DETERMINATION OF CORRESPONDING POINTS IN THE EFFICIENCY DIAGRAMS OF GEOMETRICALLY SIMILAR KAPLAN TURBINES

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Symbols

Superscripts: ' refers to model.
" indicates the prototype or another model. Quantities without a superscript apply to any member of the turbine family.

A = Point on the efficiency diagram, Fig. 1
C_L, C_D = Effective lift and drag coefficient of the blade section, respectively
D = Runner (tip) diameter
E = Net energy per unit mass, $E = gH$
F = Function, Equ. (13)
F_u = Peripheral force coefficient, Equ. (33)
f = Function, Equ. (19)
g = Normal value of acceleration due to gravity, m/s²
H = Net head across the turbine, m
H_s = Geometric suction head above tailwater, m
k = Cavitation parameter, $k = p_r - p_v/0.5\ \varrho v_r^2$
K = Constant, $K = 6.28/60$ sec/min
l = Reference point height above runner level, where a pressure p_r prevails
M = Hydraulic torque, mkp. For the prototype: the sum of shaft torque and bearing friction, for the model: torque measured by the double-bearing brake
M_{11} = Unit torque, $M_{11} = M/HD^3$
m, m_n, m_Q = Exponents, Equs (24),(26), (28), (29)
n = Runner speed, rpm
n_{11} = Unit runner speed, $n_{11} = nDH^{-0.5}$
n_* = A value introduced upon the suggestion by OSTERWALDER, in the exponent of the HUTTON formulae, Equs (4) and (38)
n_s, n_q = Specific speed, $n_q = nQ^{0.5}H^{-0.75}$, $n_s = nP^{0.5}H^{-1.25}$
p_r = Pressure in the reference point selected at the blades, kp/m²
p_v = Saturated vapour pressure, kp/m²
p_b = Absolute pressure at the suction side, kp/m². Atmospheric pressure for the prototype, and reduced pressure for the model
P = Power output of turbine, HP
Re = Reynolds number of the turbine, $Re = D\sqrt{2gH}/\nu$
Q = Volumetric flow, m³/s
Q_{11} = Unit flow, $Q_{11} = QD^{-2}H^{-0.5}$
Q_* = Flow at optimum efficiency
u = Blade rotation velocity, m/s, Fig. 2
v = Absolute fluid velocity, m/s
v_r = Relative velocity of fluid to blade, m/s, Fig. 2
V = Assessable friction loss fraction (i.e. that applicable to scale effect calculations)
V_R = Friction loss fraction of the runner
x = Discharge coefficient, $x = Q/Q_*$
α = Air-content of fluid
β = Angle between relative velocity and the plane of rotation

Δ	= Dimensionless quantity or that derived therefrom
γ	= Specific weight of fluid, kp/m³
δ	= Dimensionless loss through the turbine, $\delta = 1 - \eta$
δ_R	= Hydraulic loss of the runner
η	= Hydraulic efficiency, $\eta = KMn/QH\gamma$
ϱ	= Fluid density, kps²/m⁴
σ	= Thoma cavitation number, $\sigma = \dfrac{\dfrac{p_b}{\gamma} - H_s - \dfrac{p_t}{\gamma}}{H}$
ν	= Kinematic viscosity of fluid.

Introduction

In the research, design, and operational practice of reaction turbines, the determination of corresponding points on the efficiency diagrams of different machines is often necessary. For example, when the efficiency and cavitation parameters measured with the model are scaled up to the prototype, corresponding points must be determined on the efficiency diagrams of both the prototype and model concerned (Fig. 1). Corresponding points must be calculated, furthermore, when the diagrams of measured values for the model and prototype are available, and the scale effect is to be determined by comparing these diagrams.

In the determination of corresponding points, such operating conditions of the turbines tested are sought for, where, within the turbines flows are dynamically similar. In most cases, however, the geometric similarity of the flow boundaries does not include roughness and clearance formation, the Reynolds numbers of the flows involved also differ and, consequently, the turbines tested do not exhibit such operational conditions where the dynamic similarity would be completely satisfied. It can be, therefore, only approximate which means, in turn, that there are several methods available for the calculation of corresponding points, in accordance with the different principles followed in approximation.

In efficiency scaling up, the corresponding points can be characterized by the following equations, on the basis of IEC recommendations [1]:

Method No I: $n''_{11} = n'_{11}$, $Q''_{11} = Q'_{11}$ (1)

Some authors suggest the same equations for stepping up the cavitation parameter as well [2]. Another method is suggested, however, for both efficiency and cavitation conversions by NECHLEBA [3, 4, 5], CHISTAKOV [6], or SMIRNOV [5], also referred to by HUTTON [7]:

$$\textit{Method No II:} \qquad \frac{n''_{11}}{n'_{11}} = \left(\frac{\eta''}{\eta'}\right)^{0.5}, \quad \frac{Q''_{11}}{Q'_{11}} = \left(\frac{\eta''}{\eta'}\right)^{0.5} \tag{2}$$

SZABÓ [9] adopts this method with the slight modification of taking into consideration in the expression Q_{11} the volumetric efficiency as well which is, obviously, of importance in case of Francis turbines. VAZEILLE [8] suggested rather generalized equation for the calculation of Q_{11}, Equs $(1)_2$ and $(2)_2$ being only special cases of it. The calculation of corresponding points can be performed by further various methods but the majority of professionals employs one of the two techniques described above.

The existence of several methods presents, occasionally, a difficult problem of selection. The present paper will discuss, therefore, which method should be preferred for efficiency and cavitation parameter step up or for comparing model and prototype diagrams. In order to restrict the scope of investigation only Kaplan turbines will be dealt with although it is quite

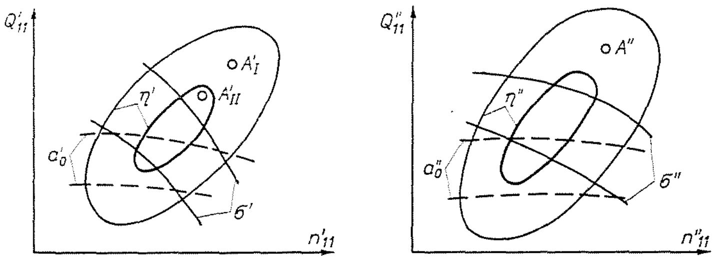

Fig. 1. Schematic diagram of model (′) and prototype (″) turbine efficiency. Points corresponding to A'' are according to method I: A'_{I}, and according to method II: A'_{II}

obvious that a number of our statements will generally apply to reaction turbines as such.

