mekhanismov. Isdatelstvo Akademi Nauk. S.S.S.R. Moscow, 1959.
5. GERONIMUS, I. L.: Geometrichesky appart teori sintesa ploskih mekhanismov. Gosudarstvennoe Isdatelstvo Fisika Matematicheskoy Literaturi. Moscow, 1962.
6. PRIMOSE, E. I.: Plane Algebraic Curves. MacMillan. London, 1955.
7. SALMON, G.: A Treatise on the Higher Plane Curves. University Press. Dublin, 1852.
8. SÁLYI, I. jun.: Mechanizmusok interpolációs tervező módszerének különleges esetei. NME. Magyarnyelvű közleményei XII. kötet. Miskolc, 1963.
9. TERPLÁN, Z.: Mechanizmusok. Tankönyvkiadó. Budapest, 1959.

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