

VARIETIES OF THE PLANETARY GEAR TRAIN TYPES

By

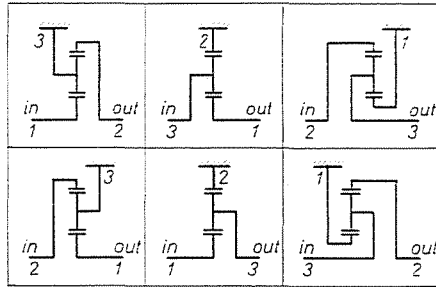
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(Received January 4, 1969)

It is a well-known fact that one can get m_w velocity ratios of six different values having only one planetary gear train (P.G.T.), since, there are six possible varieties for choosing the three main elements of P.G.T. as an input, output, and fixed elements. The main elements of P.G.T. are: two central gears and the arm. Table I shows the six varieties of connection of the main elements of the same P.G.T. to the shafts.

Table I



The simple P.G.T.-s are usually characterized by the b basic (kinematic) ratio, which is the velocity ratio between the relative angular velocities of the two central gears, when the angular velocities of the central gears are related to the angular velocity of the arm:

$$b = \frac{\omega'_2}{\omega'_1} = \frac{\omega_2 - \omega_3}{\omega_1 - \omega_3} \quad (1)$$

The index number 1 and 2 stand always for the central gears, the number 3 for the arm. One can choose either number 1 or number 2 for any of the two central gears, but if the choice had been made it must remain the same.

The b basic ratio can be calculated from the diameters of the gears. In case of auxiliary planet gears (Fig. 1), the formula is:

$$b = - \frac{D_1 D_{12} D_{112}}{D_2 D_{21} D_{221}} \quad (2)$$

The dotted lines in Fig. 1 notify that any of the central gears can be externally or internally geared (sun gear or ring gear). Calculating the value of b the values of diameters must be taken with sign: an external gear has positive sign, an internal gear has a negative one.

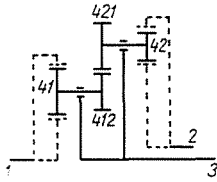


Fig. 1

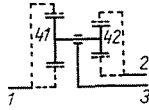


Fig. 2

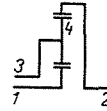


Fig. 3

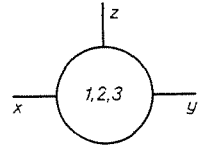


Fig. 4

The auxiliary planet gear disappears if $D_{412} = -D_{421}$ (Fig. 2), in this case

$$b = \frac{D_1 D_{42}}{D_2 D_{41}} . \quad (3)$$

Formula 2 becomes even simpler if $D_{41} = D_{42}$ (Fig. 3)

$$b = \frac{D_1}{D_2} . \quad (4)$$

It is also well-known that the same m_ω velocity ratio can be obtained by several types of P.G.T. if one chooses the right geometrical data and a convenient variety of connections for the main elements of the P.G.T. (Table 2). In this case, it is insignificant from the kinematic point of view what kind of P.G.T. type is used. Therefore, it is practical to make a generalization of the b basic ratio; that is to introduce the B general basic ratio, which can be calculated from b taking the variety of connections into account. It means that having a B general basic ratio one can find a convenient P.G.T. type of a certain b with a certain variety of connection.

In the general case, the three main elements (or their shafts) of the P.G.T. are indicated by x , y and z (Fig. 4), and the formula is:

$$B = \frac{\omega_y - \omega_z}{\omega_x - \omega_z} . \quad (5)$$

Notice that x , y , z and numbers 1, 2, 3 can be disposed in six varieties (Table III).

It goes, without saying, that if one takes shaft x as an input shaft, shaft y as an output shaft and shaft z is fixed, the B general basic ratio will equal the velocity ratio directly: $m_\omega = B$.