The different methods adaptable for the calculation of corresponding points lead to results of only slight differences and, therefore, if the operational parameters of the machines have to be known only approximately, these differences may be neglected. If, however, exact calculations are needed, the differences between the individual methods may be of considerable significance, as emphasized by NECHLEBA [5]. Let us study, for example, methods No I and No II in case of efficiency stepping up. For this purpose, let us examine point A'' of Fig. 1 where the efficiency is to be determined. Assume that the efficiency scaling up is performed by using the OSTERWALDER formula based on the HUTTON theory:

$$\frac{1-\eta''}{1-\eta'} = \frac{\delta''}{\delta'} = 1 - V\left[1 - \left(\frac{Re'}{Re''}\right)^{1/n_*}\right] \tag{3}$$

where the V and n_* values can be determined according to [10]. According to method No I, the point corresponding to A'' is A'_{I} while, according to method No II, it is A'_{II}. The efficiency measured at points A'_{I} and A'_{II}, respectively, is usually not the same and, consequently, using these two methods will lead to different results for the efficiency value at point A″. As an example, Fig. 2

presents the efficiency values of a tubular turbine after the data by OSTERWALDER. Around the point of best efficiency, the outcomes of the two different methods show an insignificant difference only, in each case. This may be the reason why the problem of corresponding points did not deserve sufficient attention earlier. Further away from the optimum, however, the difference may be of an order of magnitude corresponding to the scale effect of efficiency.

In the step up of cavitation parameters, the two methods lead to different results again because the cavitation numbers measured at points A'_{I}

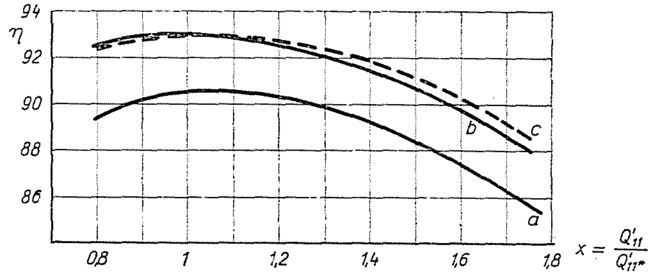

Fig. 2. Efficiency curves of a tubular turbine, based on the data of Ref [10]. The efficiency of the prototype turbine was calculated from that of the model turbine (a) by using Eq. (3) and method I, (b), and then with method II, (c)

and A'_{II}, respectively, are generally different. Comparing the difference to the scale effect reflected by the cavitation parameters calculated, for instance, by using the following NECHLEBA scale effect formula [4]:

$$\frac{\sigma''}{\sigma'} = \frac{\eta''}{\eta'} \tag{4}$$

it may be concluded that the difference is in the order of magnitude of the scale effect again, even around the point of best efficiency. Generally, therefore, it may be stated that particular attention must be paid to the correct selection of the method adopted for the calculation of corresponding points in each case where the specified accuracy of the calculation requires to take into account the scale effects.

It follows that, when using a given scale effect formula, the calculation of corresponding points must always employ the same method which had been used for the determination of the scale effect formula concerned. Thus, for example, the HUTTON efficiency scale formulae have usually method No I associated [1], whereas the NECHLEBA cavitation scale formula is generally connected to method No II [4]. When using a given scale effect formula, therefore, the problem of selecting the method to be adopted can be settled unequivocally.

Thus the actual problem consists of deciding which methods should be employed for the determination of corresponding points in the further development of scale effect formulae as well as in the research thereon.

In deciding which of methods I and II should be adopted, assistance may be obtained by knowing what principles would be manifested through their respective employment from the viewpoint of approximating the dynamic similarity of the flows within the turbines.

Let us consider two turbines which satisfy the geometric similarity requirements given in the IEC recommendations [1]. The operating conditions represented by corresponding points on the efficiency diagrams are called hence "corresponding operating conditions". Whichever method is used for the calculation of corresponding points, in order to ensure geometric similarity as far as possible, the runner blade angle should be the same for both turbines under corresponding operating conditions. Using the definitive equations of n_{11} Q_{11} and η it is easy to show for the corresponding operating conditions determined by method No I and No II that the following Eqs hold good, respectively

Method No I:

$$\frac{\frac{Q''}{D''^2}}{n'' D''} = \frac{\frac{Q'}{D'^2}}{n' D'}, \qquad \frac{H''}{(n'' D'')^2} = \frac{H'}{(n' D')^2} \tag{5}$$

Method No II:

$$\frac{\frac{Q''}{D''^2}}{n'' D''} = \frac{\frac{Q'}{D'^2}}{n' D'}, \qquad \frac{M''}{(n'' D'')^2} = \frac{M'}{(n' D')^2} \tag{6}$$

The methods can be characterized on the basis of these Eqs as follows: Although the dynamic similarity cannot be complete, due to the differences in Reynolds number, relative roughness, etc., the same ratios of the average characteristics of flows given above may be ensured for both turbines, at least. Quantities Q/D^2 and nD are of a velocity dimension, Q/D^2 is proportional to the axial component of the average velocity of the liquid flowing across the runner, and nD to the peripheral velocities. The first equation required by both methods for the determination of corresponding operating conditions provides for the equality of these velocity proportions, which can be correlated to the velocity triangles representing the average flow conditions of the runner blades [5, 12], see Fig. 3. The requirement may be interpreted so as to ensure the equality of the base to height ratios in these triangles. The difference between the two methods is manifested in the second requirements. According to method No I, H is proportional to the square of velocity Q/D^2

or nD, whereas, according to method No II, M is proportional to the square of the same velocities. The importance of these proportions can be readily realized in the special case when they are replaced by equalities. Let us study, therefore, the case when

$$D'' = D', \; n'' = n' \tag{7}$$

apply to the turbines operating under corresponding operating conditions.

