ANALYSIS OF THE DIMENSIONAL PROPERTIES OF RIB KNIT FABRICS

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(Received November 29, 1968)

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Introduction

The field for knitted fabrics has been extending rapidly in recent years. Knitted fabrics due to their structural properties show a high order of extensibility which renders them more attractive for certain end uses than woven fabrics prove to be.

The single yarn system building up the fabric by successive intermeshings with itself forms a sheet-like body. When subjected to a tensile force of whatever direction, the yarn lying in the fabric in sections of relatively high curvature (shown in Fig. 1) permits to start deformations exclusively by structural rearrangement before a considerable extension of the yarn, i.e. of the fibres contained in the fabric. In that case a relative slipping of the contacting yarns takes place, yarn sections parallel to the tensile force straighten, while others take their place along a radius of lower curvature. Accordingly, a considerable change in the dimensions of the fabric can be observed already in the initial stage of deformation.

This overall characteristic property of the knitted fabric cannot be neglected in design. In particular, in the case of fully fashioned products it is of interest to approach the required dimensions as much as possible both in order to reduce material waste and to eliminate certain technological processes (e.g. cutting). During the knitting process the dimensions of the fabric show substantial deviations from those required, as during fabric formation various forces arise, deforming the fabric as soon as it is formed. After unloading, further deformations occur in the fabric structure, until at last it acquires the structure and dimensions corresponding to those of the minimum stress state. In that ideal case the dimensions of the knitted fabric agree with those required. Consequently, design has to be based on the deformed fabric structure.

Thus, in the investigation of the dimensional properties of the product, fabric structure must be considered not only in its state of rest which follows the relaxation after the knitting process, but also in its deformed state corresponding to the knitting conditions.

Our investigations were concerned with rib knit fabrics which could be considered as basic fabric having the simplest structure of weft-knitted double-knit fabrics. (In order to render the loop diagram more descriptive it is shown in a somewhat elongated form in Fig. 2.) This type of fabric is often used for producing fully fashioned fabric sheets requiring exact dimensions.

In the technical literature there are many calculations and investigations dealing with the determination of the values of the main dimensions of fabrics

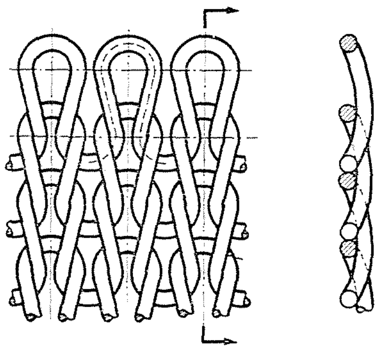

Fig. 1. Loop diagram and section of a single face side base fabric

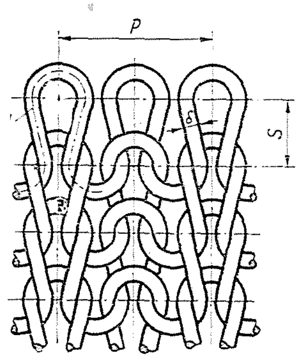

Fig. 2. Loop diagram of a rib knit fabric

in dependence of the diameter of the yarn (δ). These dimensions are: the wale-spacing (P), the course spacing (S), and the yarn length (l). (Notations see in Fig. 2.) The theoretical relationships of the most simple fabric structure, of the so-called single needle bar fabrics (see Fig. 1) have been established ([2, 4, 7, 8, 9, 14]). Rib knit fabrics can be characterized either by correcting the relations derived for single needle bar fabrics on the basis of the differences to be found between the two types of fabric ([1, 5, 6, 12, 13]), or by generalizing the formulae obtained, to such an extent that they may be applied more or less to any type of weft-knit fabrics [10]. These investigations are primarily based on geometrical analysis, as the various types of fabrics showing otherwise also different properties have been developed on the basis of the differences in the geometrical arrangement of the yarn.

The numerical values obtained by the use of the relationships mentioned substantially differ from both theoretical and empirical values depending on the methods applied. This can be attributed to the fact that by modifying the formulae valid for other types of fabric and due to the approximations and suppositions used in connection with the fabric chosen, inevitably errors arise.