Table II

	$D_1 = 4$ $D_2 = -16$ $m_{\omega} = -0,25$
	$D_1 = 6$ $D_2 = 6,57$ $D_{41} = 3$ $D_{42} = 2,43$ $m_{\omega} = -0,25$
	$D_1 = 6$ $D_2 = 2,58$ $D_{41} = 3$ $D_{42} = 6,42$ $m_{\omega} = -0,25$
	$D_1 = 6$ $D_2 = 2$ $D_{41} = 3$ $D_{42} = 4$ $m_{\omega} = -0,25$
	$D_1 = 4$ $D_2 = -20$ $m_{\omega} = -0,25$
	$D_1 = -18$ $D_2 = -20,6$ $D_{41} = 6$ $D_{42} = 8,6$ $m_{\omega} = -0,25$

Table III

It can be stated that the B general basic ratio is more significant in cases when none of the main elements of the P.G.T. are fixed, that is when all three can rotate. This occurs, for instance, in hydromechanical transmissions (Fig. 5). Obviously, the versions, shown in Fig. 5 are identical from a kinematic

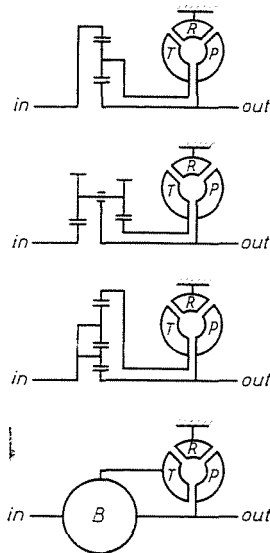


Fig. 5

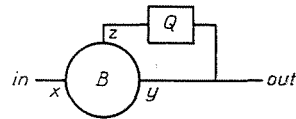


Fig. 6

point of view with each other if the values of B are the same although different types of P.G.T. are used. These kinds of hydromechanical transmission can be easily investigated kinematically by Formula 5.

P.G.T., element z of which is not fixed but connected to element y by some connecting element Q (Fig. 6), and consequently, the angular velocity of element z had been made dependent on the angular velocity of element y , can be called *side-connected* P.G.T. In Fig. 5, a hydraulic torque converter is the *connecting element*. In practice, any kind of machines and mechanisms being able to transmit power or motion (mechanic, hydraulic, electric, magnetic, etc. devices) can be used as a connecting element.

The velocity ratio of a side-connected P.G.T. can be determined by the formula [17]:

$$m_{yx} = \frac{\omega_y}{\omega_x} = \frac{B}{1 + \frac{B-1}{m_{yz}}}, \quad (6)$$

where m_{yz} is the velocity ratio of the connecting element: $m_{yz} = \frac{\omega_y}{\omega_z}$, which can be either constant or variable. In Fig. 5, for instance, where the turbine is connected to shaft y :

$$m_{yz} = m_H = \frac{\omega_{\text{Turbine}}}{\omega_{\text{Pump}}}$$

In case when the pump is connected to shaft y :

$$m_{yz} = \frac{1}{m_H} = \frac{\omega_{\text{Pump}}}{\omega_{\text{Turbine}}}$$

When shaft x is the input shaft (forward-connected P.G.T., Fig. 6) then $m_\omega = m_{yx}$, in the opposite case (backward-connected P.G.T.) $m_\omega = 1/m_{yx}$ (Fig. 7).

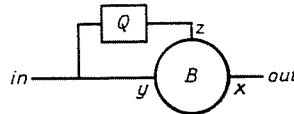


Fig. 7

It often occurs that a fixed P.G.T. is taken as a connecting element of a side-connected P.G.T. (Fig. 8). In that case Formula 6 is as follow:

$$m_{yx} = \frac{B'}{1 + \frac{B' - 1}{B''}} \tag{7}$$

if $y'' = y' = y$.

In practice, one can also meet cases, when a side-connected P.G.T. is taken as a connecting element of another side-connected P.G.T. (e. g. the third speed of the Wilson transmission, Fig. 9).

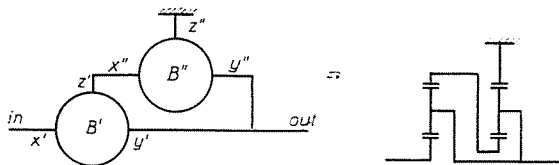


Fig. 8

Not only in the examples above, but in every case, the analysis of the P.G.T.-s is simpler and more general if one uses the B general basic ratio instead of b .