These conditions are encountered also in practice when, for example, the effect of Reynolds number variations is tested with the same model turbine, but using fluids of different viscosities [10]. Again the scale effect due to relative roughness variations can be tested by making certain surfaces of the model turbine rougher, and conducting efficiency measurements with the same turbine, by using the same velocity.

With Eqs (7) satisfied, the laws of proportionality will be reduced to equalities:

$$\text{Method No I: } Q'' = Q', \; H'' = H' \tag{8}$$

$$\text{Method No II: } Q'' = Q', \; M'' = M' \tag{9}$$

Thus, in this special case, the two methods can be characterized by the fact that, in spite of the scale effects, both require equal flows and, in addition, method I requires head equality whereas method II the equality of moments. If, owing to the scale effects, the efficiencies are unequal due to the scale effects then, since the equality of the other variables is ensured, in case of method No I the moments, and in case of method No II the heads will be different.

With these considerations known, it can be often decided which method leads to the better approximation of dynamically similar flows. Let us study, for example, the scale effect due to the variations in the relative roughness of the draft tube, with the other turbine parameters kept unchanged. According to the usual approximation in hydraulics, the flows in the individual parts of the machine are considered as independent. If, consequently, the reaction of the draft tube flow conditions on the flow around the runner is neglected then, assuming equal diameter, speed, and discharge, the identical flow conditions at the runner under corresponding operating conditions are ensured by the equality of the moments, while the draft tube efficiency variations will be manifested by the change in the head. These features are characteristic of method II thus, in testing this very scale effect, method No II can be proposed.

With the relative roughness of the guide vanes and spiral casing varied, similar considerations lead to method II again. With, however, the roughness of the runner blades modified, it would be reasonable to assume that the variations were reflected mainly by the moments, and only to a limited extent by the head, whereby method No I might represent a better approximation.

A complex situation is encountered, if the Reynolds number of the turbine varies since, in this case, the flow will vary in each part of the machine. It is reasonable to assume that, with the diameter, speed and flow being constant, upon the effect of Reynolds number variations, the shear stress over the runner blades will change as reflected by the moments, whereas the variation of the hydraulic losses will be manifested in the head. Consequently, in this case a third method which takes into consideration both head and torque changes will be required.

The importance of scale effects due to the variation of the Reynolds number is reflected by its being taken into consideration in most scale effect formulae whereas the effect of, for example, the relative roughness is generally neglected.

This paper has been written in attempt to provide a method adaptable in cases characterized by the variation of the Reynolds number for the calculation of corresponding points. The definitions of the fundamental concepts in scale effect calculations, the general forms of scale effect formulae, and some simple assumptions are used to derive even more special formulae leading, finally, to the expressions of method No III suitable for the practical application. The numerical values are estimated using the HUTTON loss analysis including the test results of FAUCONNET [5] and the results published by OSTERWALDER [10].

Scale effects

The theory of scale effect on water turbines is based on some traditional aspects and fundamental laws involving also their definition [12], [13]. Some of these will be reviewed here briefly because of their close connection to the concept of corresponding points.

From industrial aspects, one of the most important discoveries in the theory of hydraulic machines was the determination of the laws of hydraulic similarity whereupon the characteristics n_{11}, Q_{11} were then introduced. During the initial stage of development, the $n_{11} - Q_{11} - \eta$ diagrams were considered as applicable to the entire family of the geometrically similar turbines, and to all head. At this stage of approximation, the corresponding operating conditions of similar turbines may by characterized by the fact that all the dimensionless variables obtained from the main characteristics of operating conditions, that is, from variables D, ϱ, n, Q, M, E, for example

$$\frac{nD}{E^{0.5}}, \quad \frac{Q}{E^{0.5} D^2}, \quad \frac{M}{\varrho E D^3}, \quad \frac{KnM}{\varrho EQ}, \quad \frac{Q}{nD^3}, \quad \frac{nQ^{0.5}}{E^{0.75}} \tag{10}$$

are constant, while the head and the diameter vary [12]. With the equation $E = gH$ taken into consideration the row designed as (10) reveals all the n_{11}, Q_{11}, M_{11}, ... variables as multiplied by the different powers of the acceleration due to gravity. The characteristics which can be obtained from dimensionless quantities by multiplication by universal constants having dimensions are hence called "characteristics derived from dimensionless quantities". The law referred to above apply to the derived characteristics as well, whereby, expressing it with the characteristics used in practice, the corresponding operating conditions are characterized in this stage of approximation by

$$n_{11} = \text{const.},\ Q_{11} = \text{const.},\ M_{11} = \text{const.},\ \eta = \text{const},\ \ldots \tag{11}$$

The recognition that the efficiency of larger turbines is generally higher has led to efficiency scaling up. In this stage of approximation, the $n_{11} - Q_{11} - \eta$ diagrams apply only to cases of a given diameter and head, and for another head or diameter they must be scaled up. Efficiency changes, however, necessarily result in variations of the other characteristic quantities as well. The following relation holds, for example:

$$\eta = K \frac{M_{11}\, n_{11}}{Q_{11}} \tag{12}$$

where K is constant, and therefore, the efficiency variation will lead to a simultaneous change in at least one of the quantities n_{11}, Q_{11} and M_{11}.

Papers concerning scale effects often present such a definition of the scale effects on efficiency where the problem represented by the variation of the latter characteristics is left unsolved. From practical aspects, these definitions may be regarded as deficient since, as demonstrated above, the efficiency of the prototype cannot be calculated unequivocally without the determination of the n_{11} and Q_{11} variations. When defining scale effects, it is reasonable therefore to consider all characteristics.

As the basis of the scale effect theory on water turbines, the laws expressing the constancy of the dimensionless quantities derived from the main characteristics of the operating conditions as well as the invariability of the quantities derived therefrom by multiplication of constants with dimensions are accepted here. The scale effects are the deviations from these laws which means that the variation of any such characteristic value represents a scale effect. The numerical expressions of the scale effects are called scale formulae, thus Eq. (2) are the formulae of the n_{11} and Q_{11} scale effects.