Thus, it appears more convenient to start directly from the given loop arrangement of rib knit fabrics.

Investigation of rib knit fabrics in state of rest

In the investigation of any type of fabric the properties of the smallest repeat element of the fabric structure has to be determined as this permits to make generalization for fabric pieces of any dimension required.

After the rest which follows the knitting process, and on completion of the deformation relaxation, rib knit fabrics exhibit the structure shown in Fig. 3. The smallest repeat unit of the fabric can be included in a column

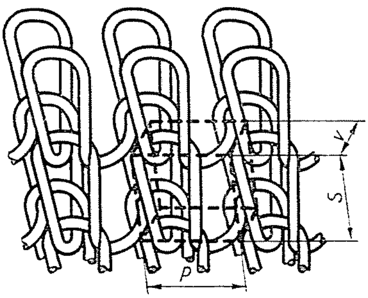

Fig. 3. Spatial arrangement of a rib knit fabric in unloaded state of rest

determined by the wale spacing, course spacing, and the thickness of the fabric. (With the notations of the figure the edge lengths of this column be P, S, v.) This unit includes the parts of a face side stitch and those of a reverse side one. As there is no difference in the appearance of the face side stitch and the reverse side stitch of a rib knit fabric ("double face side fabric") face side stitches and reverse side stitches have the same dimensions. This permits a further simplification of the analysis, i.e. to consider only the half of the repeat element, a single stitch, and to choose the volume of the column as $V = P \cdot S \cdot v/2$.

The investigation of the dimensional properties of the fabric requires the determination of the functions $P = f_1(l, \delta)$, $S = f_2(l, \delta)$ and $S/P = f_3(l, \delta)$. These relationships express the influence of the yarn length (l) contained in a stitch and that of the yarn diameter (δ), on the dimensions of the wale spacing (P), on the course spacing (S), and on the ratio of these latter respectively.

As mentioned already in the introduction, the potential energy accumulated during knitting, causes the yarn to be inclined to take its position along an arc of minimum curvature. (This trend can be clearly observed when unravelling a fabric immediately after knitting.) This condition, however, can be realized but to a certain extent, as adjacent stitches hinder each other. Thus, the trend to reach the minimum stress state with a given value of the yarn length contained in the loop results in a maximum volume of the loop.

Let us write the volume of the column including the stitch in dependence of the wale spacing, course spacing and yarn diameter, and determine the values of the wale spacing and course spacing with respect to the maximum volume of the column.

In expressing the stitch length let us consider the midline of the yarn (Fig. 4). The actual space curve will be substituted by circular arcs and straight sections, where the needle loop ($\widehat{JK}$) is a semi-circular arc of diameter d_f, the

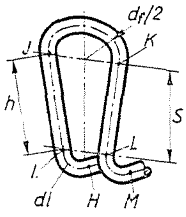

Fig. 4. Position of a stitch in a rib knit fabric in state of rest

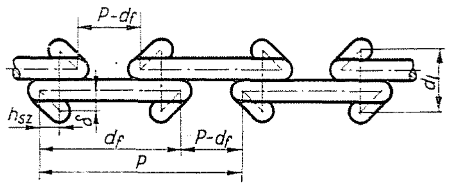

Fig. 5. Simplified top view of a rib knit fabric

two loop shanks ($\overline{IJ}$ and $\overline{KL}$) represent the straight segment (h), the two sinker loops ($\widehat{HI}$ and $\widehat{LM}$) are the semi-circles of diameter d_l lying in the planes normal to the plane of the needle loop. Thus the yarn length contained in a stitch is:

$$l = \frac{d_f \pi}{2} + 2h + 2\frac{d_l \pi}{4}.$$

The diameters of the circular arc of the needle loop and of sinker loop can be determined with the aid of the top view in Fig. 5. Assuming a linear relationship between the diameter of the needle loop and the stitch spacing, the value d_f can be expressed in function of the wale spacing P:

$$d_f = kP,$$

where k is the proportionality factor. (According to our measurements, linear relationship can be assumed for fabrics in unloaded state of rest.) In the case of unchanged wale spacing when there is a contact between adjacent needle loops

$$d_{f\,\max} = P - \delta$$

because the diameter of the outer arc of the needle loop is equal to the wale spacing. Thus

$$k_{\max} = \frac{P - \delta}{P}.$$

The lowest value of k is determined by the position where there is a close contact between the circular arc of the needle loop and the sinker loops of the stitch formed in the next course, i.e.