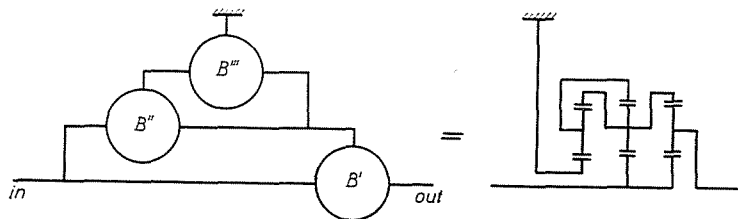


Fig. 9

The function $B = f(b)$ must be investigated more closely. This function is usually given in tables (e.g. Table III), which contain separate formulas. These tables make an impression as though there were no common base for different types of P. G. T. and for different varieties of connection; as though each type and variety of connection were independent from others. In fact, there is a very close connection between them: all types of P. G. T. with all varieties of connection can be derived from only one common "ancestor".

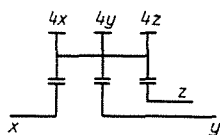


Fig. 10

First, for the sake of simplicity, derive the P. G. T. types without auxiliary planet gears. In Fig. 10, one can see the basic type. The derivation is a process of changing diameters of certain gears. By changing the diameter of an external gear ($D > 0$) one can get an internal gear ($D < 0$) as it is shown in Fig. 11.

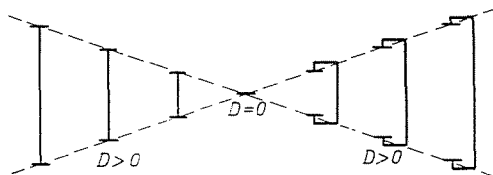


Fig. 11

Returning to Fig. 10 it can be stated that it shows not simple but connected (or rather united) P.G.T.-s, since it has not two but three central gears. These united P.G.T.-s can be separated into two simple P.G.T.-s (Fig. 12). This is a case when a simple P.G.T. is the connecting element of a side-connected simple P.G.T. Since element z is not fixed, Formula 7 gives the velocity ratio

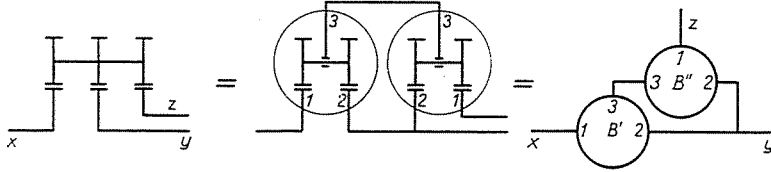


Fig. 12

related to the angular velocity of element z , that is, in this case, the m_{yz} velocity ratio is equal to B general basic ratio of the united P.G.T.-s:

$$B = \frac{B'}{1 + \frac{B' - 1}{B''}} \quad (8)$$

Taking into account the indexes from Fig. 10 and using Table III and Formula 3 one can write:

$$B' = b' = \frac{D'_1 D'_{42}}{D'_2 D'_{41}} = \frac{D_x D_{4y}}{D_y D_{4x}}, \quad (9)$$

$$B'' = 1 - b'' = 1 - \frac{D''_1 D''_{42}}{D''_2 D''_{41}} = 1 - \frac{D_z D_{4y}}{D_y D_{4x}} \quad (10)$$

Substituting Formulas 9 and 10 in Formula 8 after reduction

$$B = \frac{D_{4z} - \frac{D_z}{D_y} D_{4y}}{D_{4z} - \frac{D_z}{D_x} D_{4x}} \quad (11)$$

Having Formula 11, one may investigate the effect of decreasing the diameter of one of the planet gears to zero. In Fig. 13, another united P.G.T. is shown. Take $D_{4M} = 0$ (sketch *b*). In this case, $r_M = r_3$ and $\omega_M = \omega_3$, which means, that a central gear becomes an arm when the diameter of the planet gear being in mesh with it, decreases to zero (sketch *c*).

Returning to the basic type (Fig. 10), decrease the diameters of the planet gears one by one (Fig. 14). When $D_{4x} = 0$, the element x becomes arm, elements y and z remain central gears (sketch *a*). In case $D_{4y} = 0$, element y becomes arm. The type of P.G.T. is the same as before but the connection of its main elements with shafts x , y and z has been changed (sketch *b*). The result is similar when $D_{4z} = 0$ (sketch *c*).