By such an interpretation of the scale effects, the calculation of corresponding points requires two scale formulae. For example, when using the $n_{11} - Q_{11} - \eta$ diagrams, the n_{11} and Q_{11} scale effect formulae are needed.

In order to calculate the efficiency of the prototype turbine unequivocally, three scale formulae: those of n_{11}, Q_{11} and η are required. It is easy to show that the dimensionless characteristics derived from the main characteristics of the operating condition as well as those derived therefrom can all be expressed in function of n_{11}, Q_{11} and η, for example M_{11} from Eq. (12). Consequently, the scale effect formulae of all the quantities in question can be determined from those of n_{11}, Q_{11}, and η.

The series of dimensionless characteristics can be completed with other variables as well. Including the Thoma cavitation number (σ) here, the fundamental law of the cavitation scale effect theory will be the equation expressing the invariability of sigma, in conformity with the above definition. This is the well-known law of similarity introduced by THOMA. The cavitation scale effects are actually the variations of sigma as interpreted, among others, by HOLL and WISLICENUS [13]. Simultaneously to the efficiency variations, sigma will also reflect a scale effect according to NECHLEBA [4]. For unequivocal cavitation scaling up, the scale effect formulae of n_{11}, Q_{11}, and sigma are required.

In our investigations, the scale effects were interpreted only for dimensionless characteristics, and for those derived therefrom. This is quite suitable for pratical purposes since the quantities with dimensions can be calculated from their dimensionless ratios.

The concept of scale effects covers, furthermore, the determination of the reasons they might be attributed to. Originally, the term scale effect was used to indicate the effects due to the variation of the characteristic length of the machine. Later on, however, the meaning of this expression was modified. Scale effects may be due to any variation of the flow conditions within the turbine, which is usually characterized by a dimensionless factor. The permissible differences from geometrical similarity between prototype and model, as far as the shape of the flow boundaries, their surface roughness, and the size of the clearances are concerned, are determined by IEC specifications [1]. Geometric similarity is understood here as that within these limitations. In addition, the most important dimensionless characteristics typical of the flow conditions are the Reynolds number, and, for cavitation step up the Froude number and the air content. Accordingly, variations in the turbine head and fluid quality (viscosity, air content) cause scale effects even if the geometric scale does not vary [13].

General scale formulae

Let us consider a turbine family which consists of geometrically similar turbines, and an optional quantity Δ, the scale effect of which will be investigated. It may be for example η, or one of the characteristic cavitation numbers. Then the general scale formula for Δ is:

$$\Delta'' = F_{\Delta}(\Delta', n'_{11}, Q'_{11}; Re'', Re', e'', e', \ldots) \tag{13}$$

where Δ'' and Δ' regard two members of the turbine family, respectively. The variables n'_{11}, Q'_{11} are introduced in function F_{Δ} to offer an opportunity for the calculation of the scale effects in different ways at different points of the $n'_{11} - Q'_{11} - \eta'$ diagram, and the variables Re, e, ..., represent the Reynolds number, relative roughness, and all the factors characteristic of the variation of flow conditions in the members of the turbine family. As an example, Eq. (3) multiplied by δ' may be referred to, representing a scale formula for δ, depending on n'_{11}, Q'_{11}, since V is a function of Q'_{11}.

On the same line of reasoning, the general scale formulae for the calculation of the corresponding points are of the following form:

$$n''_{11} = F_n(n'_{11}, Q'_{11}; Re'', Re', e'', e', \ldots)$$
$$Q''_{11} = F_Q(n'_{11}, Q'_{11}; Re'', Re', e'', e', \ldots) \tag{14}$$

For a complete scaling up of Δ, all the three functions F_{Δ}, F_n, F_Q are to be known.

For simplicity the scaling up of Δ', n'_{11}, Q'_{11} pertaining to the turbine characterized by Re', e', ..., resulted in Δ'', n''_{11}, Q''_{11} for the turbine of Re'', e'', ... will be indicated by the symbol $\Delta', n'_{11}, Q'_{11} \rightarrow \Delta'', n''_{11}, Q''_{11}$. Only special functions are suitable for scaling up, since some logical requirements should be satisfied as follows:

(i) If the flow conditions are the same for both turbines, that is: $Re'' = Re'$, $e'' = e'$, ..., then $\Delta'' = \Delta'$, $n''_{11} = n'_{11}$, $Q''_{11} = Q'_{11}$.

(ii) In case of two turbines considered, if scaling up one to the other and the latter to the original gives the quantities

$$\begin{array}{cc} Re',\ e',\ \ldots & Re'',\ e'',\ \ldots \\ \left.\begin{array}{r} \Delta', n'_{11}, Q'_{11} \longrightarrow \\ (\Delta')^*, (n'_{11})^*, (Q'_{11})^* \longleftarrow \end{array}\right\} & \Delta'', n''_{11}, Q''_{11} \end{array}$$

then

$$(\Delta')^* = \Delta', (n'_{11})^* = n'_{11}, (Q'_{11})^* = Q'_{11}.$$

(iii) When three turbines are examined with the following stepping up calculations:

$$\begin{array}{l} Re', e', \ldots \qquad Re'', e'', \ldots \qquad Re''', e''', \ldots \\ \Delta', n'_{11}, Q'_{11} \left\{ \begin{array}{l} \rightarrow \Delta'', n''_{11}, Q''_{11} \rightarrow \Delta''', n'''_{11}, Q'''_{11} \\ \longrightarrow (\Delta''')^*, (n'''_{11})^*, (Q'''_{11})^* \end{array} \right. \end{array}$$

then

$$(\Delta''')^* = \Delta''', (n'''_{11})^* = n'''_{11}, (Q'''_{11})^* = Q'''_{11}$$

By analogy to the equivalencies frequently used in mathematics the above relations could be referred to as the scaling up of Δ and the calculation of the corresponding points are: (i) reflexive, (ii) symmetrical, (iii) transitive.

These requirements could be expressed by the functions F_Δ, F_n and F_Q for example the transitivity means for F_n, that

$$F_n(n'_{11}, Q'_{11}; Re''', Re', \ldots) = F_n(F_n(n'_{11}, Q'_{11}; Re''', Re'; \ldots), \\ F_Q(n'_{11}, Q'_{11}; Re'', Re', \ldots); Re''', Re'', \ldots) \tag{15}$$

and similar relations hold for F_Q and F_Δ too.