$$k_{\min} = \frac{3\,\delta}{P}\,.$$

Thus within the range of $k_{min} \leq k \leq k_{max}$ the value of the proportionality factor k is theoretically interpreted.

The diameter of the midline of the sinker loop can be calculated also on the basis of Fig. 5.

$$d_l = 3\,\delta\,.$$

The position of the loop shanks is shown in Figs 4 and 5. Accordingly (with the notations of the two figures) the loop shank can be represented by the body diagonal of a column of edge lengths S, δ and h_s, where

$$h_s = \frac{d_f - (P - d_f)}{4}\,.$$

With the usual fabric characteristics and the factor k the stitch length can be expressed as:

$$l = BP + 2\sqrt{S^2 + P^2 A^2 + \delta^2} + D\,\delta\,,$$

where

$$A = \frac{2\,k - 1}{4}\,, \qquad B = \frac{k\,\pi}{2}\,, \qquad D = \frac{3\,\pi}{2}\,.$$

The value of the wale spacing with respect to the maximum volume of the column including the stitch (Fig. 3) can easily be determined by substituting in the expression of the volume

$$V = PS\,\frac{d_l}{2}$$

the values of d_l and S (the latter from the relationship valid for the stitch length) and calculating the position of the extreme value of the relationship

$$V = P\,\frac{3\,\delta}{2}\sqrt{P^2\,\frac{B^2 - 4\,A^2}{4} - P\,\frac{(l - D\delta)\,B}{2} + \frac{(l - D\delta)^2 - 4\,\delta^2}{4}}$$

Using the cover factor $\sigma = \frac{l}{\delta}$ characteristic for knitted fabrics, we get for the wale spacing and course spacing yielding maximum stitch volume:

$$P_{V\max} = \frac{\frac{3}{2}(\sigma - D)\,B \pm \sqrt{(\sigma - D)^2\left[\frac{9}{4}B^2 - 2\,(B^2 - 4\,A^2)\right] + 8\,(B^2 - 4A^2)}}{2\,(B^2 - 4\,A^2)}$$

$$S_{V\max} = \delta\sqrt{F^2\frac{B^2 - 4\,A^2}{4} - F\frac{(\sigma - D)\,B}{2} + \frac{(\sigma - D)^2 - 4}{4}}$$

where

$$F = \frac{P_{V\max}}{\delta}$$

and the relationship of the dimensional properties of the stitch takes the form:

$$\left[\frac{S}{P}\right]_{V\max} = \frac{\sqrt{F^2\frac{B^2 - 4\,A^2}{4} - F\frac{(\sigma - D)\,B}{2} + \frac{(\sigma - D)^2 - 4}{4}}}{F}\,.$$

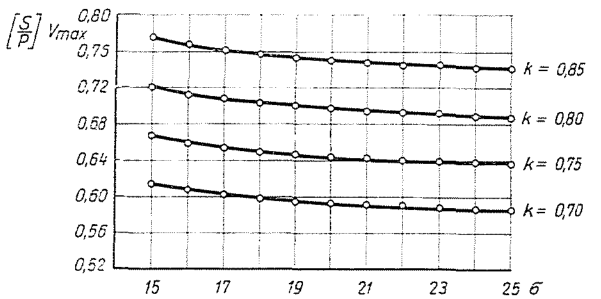

Fig. 6. Theoretical relationship of the cover factor and the dimensional properties of an unloaded rib knit fabric

It is to be seen from the formulae that the relation between the dimensional properties of the stitch depends exclusively on the cover factor.

