FITNESS OF THE STEFAN–BOLTZMANN FORMULA IN FIRING TECHNIQUE

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1. Validity of the Stefan—Boltzmann formula

Solid bodies are known to emit a heat quantity proportional to the fourth power of their absolute temperature to their environment. This law has been proved both experimentally and analytically. Thermal radiation of bodies propagates through electromagnetic waves depending on wavelength:

$$I_\lambda \equiv \frac{C_1}{\lambda^5} \cdot \frac{1}{e^{C_2/T\lambda}} \text{ kcal/m}^2 \cdot \text{h. Å}$$

Thus it means the energy emitted within a narrow wavelength band. In the range $0 < \lambda < \infty$ the total emitted energy is the integral of the curve, hence the area under the curve:

$$\int_0^\infty I_\lambda \, d\lambda = \sigma \cdot T^4 \quad \text{kcal/m}^2 \cdot \text{h}$$

Much of the heat radiation belongs to the infrared range, in spite of this, much importance is due to the light emission in the visible range, accessible to the optical instruments available. Light radiation emits slight energy to the environment, but it counts with the total emitted energy.

Condition of the stationary state is that the temperature of bodies involved in the radiation heat exchange is constant in time and uniform throughout the body. In this case the radiation heat exchange between absolute black bodies is ruled by the Stefan—Boltzmann relationship without restriction:

$$q_s = C\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] \text{kcal/m}^2 \cdot \text{h}$$

To insure stationary state, heat energy must be supplied for the body of higher temperature, so that temperature T_1 of the emitting body is kept constant, hence energy supplied to heat Body 1 should equal that emitted by it. The same is valid to Body 2 of temperature T_2 to the sense, namely, its temperature is constant if the delivered energy is identical to the absorbed energy. The process can schematically be described as:

$$\text{heating} \rightarrow T_1 \rightarrow \text{radiation} \rightarrow T_2 \rightarrow \text{cooling}$$

For T_1 = constant and T_2 = constant, heating = cooling (no heat loss)

For a non-uniform energy supply the temperature of heat exchanger media varies and no stationary state can be spoken of. In such cases the Stefan—Boltzmann relationship is only valid with restrictions, after certain considerations.

2. Influence of the burning process

Consideration will be given to the special case of "heating" due to the chemical transformation — *burning* — of Material 1, and heat evolution, confined in space, varies with time and location.

Also for heat evolution simultaneous to burning, emitted heat is assumed to be proportional to the fourth power of temperature, though this assumption in not duly proved. The Stefan—Boltzmann law gives a close approximation, intermediating a multiplying factor $0 < \bar{\varepsilon} < 1$ for flames with radiation intensity ratios decreasing uniformly in any wavelength band ("gray" bodies).

Radiation laws for high temperature combustion products — gases — have been determined experimentally. These differ from solid bodies by radiating selectively according to wavelengths, partly absorbing and partly transmitting incident heat — depending on their material composition — and the emitted energy is not proportional to the fourth power of the absolute temperature any more.

3. Variation of the emissivity coefficient

Speaking of the radiation of combustibles involved in the burning process and of their by-products, deviations due to heat evolution simultaneous to the radiation heat transfer is expressed by the value of the emissivity coefficient. In this case, the Kirchhoff law cannot be considered valid any more, since, according to the Kirchhoff law, the emissivity coefficient describes the attitude against heat from external sources, but now, an additional point of view to be taken into consideration is the percentage of heat evolving from chemical transformation serving to heat the radiating medium or that transmitted to the environment during heat evolution. This ratio depends both on the material composition and on the actual state of the burning material.

Most of the problem is due to the fuel finely dispersed in the gaseous medium, radiation properties of the disperse medium being dependent not only on the composition of combustion materials, but also on the solid particle size in the disperse medium, on the particle size distribution and concentration. Luminous flames are likely to be such disperse media containing suspended solid particles from submicron size (soot) to 500 μ (pulverized coal).

Emissivity coefficient can only be determined with difficulties, it depending on the actual state of chemical process in the medium. PRÉVOST empirically stated the emissivity of an infinitesimal volume to be the function of processes in it.