Using functional equations such as Eq. (15), or the relationships given above, all scale formulae could be checked with respect to the requirements (i), (ii) and (iii). It is easy to show that the basic equations of method No I and No II satisfy all of these requirements. However, one of the most popular efficiency scale formulae, that of HUTTON for the best efficiency, recommended by the IEC too [1]:

$$\frac{\delta''}{\delta'} = 0.3 + 0.7\left(\frac{Re'}{Re''}\right)^{0.2} \tag{16}$$

fails symmetry and transitivity. The lack of symmetry is not inconvenient in practice, since always the results measured under the smaller Reynolds number are scaled up to the turbine of the higher Reynolds number. Missing the transitivity, however, has essential consequences. Let us consider, for example, two model turbines with the Reynolds numbers Re', Re'', respectively, for which $Re''/Re' = 10$ and by using Eq. (16) $\delta''/\delta' = 0.741$, and a prototype of the Reynolds number Re''', $Re'''/Re'' = 10$, $Re'''/Re' = 100$. Applying Eq. (16) twice

$$(\delta''')^* = \delta' \left[0.3 + 0.7\left(\frac{Re'}{Re'''}\right)^{0,2}\right] = 0.578\,\delta'$$

$$\delta''' = \delta'' \cdot 0.741 = \delta' \cdot 0.741^2 = 0.548\,\delta'$$

Since the values obtained are different, the efficiency of prototype depends on the Reynolds number of the model used for the basis of scaling up calculations. With the intention to eliminate this uncertainty a transformation of the HUTTON formula could be applied. For every turbine family a model will be pointed out, the values Re^*, e^*, ... characterizing the flow conditions in it are called the "normal" parameters. Every full-scale efficiency will be calculated from the data of the model of these normal parameters with the formula

$$\frac{\delta}{\delta^*} = 0.3 + 0.7\left(\frac{Re^*}{Re}\right)^{0,2} \tag{17}$$

It could be reasonable to use the same values for Re^* as had been applied ordinarily for the model tests on which the formula is based. For a couple of turbines this gives:

$$\frac{\delta''}{\delta'} = \frac{0.3 + 0.7\left(\dfrac{Re^*}{Re''}\right)^{0,2}}{0.3 + 0.7\left(\dfrac{Re^*}{Re'}\right)^{0,2}} = \frac{1 + \dfrac{0.7}{0.3}\left(\dfrac{Re^*}{Re''}\right)^{0,2}}{1 + \dfrac{0.7}{0.3}\left(\dfrac{Re^*}{Re'}\right)^{0,2}} \tag{18}$$

This scale formula, derived from the HUTTON one, is similar to that introduced by McDONALD [5]. It satisfies reflexivity, symmetry, and transitivity. As revealed by the numerical values given above, the difference between the full-scale efficiencies calculated from the test results of different model turbines by the HUTTON formula is not significant, consequently, the difference between the results offered by the HUTTON formula and the latter one is in most case negligible. For practical calculations the original HUTTON formula is better. For theoretical considerations, however, Eq. (18) seems to be more suitable.

With normal values fixed, the general forms of the scale formulae can be simplified. Calculating the values of Δ, n_{11}, Q_{11} from the test results of the normal model turbine at the point n^*_{11}, Q^*_{11} given on its diagram, the normal parameters, being constants, can be omitted from Eq. (13) and (14). Thus

$$\begin{aligned} \Delta &= f_\Delta (n^*_{11}, Q^*_{11}; Re, e, \ldots), \\ n_{11} &= f_n ({}^*_{11}, Q^*_{11}; Re, e, \ldots), \\ Q_{11} &= f_Q (n^*_{11}, Q^*_{11}; Re, e, \ldots). \end{aligned} \tag{19}$$

At a given point n^*_{11}, Q^*_{11}, the values of Δ, n_{11}, Q_{11} are the functions of the variables, Re, e, ... only, in accordance with the general scale formula given by COMOLET [11]. In case of different turbine families, the models of which are not geometrically similar, functions f_Δ, f_n, f_Q may be naturally different.

Scale effects due to Reynolds number changes

If nothing but the Reynolds number of the turbine varies, that is, if the other scale effects are neglected, and the basic formulae of the efficiency scaling up are considered at a point n^*_{11}, Q^*_{11} given in the efficiency diagram of the normal model turbine, then all variables but the Reynolds number may be omitted from the general scale formulae (Eq. (19)):

$$n_{11} = f_n (Re), \quad Q_{11} = f_Q (Re), \quad \eta = f_\eta (Re). \tag{20}$$

Function f_η will not be dealt with here but regarded as known since it may be calculated, for example, from Eq. (17). It will only be assumed that function f_η is increasing with Reynolds number increasing. This applies to all efficiencies obtained from the known efficiency scale formulae. It follows that there exists an inverse function of f_η and, consequently, n_{11} and Q_{11} can be expressed as the functions of η:

$$n_{11} = g_n(\eta), \; Q_{11} = g_Q(\eta) \tag{21}$$

Applying these functions to a couple of turbines

$$\frac{n''_{11}}{n'_{11}} = \frac{g_n(\eta'')}{g_n(\eta')}, \quad \frac{Q''_{11}}{Q'_{11}} = \frac{g_Q(\eta'')}{g_Q(\eta')} \tag{22}$$

These equations reveal the general form of the basic equations of methods I and II. Our reasoning thus verified that, if nothing but the Reynolds number variation is reckoned with, then the basic formulae suitable for the calculation of corresponding points can always be written in function of η.