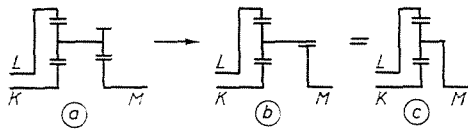
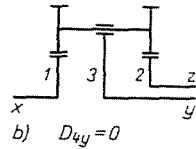
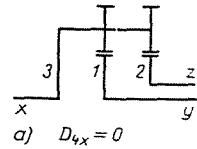


Fig. 13

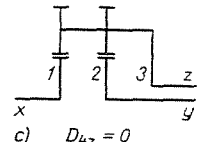


Fig. 14

Look at Formula 11 in all three cases.

Case a:

$$\begin{aligned} D_{4x} &= 0; & D_x &= D_3; \\ D_{4y} &= D_{41}; & D_y &= D_1; \\ D_{4z} &= D_{42}; & D_z &= D_2. \end{aligned}$$

Note: The indexes of the two central gears can be chosen from 1 and 2 without restriction; the inverse of them will automatically appear later.

Substituting the above equations in Formula 11, using Formula 3 one gets

$$B = \frac{D_{42} - \frac{D_2}{D_1} D_{41}}{D_{42}} = 1 - \frac{D_2 D_{41}}{D_1 D_{42}} = 1 - \frac{1}{b} = \frac{b-1}{b}. \quad (12)$$

Case b:

$$\begin{aligned} D_{4x} &= D_{41}; & D_x &= D_2; \\ D_{4y} &= 0; & D_y &= D_3; \\ D_{4z} &= D_{41}; & D_z &= D_1. \end{aligned}$$

After substitution,

$$B = \frac{D_{41}}{D_{41} - \frac{D_1}{D_2} D_{42}} = \frac{1}{1 - \frac{D_1 D_{42}}{D_2 D_{41}}} = \frac{1}{1-b}. \quad (13)$$

(Case c:

$$\begin{aligned} D_{4x} &= D_{41}; & D_x &= D_1; \\ D_{4y} &= D_{42}; & D_y &= D_2; \\ D_{4z} &= 0; & D_z &= D_3. \end{aligned}$$

After substitution,

$$B = \frac{-\frac{D_3}{D_2} D_{42}}{-\frac{D_3}{D_1} D_{41}} = \frac{D_1 D_{42}}{D_2 D_{41}} = b . \tag{14}$$

One can see that sketches 14c, 14a, 14b, and Formulas 14, 12, and 13 correspond to the upper part of Table III. By changing indexes 1 and 2, one could get the lower part of it.

After getting simple P.G.T.-s from the basic type by eliminating one of the planet gears, now change the diameters of the central gears. For the sake of brevity, the process only for sketch 14b will be shown, the conclusions will be valid for sketch 14a and 14c too.

In Fig. 15, sketch 1 corresponds to sketch 14b.

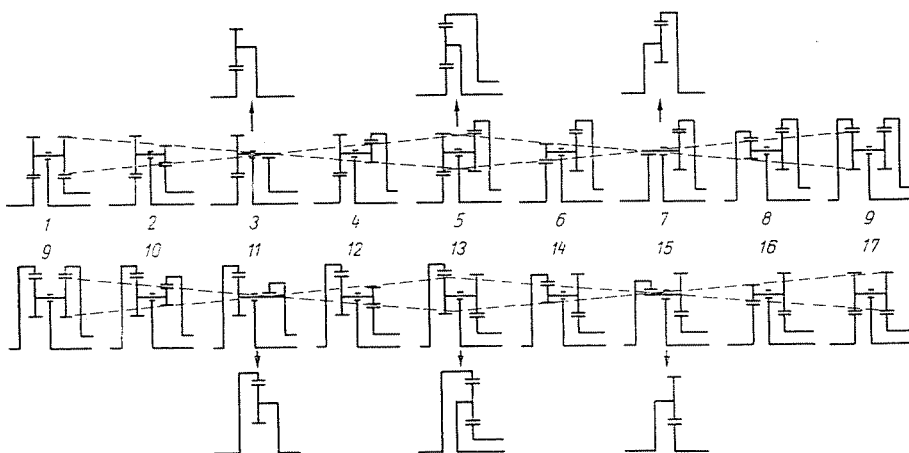


Fig. 15

In sketches 2 through 5, D_2 changes and D_1 is constant. In sketch 3, $D_2 = D_3$, that is $D_{42} = 0$, it means that element 3 has become an arm too. Only one central gear has remained: the simple P.G.T. has become an elementary