Thus, in addition to the Kirchhoff law, the PRÉVOST observation has to be taken into consideration when determining the flame emissivity coefficient for heat evolution simultaneous to burning. In case of disperse medium, emissivity coefficient at different spots also depends on the particle concentration and size. The concentration varies along the flame, while the particle size depends partly on conditions of production and partly on the fineness of produced grains.

4. Problem of radiation temperature

While the emissivity coefficient is rather a function of material properties and concentration, temperature T_1 involved in the first term of the Stefan—Boltzmann relationship ranges from the theoretical burning temperature T_0 to the temperature T_{ex} at the combustion chamber exit. MIKHEIEV suggests a mean value $T_1 = \sqrt[4]{T_0^2 \cdot T_{ex}^2}$. This approximation is a rather common one for combustion chamber temperature calculations. For boiler heating, however, often the approximation $T_1 = T_{ex}$ is applied. One may wonder whether this approximation is permissible.

LONG in "Öl- und Gasfeuerung" (1963, p. 1036) gives a comprehensive discussion of this problem, and, referring to the statements by EVANS, LOBO and HOTTEL, he states the temperature T_1 according to Stefan—Boltzmann can be considered "temperature of the combustion chamber at the exit" provided a uniform temperature distribution by intensive mixing prevails. Then the radiation temperature of the flame can be considered T_1 throughout, T_1 being also the temperature at the exit. As a conclusion, the following "radiation heat transfer efficiency", suggested by HOTTEL, is presented:

$$C_\pi = \frac{q_r - q_{rw}}{q_{r0} - q_{rw}}$$

q_r = heat effectively transferred by radiation;
q_{rw} = radiation heat transfer in thoroughly homogeneized combustion chambers;

q_{r_0} = radiation heat transfer for a one-dimensional temperature distribution.

$0 < C_\pi < 1$ being likely to vary from zero for a zero numerator if the emitted heat equals the heat calculated on the assumption of a perfectly homogeneous temperature distribution.

EVANS and LOBO state this case to be possible for large combustion chambers. In the general case, "flame temperature" is somewhere between the theoretical maximum temperature and temperature at the exit of the combustion chamber.

In the practice, empirical formulae are being used for the so-called "flame temperature" or — just to avoid problems inherent with the Stefan—Boltzmann formula — for the emitted heat ratio. Such an empirical formula is the Hudson—Orrok one:

$$\mu = \frac{\text{emitted heat}}{\text{evolved heat}} = \frac{1}{1 + \frac{A}{55,24} B}$$

where

$$A = \frac{\text{air kp}}{\text{fuel kp}}$$

$$B = \frac{\text{unburnt fuel}}{\text{irradiated heating surface in sq. m.,}}$$

as well as the Reid—Cohen—Corey formula:

$$\mu = \frac{100}{1 + CGQ}$$

G = weight of flue gas by 1000 kcal of evolved heat
C = empirical constant
Q = a factor proportional with the specific heated surface.

These are, however, close approximations. As concerns the Mikheiev mean value, the temperature named "equivalent" applied as an approximation is based on theoretical considerations. From radiation aspects, an absolute black body at temperature Θ emitting the same heat quantity as the given flame with an emissivity coefficient ε and real temperature T_1 can be considered as equivalent.

Thereby the real and correct T_1 value can be arrived at directly by measurement, since:

$$\sigma \cdot \Theta^4 = \bar{\varepsilon} \cdot \sigma \cdot T_1^4$$

where Θ is the total radiation reading off a pyrometer to be applied in emitted energy calculations. In a concrete case, for instance:

$$q = a \frac{F_1 \cdot F_2}{R^2} \cdot \frac{\sigma}{\pi} \Theta^4 \text{ kcal/m}^2 \text{ h.}$$

where

a = absorption coefficient of the instrument sensor ($a = \varepsilon$)
F_1 = surface of the "absolute black body" (actually identical to the flame surface)
R = spacing between emitting body and instrument
F_2 = instrument sensor surface.

Thus, the Stefan—Boltzmann formula is valid for flames existing in practice, in the sense that only the term $\sigma \cdot \Theta^4$, namely the heat fluxus, is true, so that the emissivity coefficient and the radiation temperature cannot be told apart.