The function in Eq. (22) can be simplified by approximation. Let us assume that functions g_n and g_Q can be expanded into a power series:

$$n''_{11} - n'_{11} = \frac{dg_n}{d\eta}(\eta'' - \eta') + \ldots, \qquad Q''_{11} - Q'_{11} = \frac{dg_Q}{d\eta}(\eta'' - \eta') + \ldots \tag{23}$$

Scale effects are usually small. For this reason, the square and higher powers of $(\eta'' - \eta')$ can be neglected in first approximation. They are already omitted in Eq. (23). Introducing the quantities

$$m_n = \frac{dg_n}{d\eta}\frac{\eta'}{n'_{11}}, \quad m_Q = \frac{dg_Q}{d\eta}\frac{\eta'}{Q'_{11}} \tag{24}$$

will lead to the following form of Eq. (22):

$$\frac{n''_{11} - n'_{11}}{n'_{11}} = m_n \frac{\eta'' - \eta'}{\eta'}, \quad \frac{Q''_{11} - Q'_{11}}{Q'_{11}} = m_Q \frac{\eta'' - \eta'}{\eta'} \tag{25}$$

These represent the first approximations of the following power functions:

$$\frac{n''_{11}}{n'_{11}} = \left(\frac{\eta''}{\eta'}\right)^{m_n}, \quad \frac{Q''_{11}}{Q'_{11}} = \left(\frac{\eta''}{\eta'}\right)^{m_Q} \tag{26}$$

The derivation of these equations does not involve any hydraulic assumptions and, therefore, they may be considered as the general formulae of the calculation of corresponding points in case of Reynolds number variations. The formulae of method I are the special cases of Eq. (26): $m_n = 0$, $m_Q = 0$, while the formulae of method II are $m_n = 0.5$ and $m_Q = 0.5$. In the general formula,

m_n and m_Q may differ for the various turbine families, and depend on at which point of the $n'_{11} - Q'_{11} - \eta'$ diagram was the corresponding point determined. Thus for the efficiency scaling up at a given point, in case of a given turbine family, the f_η formula and two constants (m_n, m_Q) are required.

Calculation of the exponents m_n and m_Q for Kaplan turbines

For convenience the m_n and m_Q exponents are calculated for Kaplan turbines in the special case corresponding to Eq. (7) when two turbines of identical diameter and speed are examined, with only their Reynolds numbers being different. From practical aspects, this may be considered as operating the same turbine with, however, different viscosity fluid like, for example, in the OSTERWALDER experiments [10].

It is the first assumption for the corresponding operating conditions, that

$$Q'' = Q' \tag{27}$$

Methods I and II have also applied this assumption. Here, however, neither head nor torque equality is required but another hydraulic assumption will be specified instead. First, however, the consequences of Eq. (27) are discussed. On the basis of Eq. (7): $Q''_{11}/Q'_{11} = n''_{11}/n'_{11}$ and, therefrom, it is obtained for the exponents in Eq. (26) that

$$m_n = m_Q = m \tag{28}$$

wherefrom, in turn, the formulae adaptable for the calculation of corresponding points are:

$$\frac{n''_{11}}{n'_{11}} = \left(\frac{\eta''}{\eta'}\right)^m , \quad \frac{Q''_{11}}{Q'_{11}} = \left(\frac{\eta''}{\eta'}\right)^m \tag{29}$$

Exponent m is in close connection with the head and torque variations. Taking into consideration that diameter, speed, and flow are identical for both turbines, the following equations apply to the first aproximation:

$$\frac{n''_{11} - n'_{11}}{n'_{11}} = -\frac{1}{2}\,\frac{H'' - H'}{H'} , \quad \frac{\eta'' - \eta'}{\eta'} = \frac{M'' - M'}{M'} - \frac{H'' - H'}{H'} \tag{30}$$

and using Eq. (25):

$$\frac{H'' - H'}{H'} = (-\,2m)\,\frac{\eta'' - \eta'}{\eta'} , \quad \frac{M'' - M'}{M'} = (1 - 2m)\,\frac{\eta'' - \eta'}{\eta'} \tag{31}$$

According to Eq. $(30)_2$, the relative efficiency variation consists of two parts: the relative changes of head and of torque. Their ratio is shown by Eq. (31): a part of $2m$ is due to head variation, and a part of (1—2 m) can be attributed to torque changes. This actually, illustrates the meaning of exponent m.

It will be noted here that a further simple hydraulic assumption and Eq. (25) permit the determination of some bounds for exponent m. In the various hydraulic structures (pipes, elbows, etc.), reduction of the Reynolds number at the same average flow rate usually has two consequences: (i) the flow losses and, (ii) the shear stresses on the flow boundaries will increase. Assuming that this law applies to the corresponding operating conditions of the turbines, then, with the Reynolds number reduced, (i) the head required for the production of the same volumetric flow would be higher and, (ii) due to the increased shear stresses acting on the blades, the torque would be lower. It follows that η will similarly decrease and, thereby, the above equations render for the exponent m that

$$0 \leq m \leq 0.5$$

Thus methods No I ($m = 0$) and II ($m = 0.5$) represent the two extreme cases.

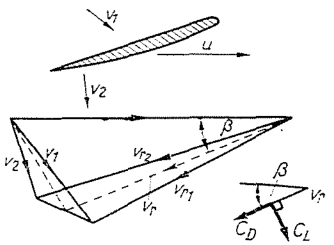

Fig. 3. Runner blade velocity diagram from Ref [5]

For the determination of exponent m another hydraulic assumption can be adapted. For this purpose, the runner blade velocity diagram of HUTTON's well-known paper [5] must be referred to which represents the average conditions on the blading (Fig. 3). Now the incidence angle of the relative velocity is assumed to be the same for the two turbines under the corresponding operating conditions:

$$\beta'' = \beta' = \beta \tag{32}$$

It is the basic idea of this assumption that in the blade cascade theory the average velocity corresponds to that at "infinity", whereby this condition provides for an identical direction for the velocity of "representative blade cascade" of the two turbines at infinity.

The assumption according to (32) may be characterized by its providing, together with our previous assumptions involving equal diameter, speed, and volumetric flow values, for congruent average runner velocity triangles.