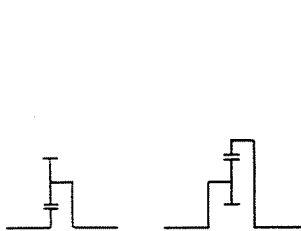


Fig. 16

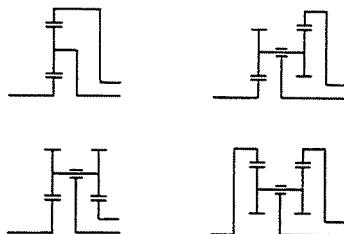


Fig. 17

P.G.T. which has always one central gear only. In sketch 4, $D_2 < 0$. In sketch 5, $D_{41} = D_{42}$, and the form of the P.G.T. can be simplified.

In sketches 6 through 9, D_1 changes and D_2 is constant. The sketch 6 shows the same type of P.G.T. as shown in sketch 4, but the geometrical sizes are different. In sketch 7, one can see an elementary P.G.T. again. In sketch 8, both D_1 and D_2 already have negative sign.

In sketches 10 through 13, D_2 will change its sign to positive, in sketches 14 through 17, D_1 will do the same. Note that in sketches 10 through 17, the

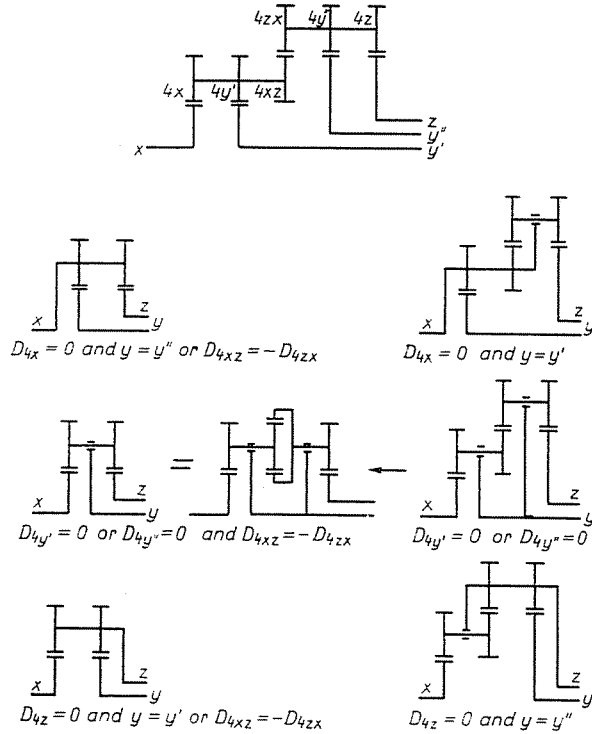


Fig. 18

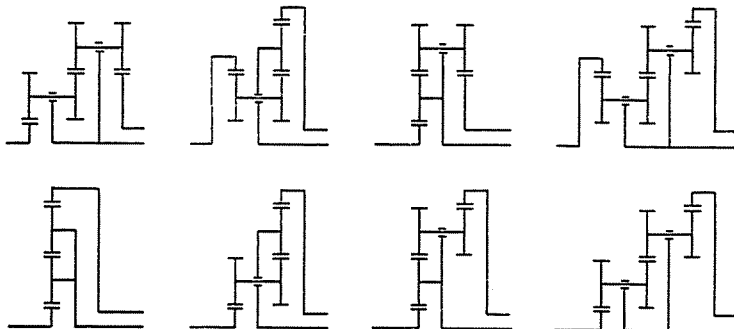


Fig. 19

same P.G.T. types are shown as in sketches 9 back to 1, but with reversed 1 and 2 indexes.