Fig. 6 shows the connection between the cover factor and the ratio $\frac{S}{P}$ for different k values. From the characteristics of the family of curves it is clearly seen that for a given value of k the value $\frac{S}{P}$ is but slightly affected by the changes of σ, within the range of the same ordinate, i.e. in the case of a fixed cover factor, however, the changes in the k value exert a decisive influence on the ratio $\frac{S}{P}$. For rib knit fabrics used in the practice, the value $k = 0.80$ is in general characteristic, thus from the curves in Fig. 6 that corresponding to this value will be chosen for the purpose of our calculations.

Examination of rib knit fabrics stretched in wale direction

Considering the fact that the stresses acting on the fabric during knitting are primarily parallel with wales, the form taken by the fabric during the knitting process practically agrees with the form taken after deformation by a fabric stretched in wale direction. On the basis of the forces acting during knitting it can be assumed that the tensile force simply rearranges the loop structure without a considerable extension of the yarn. Thus, omitting yarn extension, the changed loop structure and the corresponding fabric deformation can be theoretically determined.

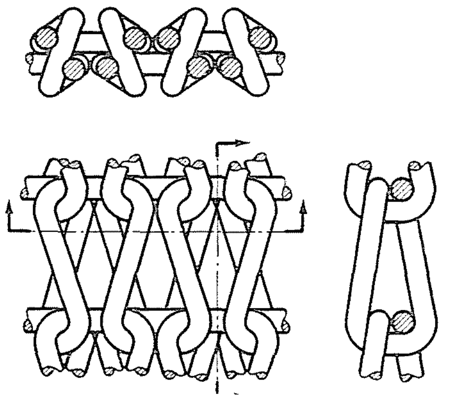

Fig. 7. Loop diagram and section of a rib knit fabric stretched in wale direction

In the extreme case of maximum deformation produced exclusively by changes in the loop structure, omitting yarn extension, knitted fabrics show the structural arrangement represented in Fig. 7. The yarn length contained in a stitch agrees with the stitch length l of an unloaded fabric in state of rest. This value, due to the changed lengths of the individual parts of the stitch, is composed of other components than those to be found in the fabric in a state of rest of minimum stress.

The tensile force causes the circular arc of the needle loop and that of the sinker loops to transform to straight segments between the points of contact of the yarn of the intermeshing stitches, while the contacting sections form curved arcs which can be substituted by a quarter of an ellipse. The plane of the needle loop can roughly be approached by a plane defined by the directions of the wale and of the course. The position of the sinker loop is not quite as unambiguous. As it is seen in top-view in Fig. 8*a*, the sinker loops may have a parallel position in which case there is no contact between the loop shanks and thus the wale spacing cannot be at its minimum because the lengths of the straight sections in the needle loop increase. The fabric structure shown in Fig. 8*b* assumes contacting loop shanks yielding the lowest possible values for the wale spacing. This requires, however, that the sinker loops be deflected by $\pm 30°$ from the plane normal to the fabric. In that case the length of the sinker loop increases as compared to the former case. (The deflection angle can be determined from the design.) It can be assumed that between the two possible extreme positions the yarn takes a position which in the case of the given loop length results in the maximum extension of the fabric and thus in maximum course spacing. This permits an approximate determination of the

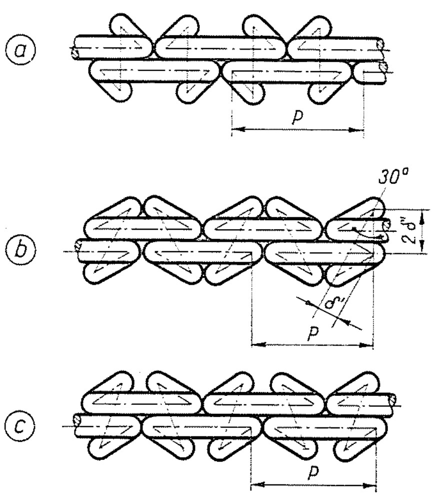

Fig. 8. Top view of the possible structural arrangement of a rib knit fabric stretched in wale direction

actual arrangement of the loop structure: for an intermediate position (Fig. 8*c*) the value of the extended course space (in the following S_n) can be, in general, expressed, and it can be calculated at which deflection angle its maximum can be attained.