5. Geometrical symmetry and distributions with respect to location

Temperature determination at the exit of the combustion chamber is a practical problem and on the basis of the combustion chamber heat balance

$$q_s' = B' \cdot V \cdot c_p \quad (T_0 - T_{ex}) \quad \text{kcal/h}$$

nothing else is needed for the knowledge of total emitted energy. Often however, temperature distribution within the combustion chamber or burning space is referred to, if, for instance, the local heat load on the heating surface is required, that may reach unduly high values. In existing firing systems there is a nearly circular symmetric flow of flame and combustion products, so that one-dimensional flow pattern and temperature variation curve can be spoken of. Tests refer to the cases of uniform fuel input, so that the variation of burning — heat emission can be followed up, in function of either travel or time. There are no sufficient measuring facilities and methods for instationary, transient states.

As against circular symmetric flame flows, testing of asymmetric flame evolution requires of course much more measuring spots, and just these asymmetries involve risk of danger from operation aspects, such as oil burners in front firing, four corner burner pulverized fuel firing but only with three burners etc.

In such cases of geometrical asymmetry, the absolute value of heat evolution and the ratio of heat evolution to heat emission depends on the given spot.

In addition to "temperature distribution" other kinds of distribution can be spoken of, that are related to the temperature distribution.

1. When examining *temperature distribution*, distinction should be made between that calculated and measured. For the first case, there are some approximations available (e.g. to calculate the radiation from short flame sections normal to the flame axis).

For measured temperature distributions, the measurement method has to be indicated. It should be considered namely that the measured true body temperature value is independent of the applied measurement method only for absolute black bodies. For real bodies, different measuring methods (colour temperature, brightness temperature, total radiation temperature, temperature obtained by exhaustion pyrometry) yield different values. What is more, determination of the temperature distribution vs. location should be made by the same method in any point, and temperatures obtained simultaneously in several points should be considered as correlated, temperature oscillating even in stationary state because of flame pulsation. Brightness temperature measured by green filter is of practical advantage (wavelength of 5300 to 5400 Å), it better approaching the true flame temperature.

Up to now, heat transfer by convection in combustion chamber has been neglected beside radiation (permissible for boilers), just as the geometry of surfaces involved into the heat exchange by radiation (angular factor).

2. Also *total radiation distribution* with respect to place can be spoken of, proportional to the product of the fourth power of the true temperature by the emissivity coefficient, easy to measure in common but cannot be distinguished without several, simultaneous measurements. Because of the variation of the emissivity coefficient, the radiation distribution is not certainly coincident with the temperature distribution.

3. From the variation with respect to place of the *emissivity coefficient*, conclusions can be drawn on the state or run of burning process. Temperature measuring methods are available, assuming some correlations between different methods, based on the Planck or Wien law. Since the emissivity coefficient may also depend on wavelength e.g. for soot, for any local co-ordinate an emissivity coefficient distribution may be ordered as a function of wavelength.

4. From the above it is evident that the emissivity coefficient variation causes the flame *intensity distribution* curves not to fit the Planck curves for absolute black bodies at a given point. Their continuity is affected by the selectivity of gas radiation, namely the gases are known to emit striped spectra superimposing the continuous radiation curves for solid bodies or disperse media.

Summary

The Stefan—Boltzmann formula, derived from the Planck—Wien law, is valid for absolute black bodies. It fits also "gray" radiation (the emissivity coefficient independent of wavelength), leading to integrated values for heat exchange by radiation in combustion chambers. It gives also a fair approximation for exit temperatures of combustion chamber.

The Stefan—Boltzmann formula is valid with restrictions to the determination of radiation of luminous flames and combustion products, much affected by the place, wavelength and time dependence of the emissivity coefficient.

As concerns space distribution, no approximations are known but for simple cases, temperature notions being correlated to measuring methods. There is no exact method to determine the true temperature of luminous flames. According to Kutateladse, the true temperature is confined by two limits: the lower limit is the colour temperature, and the upper is given by the Na-reversing method.

In practice, optical pyrometry, spectroscopy are likely to help determination of temperature, total radiation and emissivity coefficient distributions. They are important in that they help to learn variation of the burning process with time and place, no other method being available up to now. Knowledge of these distributions is of importance for the estimation of effect and variation of firing boundary conditions.

For high intensity burning processes, study of temperature and radiation distributions is of interest from the aspect of local load shape.

References

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