On the basis of Fig. 15, it can be stated that two types of elementary P.G.T. and four types of simple P.G.T. can be derived from type 14b, each with two varieties of connection. If one made the same process of derivation for types 14a and 14c, one would have gotten the same types of P. G.T. but with two other couples of varieties of connection.

The types of elementary and simple P.G.T. derived from the basic type, shown in Fig. 10, are illustrated in Figs 16 and 17. These types are all without auxiliary planet gears. If one starts from the basic type given in the top of Fig. 18, instead of Fig. 10, one will get all types of the elementary and simple P.G.T. *with* or *without* auxiliary planet gears. The process is similar to the process illustrated above, so only the main conclusions will be reported.

The basic type given in Fig. 18 has four central gears, two of them have index y (y' and y''). At the same time, one can take into account only one of elements y ; otherwise the mechanism becomes overspecified. The left side of Fig. 18 corresponds to the well-known sketches 14a, 14b, and 14c. The right side of Fig. 18 illustrates the basic type of P. G. T. with auxiliary planet gears in three varieties of connection.

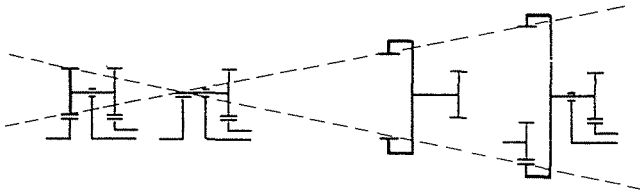


Fig. 20

If one takes into account element y' the formula, convenient to Formula 11, is as follows:

$$B = \frac{D_{4z} + \frac{D_z}{D_{y'}} D_{4y'} \frac{D_{4zx}}{D_{1xz}}}{D_{4z} + \frac{D_z}{D_x} D_{4x} \frac{D_{4zx}}{D_{4xz}}} \quad (15)$$

If one takes into account element y'' ,

$$B = \frac{D_{4z} - \frac{D_z}{D_{y''}} D_{4y''}}{D_{4z} + \frac{D_z}{D_x} D_{4x} \frac{D_{4zx}}{D_{4xz}}} \quad (16)$$