In order to simplify the calculations, a section including a half stitch (Fig. 8*c*) is shown separately in Fig. 9. The diameter δ' given in the figure does not agree with the value δ discussed with respect to the unloaded fabric, as the tensile force causes the average diameter of the yarns gathering close to one another to decrease. Empirically [3, 15] the ratio of the two diameters can be written as:

$$\frac{\delta'}{\delta} = 0.8$$

With notations in Fig. 9, be j the half of the longer axis of the ellipse approximating the arc section of the needle loop and the sinker loops; δ' the half length of the shorter axis; u and j the lengths of the straight segment at the needle loop, and at the sinker loop, respectively. For the length of the needle loop and the sinker loop (l_f and l_l) respectively, the following approximations can be written [11]:

$$l_f = 2\left(0.9827\,j + 0.3110\,\delta' + 0.2867\,\frac{\delta'^2}{j}\right) + u$$

$$l_l = 2\left(0.9827\,j + 0.3110\,\delta' + 0.2867\,\frac{\delta'^2}{j}\right) + j\,.$$

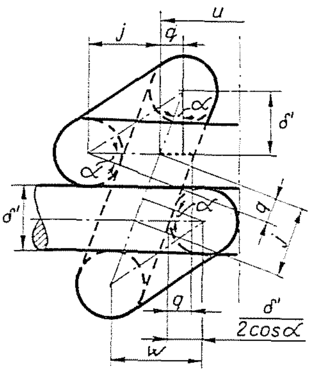

Fig. 9. Enlarged view of the half design unit shown in top view in Fig. 8b

The lengths of the loop shanks can be calculated on the basis of the body diagonal of a column of side lengths S_n, δ', w:

$$l_s = \sqrt{S_n^2 + \delta'^2 + w^2}\,.$$

The notations of the formula according to Fig. 9 are as follows:

$$j = \frac{\delta'}{\cos\alpha};$$

$$w = j + q\,,$$

where

$$q = \delta'\,\mathrm{tg}\,\alpha\,,$$

thus

$$w = \delta'\,\frac{1+\sin\alpha}{\cos\alpha};$$

$$= 2\left(\frac{\delta'}{2} - q + \frac{\delta'}{2\cos\alpha} + \frac{\delta'}{2}\right)$$

(along the dotted line in Fig. 9).

For the total stitch length we get:

$$l = 4\left(0.9827\,\frac{\delta'}{\cos\alpha} + 0.3110\,\delta' + 0.2867\,\delta'\cos\alpha\right) +$$
$$+ 2\,\delta' + \frac{2\,\delta'}{\cos\alpha} - 2\,\delta'\,\frac{\sin\alpha}{\cos\alpha} +$$
$$+ 2\sqrt{S_n^2 + \delta'^2\left[\left(\frac{1+\sin\alpha}{\cos\alpha}\right)^2 + 1\right]}\,.$$

From the relationship the S_n value can be expressed in dependence of α and thus the maximum of S_n can be calculated. In determining the position of the extreme value we obtain a tenth order mixed equation for $\cos\alpha$, and solving the same with Newton's approximation, the α values, i.e. the positions of the extreme values corresponding to practical parameters l and δ', can be found in the following Table:

Table 1

α values of the deflection angle of the sinker loops for different values of the stitch length and the yarn diameter

l mm \ δ' mm	0.20	0.25	0.30	0.35	0.40
5.0	14.6333	11.8774	9.0126		
5.5	15.5787	13.1551	10.5538	8.2047	
6.0	16.3178	14.1749	11.7227	9.3907	5.4946
6.5	16.9652	15.0401	12.4847	9.8892	8.6688

From the calculated α values, the dimension ratios of the stitch in the fabric stretched in wale direction can be determined. The value of the course spacing S_n can be calculated by substituting the tabulated α values into the function $S_n = f(\alpha)$. The wale spacing P_n of the stretched fabric is equal to the width of the needle loop, as the stretching brings the needle loops into touch with each other. Accordingly, (with notations in Fig. 9) the wale spacing can be calculated as follows:

$$P_n = u + 2j + \delta'$$

or using the respective relationship

$$P_n = 0.8\,\delta\left(3 + \frac{3 - 2\sin\alpha}{\cos\alpha}\right).$$

Thus, the value of wale spacing depends on the stitch length as far as the value α is influenced by it.