Now the torque variation will be estimated on the basis of our hitherto assumptions. According to Fig. 3, the peripheral force coefficient for either of the two turbines is:

$$F_u = C_L \sin \beta - C_D \cos \beta \tag{33}$$

and, therefore,

$$\frac{M'' - M'}{M'} = \frac{F''_u - F'_u}{F'_u} = \frac{(C''_L - C'_L)\sin\beta - (C''_D - C'_D)\cos\beta}{C'_L \sin\beta - C'_D \cos\beta} \tag{34}$$

Variation of the Reynolds number has, in the flow around the profiles under invariable incidence angle conditions, a number of different consequences: variations of the boundary layer thickness lead to changes in the average rate of the flow velocity outside the boundary layer and, at the trailing edges of the profiles, the theoretical rear stagnation point will be displaced [12]. Although these effects ought to be analyzed in detail, in scale effect calculations they are usually neglected [5, 10], and the flow outside the boundary layer is considered as invariable. In conformity to this assumption,

$$C''_L = C'_L \tag{35}$$

is accepted. Factor C_D can be expressed, according to the analysis by HUTTON [5], with the runner losses δ_R:

$$C_D = C_L \delta_R \sin\beta \cos\beta \tag{36}$$

whereby the torque change will be:

$$\frac{M'' - M'}{M'} = \frac{-(C''_D - C'_D)\cos\beta}{C'_L \sin\beta - C'_D \cos\beta} = \frac{-(\delta''_R - \delta'_R)\cos^2\beta}{1 - \delta'_R \cos^2\beta} = \\ = \frac{\eta' \cos^2\beta}{1 - \eta'_R \cos^2\beta} \; \frac{\delta''_R - \delta'_R}{\delta'' - \delta'} \; \frac{\eta'' - \eta'}{\eta'} \tag{37}$$

Loss variations, on the other hand, can be calculated from the following equations:

$$\frac{\delta''_R}{\delta'_R} = 1 - V_R\left[1 - \left(\frac{Re'}{Re''}\right)^{\frac{1}{n_*}}\right], \quad \frac{\delta''}{\delta'} = 1 - V\left[1 - \left(\frac{Re'}{Re''}\right)^{\frac{1}{n^*}}\right] \tag{38}$$

$$\frac{\delta''_R - \delta'_R}{\delta'' - \delta'} = \frac{\delta'_R V_R\left[1 - \left(\frac{Re'}{Re''}\right)^{\frac{1}{n_*}}\right]}{\delta' V\left[1 - \left(\frac{Re'}{Re''}\right)^{\frac{1}{n^*}}\right]} = \frac{\delta'_R V_R}{\delta' V} \tag{39}$$

Eq. (38) have a value of $n_* = 5$ in HUTTON's paper [5]. Here, they have been given a general form as suggested by OSTERWALDER [10]. Eq. (39) reveals that n_* had been excluded from the calculation and, therefore, the derivation applies to any arbitrary n_* value. Comparing Eqs (31) and (37), then substituting the expression of Eq. (39), the value of m will be:

$$m = \frac{1}{2}\left(1 - \frac{\eta' \cos^2\beta}{1 - \delta_R' \cos^2\beta} \frac{\delta_R' V_R}{\delta' V}\right) \tag{40}$$

The quantities of Eq. (34) are given by HUTTON in the function $x = Q'/Q'_*$ where Q'_* pertains to the optimum efficiency (Ref [5], Figs 5, 6, and 7). Thus, by assuming that the value of $\cos \beta = 0.96$, exponent m can be determined for an average efficiency curve calculated from Fig. 5 in [5] also shown here in Fig. 4. The values thus obtained are presented in Fig. 4. In addition, this figure illustrates the values calculated from the test results obtained by Fauconnet (see the Discussion on [5]) and from the test data published by

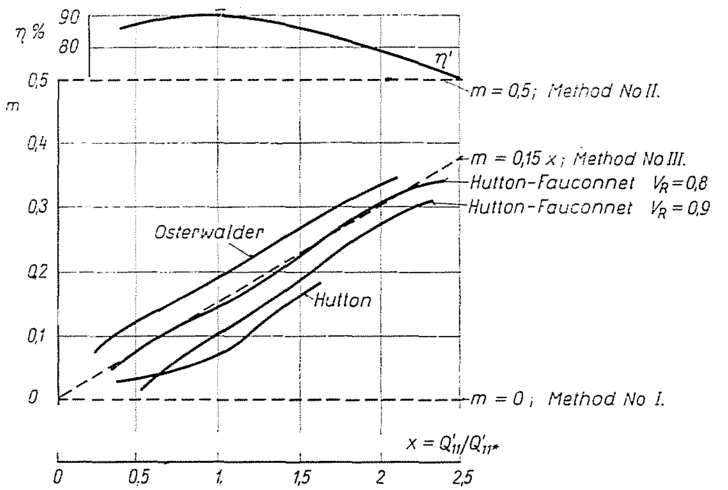

Fig. 4. The model efficiencies assumed for the calculation of m, according to [5], and the value of exponent m in function of the discharge ratio

OSTERWALDER [10], Fig. 6. The character of the curves thus plotted is identical. Their differences may be attributed to the fact that these authors tested different turbine families. A mean value can be obtained by making use of the straight line plotted in the same figure, whereby the values of exponent m can be estimated for any Kaplan turbine family. Accordingly, the fundamental formulae of the new method are:

Method No III:

$$\frac{n''_{11}}{n'_{11}} = \left(\frac{\eta''}{\eta'}\right)^m, \quad \frac{Q''_{11}}{Q'_{11}} \left(\frac{\eta''}{\eta'}\right)^m$$
$$m = 0.15\,x, \quad x = \frac{Q'}{Q'_*}. \tag{41}$$

Some numerical values deserve mention. At the point of best efficiency ($x = 1$), Eq. (41) renders $m = 0.15$ which means, according to Eqs (31), that

30 per cent of the efficiency variations are due to head variation, and 70 per cent to torque changes. In case of large discharge rates for $x = 2$ the opposite situation will exist as now $m = 0.3$ and consequently, the respective percentages of head and torque will be 60 and 40 per cent.

Efficiency scale effect calculations

In case of Kaplan turbines, when the efficiency scale effects are examined, the calculation of corresponding points for the solution of both scaling up and comparison problems is best performed, according to the above analysis, by using method No III. Although the HUTTON efficiency scale formulae were employed earlier with method I, their employment may be suggested when using method III as well. It seems that the basic assumptions accepted for the derivation of these formulae [5, 10] approximate much closer the assumptions adopted in the derivation of method III than the relationships in the application of method I. It should be noted, however, that in case of smaller discharge ratios the difference between the results obtained by any of the two methods, are negligible.