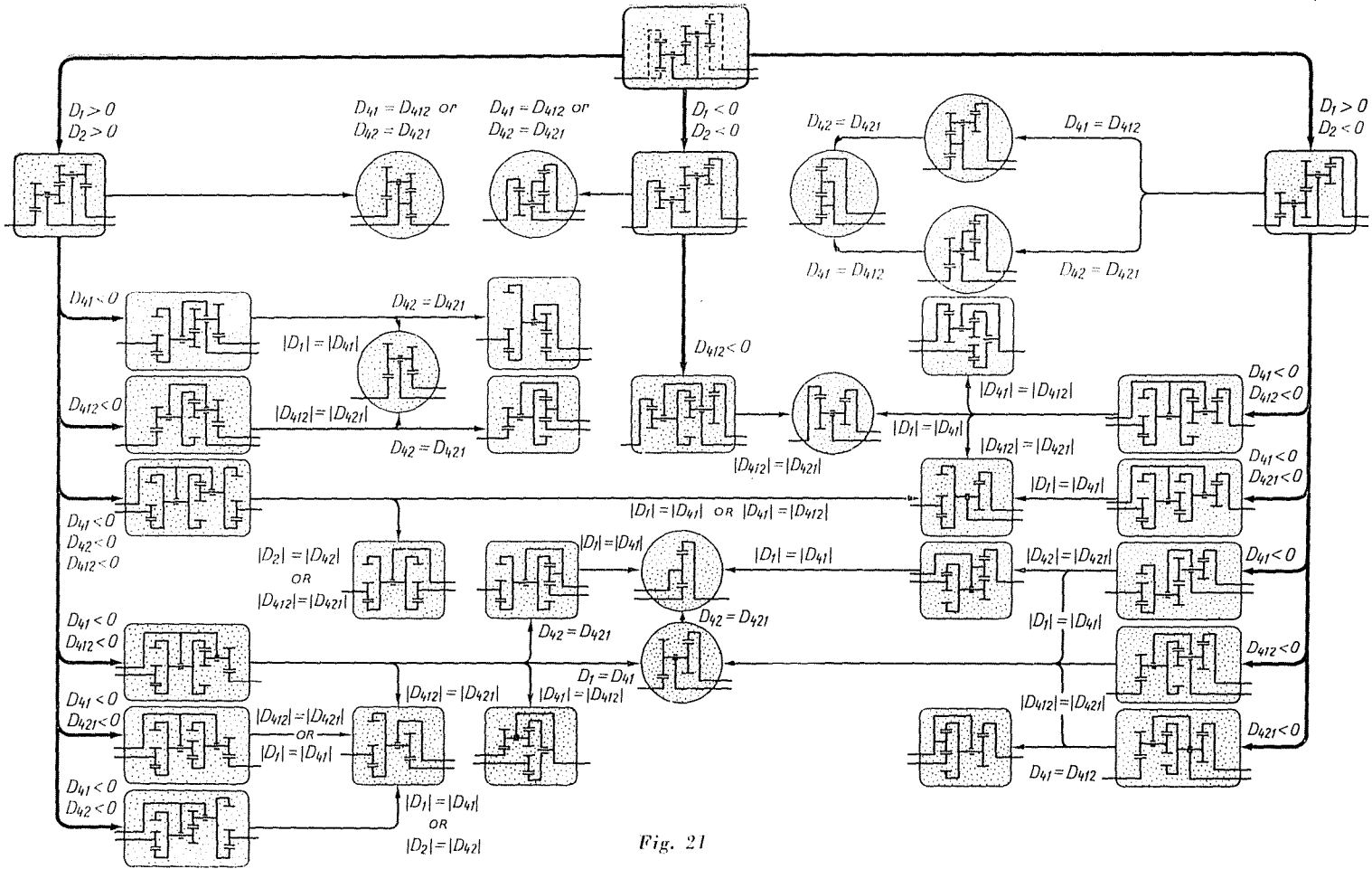


Fig. 21

Naturally, Formulas 15 and 16 include Formula 11; one can check it by substituting $\frac{D_{4zx}}{D_{4xz}} = -1$ in them.

Making the process of derivation illustrated in Fig. 15 with the right side of Fig. 18, one gets further eight types of simple P.G.T., each of them with six varieties of connection. These new types are shown in Fig. 19.

Till now cases when the planet gears can also become internal gears ($D_4 < 0$) have not been examined. It would take up too much space to illustrate the process of derivation in case of internal planet gears. Instead, some points of the process are only shown in Fig. 20, and the conclusion is reported

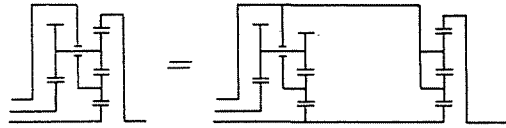


Fig. 22

that the simple P.G.T. with internal planet gears has 22 types, each with six varieties of connection.

In conclusion, it can be said that two types of elementary P.G.T. and 34 types of simple P.G.T. exist in all. The 34 types are: four types without auxiliary planet gears, eight types with auxiliary planet gears, all these with external planet gears, and twenty two with internal planet gears. This agrees with the conclusion of an investigation made in another way [20] which resulted in the family tree of the simple P.G.T. (Fig. 21). One can see from the family

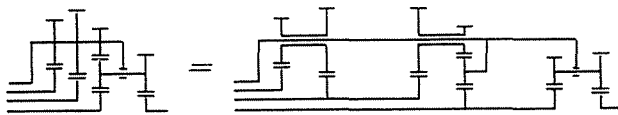


Fig. 23

tree which of the special type of P.G.T. had been derived from which of the more general type. In the family tree, either the proportions or the signs of the various diameters as related to the certain types are also indicated. Fat arrows show the types made by changing the sign of diameters, thin arrows show the types made by reducing the number of the planet gears.

In all cases when connected or united P.G.T.-s appear, recognizable by more than two central gears, it can be separated into two or more simple P.G.T.-s illustrated in the family tree (Figs 22 and 23).

Summary

All types of planetary gear train with all varieties of connection can be derived from only one common "ancestor". The derivation is a process of changing diameters of certain gears: by changing the diameter of an external (central or planet) gear one can get an internal gear and vice versa. In conclusion, it is said that two types of elementary planetary gear train and 34 types of simple planetary gear train exist in all, each with six varieties of connection.

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