In the relationships valid for the course spacing and the wale spacing, the value δ is included. In expressing the dimensional relations of the stitch, a simplification by δ can be introduced, which means that the structural arrangement of a fabric stretched in wale direction is not directly influenced by the yarn diameter. The expression for the ratio $\frac{S_n}{P_n}$ contains only the cover factor and α:

$$\frac{S_n}{P_n} = \frac{\left(\frac{1}{4}\sigma^2 + U\sigma + R\right)^{1/2}}{Z},$$

where

$$R = 0.64\left[22.3851 + \frac{1.8628}{\cos^2\alpha} + \frac{9.6198}{\cos\alpha} + 1.8601\cos\alpha + 0.3288\cos^2\alpha - 3.2440\frac{\sin\alpha}{\cos\alpha} - 7.9308\frac{\sin\alpha}{\cos^2\alpha} - 1.468\sin\alpha + \frac{\sin^2\alpha}{\cos^2\alpha}\right];$$

$$U = 0.8\left[\frac{\sin\alpha}{\cos\alpha} - 1.6220 - \frac{1.9654}{\cos\alpha} - 0.5734\cos\alpha\right];$$

$$Z = 0.8\left[3 + \frac{3}{\cos\alpha} - \frac{2\sin\alpha}{\cos\alpha}\right].$$

The values $\frac{S_n}{P_n}$ corresponding to the tabulated l and δ' values are shown in Fig. 10 directly in dependence of the cover factor.

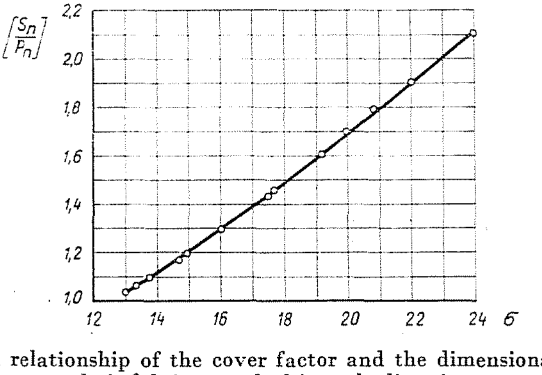

Fig. 10. Theoretical relationship of the cover factor and the dimensional properties in a rib knit fabric stretched in wale direction

The dimension ratios of the stretched fabric are seen to show an almost linear relationship. With higher values of σ, i.e. in the case of a fabric of looser structure, the quotient of the course spacing by the wale spacing will be higher. Thus, structural arrangement is influenced to a substantially greater extend by the cover factor in a fabric stretched in wale direction than in the case of an unloaded fabric in state of rest.

Knowing the dimension ratios of the fabric both in the unloaded state and when stretched in wale direction (Figs. 6 and 10) it is possible "to design" fabrics of given cover factor more accurately with respect to the deformations after knitting, resulting in a better approximation of the required dimensions by the final form of the fabric.

The numerical values given are, of course, valid only for rib knit fabrics, but the experimental method and the considerations underlying the derivations may be applied for other types of fabric.

Summary

An experimental method is described by which the relationships of the most important characteristics of rib knit fabrics extensively used in the hosiery and knitting industry can be determined. Starting directly from the spatial arrangement of rib knit fabrics the relation between the wale spacing and course spacing in dependence of the cover factor can be derived. Since fabric structure attains its state of minimum tension of rest after a considerable deformation occurring after the knitting process, it appeared more convenient to determine the relationships of the parameters required for the structural arrangement in the states of deformation and after relaxation, respectively. The use of the diagrams obtained as a result of the derivations facilitates the design of rib knit fabrics.

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