In case of Francis turbines, if by the determination of corresponding operating conditions it is intended to follow the principles of the above analysis, that is, to consider the flow outside the boundary layer as invariable with the Reynolds number varied, and to take only the changes within the boundary layer into account, then the condition of the above analysis as specified by Eq. (27) will already have to be doubted. Due to the variation of the boundary layer thickness, the flow outside it can be invariable within the narrow ducts of the runner only if the volumetric flow would also vary. Thus, in case of Francis turbines, all that is known about the calculation of corresponding points by this theory is its following Eq. (26). For the determination of the values of exponents m_n and m_Q, however, is no suitable loss analysis available.

Cavitation scale effect calculation

If the incipient cavitation number measured by means of a model or that pertaining to efficiency breakdown are to be converted for the prototype turbine then, because of the Reynolds and Froude number variations and other factors such as the effect of air content, certain scale effects must be reckoned with. Let us examine first the cavitation scale effect due to Reynolds number variations. This can be calculated by using the NECHLEBA formula (Eq. (4)), applied together with method II. Cavitation scale effect calculations however, are based again on the assumption that the flow conditions outside the boundary layer do not vary. For this reason, it is much better to use method III for the calculation of corresponding points. Following NECHLEBA's derivation [4] reveals that he employed the equations of method No II only in the last step. Performing this with an arbitrary exponent m, the NECHLEBA formula will assume the following form:

$$\frac{\sigma''}{\sigma'} = \left(\frac{Q''_{11}}{Q'_{11}}\right)^2 = \left(\frac{\eta''}{\eta'}\right)^{2m} \tag{42}$$

This formula applies to Kaplan turbines, and should employ the m-value occasionally selected for the calculation of corresponding points.

Author has suggested two further scale effect formulae for stepping up the cavitation parameters of Kaplan turbines, similarly by making use of method No II [16]. As done with the NECHLEBA formula, these also can be modified for method No III. The formulae thus transformed are:

$$\frac{\sigma''+1}{\sigma'+1} = \left(\frac{Q''_{11}}{Q'_{11}}\right)^2 = \left(\frac{\eta''}{\eta'}\right)^{2m} \tag{43}$$

$$\sigma''+1 = (\sigma'+1)\left(\frac{\eta''}{\eta'}\right)^{2m} + \frac{l''}{H''}\left[1 - \frac{H''\,D'}{H'\,D''}\left(\frac{\eta''}{\eta'}\right)^{2m}\right] - (k'-k'')\frac{v_r''^2}{2g\,H''} \tag{44}$$

Eq. (43) applies to the scale effect due to Reynolds number variations, that is, it may be used for the same purpose as served by the NECHLEBA formula. It is obtained by modifying one of the assumptions in the derivation of the NECHLEBA formula (16). It seems that in case of turbines with a lower head the employment of the latter formula is much more reasonable, particularly as it renders a higher sigma value for the prototype than that offered by Eq. (42) and, therefore, its application provides for a much higher safety.

Eq. (44) is a general cavitation scale formula where the first member indicates the NECHLEBA scale effect due to the variation of the Reynolds number, the second one represents the correction given by the difference of the Froude numbers, and the third member reflects the other scale effects, such as the influence of the Reynolds number on bubble formation or the effect of air entrained by the water. The application of method III permits to treat the latter scale effects like those encountered with the blade cascades of constant incidence angle. That is why this member includes the cavitation coefficient k expressing the local cavitation flow conditions of the runner blade lattice as against the sigma cavitation number characteristic of the cavitation conditions of the machine proper. It deserves mention as an example that, on the basis of the well-known scale effect formula by SHMUGLIAKOV, the influence of the air content on the $(k' - k'')$ factor can be estimated [14] as follows:

$$k' - k'' = \frac{6}{\left(\frac{n_{11}}{100}\right)^2} \left(\frac{\sqrt{\alpha'}}{H'} - \frac{\sqrt{\alpha''}}{H''}\right) \tag{45}$$

Thus, when stepping up the incipient cavitation number of blade cavitation, the adaptation of method III and of Eqs (44) as well as (45) can be suggested. When, on the other hand, converting the cavitation number characteristic of the efficiency breakdown, the effect of air content may be neglected according to VUSKOVIC [15] and, consequently, the approximation $k' - k'' = 0$ may be employed. An objective of cavitation scale effect research may be in the next future to find a more accurate approximation for the $k' - k''$ value.

Application for the calculation of scale effect on specific speed

Problems concerning specific speed were discussed by JONES [16], BARR [17], BOREL [18], and others in various respects. As a corollary of above considerations scale formulae can be deduced for the specific speeds too. Using one of the three methods discussed in this paper for the determination of corresponding points, it can be shown that the specific speed values of two turbines of the same family calculated at corresponding points satisfy the following relationships:

$$\frac{n''_q}{n'_q} = \frac{n''_{11}\sqrt{Q''_{11}}}{n'_{11}\sqrt{Q'_{11}}} \left(\frac{\eta''}{\eta'}\right)^{\frac{3m}{2}} \tag{46}$$

$$\frac{n''_s}{n'_s} = \frac{n''_q\sqrt{\eta''}}{n'_q\sqrt{\eta'}} = \left(\frac{\eta''}{\eta'}\right)^{\frac{3m+1}{2}} \tag{47}$$

Thus method No I $(m = 0)$ offers no scale effect on n_q, but it does on n_s. Using the other methods, however, scale effects on both n_q and n_s are to be reckoned with.

Summary

Scaling up the efficiency and cavitation parameters, or the comparison of their measured values is performed at the corresponding points of the prototype and model turbine efficiency diagrams. Of the several methods known for the calculation of corresponding points, two is discussed here. Method No I: n_{11} and Q_{11} are constants, method No II: n_{11} and Q_{11} are proportional to the square root of the efficiency. For the discussion of the scale effect due to Reynolds number changes, however, a third method is required. Following the definition of the turbine scale effects, and the derivation of the general forms of scale effect formulae, those of method No III will be determined on the basis of the HUTTON loss analysis, and the test results obtained by FAUCONNET and OSTERWALDER.

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