EXAMINATION OF UNSTEADY GAS FLOW IN THE SUCTION PIPE OF RECIPROCATING COMPRESSORS

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Preface

The study of the unsteady gas flow in the suction pipes of reciprocating compressors enables the determination of the most important parameters characteristic to their operation.

The proposed calculation process makes it possible to establish the way in which the volume of gas streaming into the cylinder space during each suction stroke can be influenced by varying the characteristics of the compressor and the attached suction system, and the calculation results allow conclusions to be drawn as to the stresses imposed on the compressor valve as well as the optimum hydraulic design of the valve. The process furthermore helps to determine the detuning conditions of the detrimental phenomena of resonance arising in the suction system.

The calculation process presented herebelow relies upon methods evolved for the examination of other reciprocating-type machines, primarily of Diesel engines [1, 2] complementing them with information on the peculiarities of compressor operation.

Determination of the flow in the suction pipe

The aim of the calculation has been to determine the changes in time of the physical parameters of flow in the suction pipe.

Assuming constant entropy of the medium streaming in a gas flow regarded to be one-dimensional, the following relationships may be written down:

$$\Phi_{tt} + 2\Phi_x \Phi_{xt} + \Phi_{xx}(\Phi_x^2 - A^2) = 0,$$

where

$$\Phi_x = U = \frac{\partial \Phi}{\partial x}$$

and

$$\Phi_{xt} = \frac{\partial U}{\partial Z},$$

In view of the fact that it is preferable to carry out the numerical calculations by a computer, the method of the characteristics elaborated by NEUMANN [3] seemed best suited to solve the above quasi-linear hyperbolic partial differential equation of the second order, with two variables.

The slope of the projections of the characteristic lines on the physical plane $[X, Z]$, i.e., the velocity of the elementary shock waves can be calculated in the following way:

$$\frac{dX}{dZ} = U \pm A. \tag{1}$$

The relationship between the velocity differences of gas flow (U) and the variations in the sound velocity along the characteristic lines can be written in the following form:

$$dU = \pm \frac{2}{\varkappa - 1} dA \tag{2}$$

The following relationships are obtained as the solution of the differential equation (2):

$$A + \frac{\varkappa - 1}{2} U = \text{constant} = \lambda$$

$$A - \frac{\varkappa - 1}{2} U = \text{constant} = \beta$$

The values of A and U can be determined from the equations:

$$A = \frac{\lambda + \beta}{2} \tag{3}$$

$$U = \frac{\lambda - \beta}{\varkappa - 1} \tag{4}$$

Making use of equations (3) and (4), the relationship (1) can be transformed in the following manner:

$$\left|\frac{dX}{dZ}\right|_{\lambda = \text{const.}} = \left[\frac{\varkappa + 1}{2(\varkappa - 1)}\right]\lambda - \left[\frac{3 - \varkappa}{2(\varkappa - 1)}\right]\beta \tag{5}$$

and

$$\left|\frac{dX}{dZ}\right|_{\beta = \text{const.}} = \left[\frac{3 - \varkappa}{2(\varkappa - 1)}\right]\lambda - \left[\frac{\varkappa + 1}{2(\varkappa - 1)}\right]\beta \tag{6}$$

The suction pipe, taken to be of unit length, is divided into lengths of ΔX, the change of state taking place at time Z to intervals ΔZ (See Fig. 1). Assuming that the values of λ and β at the interfaces of the ΔX lengths of the pipe at time Z_i are known, we draw the tangents of the characteristics setting out from the points marked with r and $(r + 1)$ (taken to be identical with the characteristics along a short length, and regard accordingly the values of λ_r and β_{r+1} to be constant. This means that in the intersections of the characteristics and the interfaces of the sections, the values of λ_r and β_{r+1}

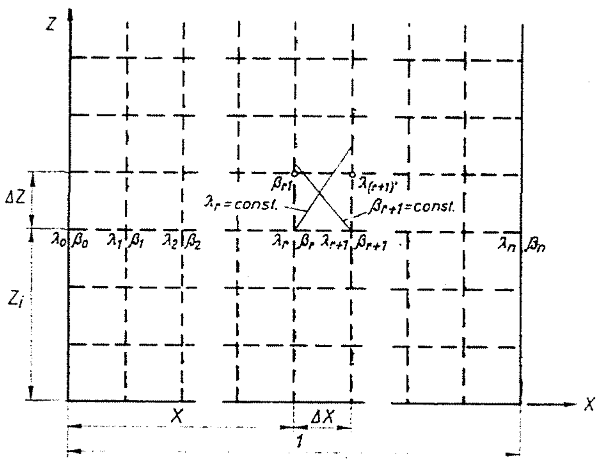

Fig. 1

are regarded to be valid. In the points r' and $(r + 1)'$, on the other hand the determination of $\lambda_{(r+1)'}$ and $\beta_{r'}$ is carried out by linear interpolation, in the following manner:

$$\lambda_{(r+1)'} = \frac{\Delta Z}{\Delta X}\left[\frac{dX}{dZ}\right]_{\lambda_r}(\lambda_r - \lambda_{r+1}) + \lambda_{r+1} \tag{7}$$

and

$$\beta_{r'} = \frac{\Delta Z}{dX}\left|\left[\frac{dX}{dZ}\right]_{\beta_{r+1}}\right|(\beta_{r+1} - \beta_r) + \beta_r. \tag{8}$$

When choosing the value of ΔZ with a given ΔX, the stability criterion must be taken into consideration according to which:

$$\left|\frac{\Delta Z}{\Delta X}\right|_{\max} < \frac{1}{\left|\left[\frac{dX}{dZ}\right]_\beta\right|_{\max}} < \frac{1}{\left|\left[\frac{dX}{dZ}\right]_\lambda\right|_{\max}}.$$

Along the lines $[X = 0, Z]$ and $[X = 1, Z]$ in the mesh points, only the values of β_0 and λ_n can be obtained in this way, while the values of λ_0 and β_n can be determined from the boundary conditions to be described later. For the solution of the values of λ and β along the line $[X, Z = 0]$ as the initial data must, naturally, be known.

Boundary conditions

The block schematics of the compressor and the attached suction system are shown in Fig. 2. Gas, at atmospheric pressure, passes through the suction pipe having a rounded-off suction orifice then through the free-controlled Hörbiger valve, to reach the cylinder space. The model corresponds to a single-stage air compressor which draws air from the atmosphere without a filter.

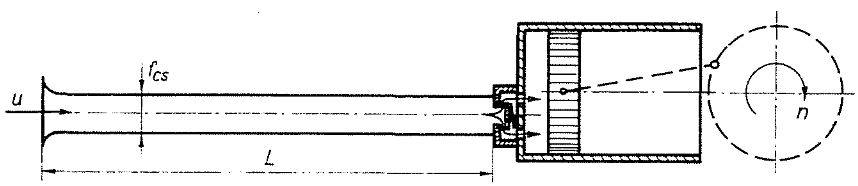

Fig. 2

The piston path within the stroke taken as given, is determined by a crank drive having a driving rod which, for an approximation, is regarded to be infinitely long and rotating at a fixed speed.

Flow in the suction pipe is produced by the piston movement while the piston and the valve moving in the cylinder of given geometry define the boundary conditions at one end of the suction line. At the opposite end of the suction pipe, the variations of the physical characteristics are determined by the rounded-off suction orifice and the infinitely large air space at atmospheric pressure, attached to it.

The difference between this model and the models representing the suction systems of reciprocating machines of other systems consists of the free-controlled valve which causes marked differences in the boundary conditions on the cylinder side.

Let us now look into the relationships expressing the boundary conditions, one by one.

1. *Open pipe end*

a) In the case of inflow ($\beta < 1$)

$$\lambda = \frac{3-\varkappa}{\varkappa+1}\beta + 2\sqrt{\frac{\varkappa-1}{\varkappa+1}\left[1 - \frac{2\beta^2}{\varkappa+1}\right]^{1/2}}. \tag{9}$$

b) In the case of outflow ($\beta > 1$)

$$\lambda = 2 - \beta$$

2. *Cylinder side*

a) With inflow into cylinder $\left(\lambda > P^{\frac{\varkappa-1}{2\varkappa}}\right)$.

$$\beta = \lambda - \left\{ \frac{2(\varkappa-1)\left[\left(\frac{\lambda+\beta}{2}\right)^2 - P^{\frac{\varkappa-1}{\varkappa}}\right]}{\frac{\left(\frac{\lambda+\beta}{2}\right)^{\frac{4}{\varkappa-1}}}{P^{\frac{2}{\varkappa}} D_7 \cdot \varphi^2} - 1} \right\}^{1/2} \tag{10}$$

Pressure and temperature prevailing in the cylinder can be calculated from the following differential equations:

$$\frac{dP}{dZ} = -\frac{\varkappa}{W}\left\{D_5 P \sin(2D_5 Z) - \frac{d\delta_{\text{in}}}{dZ}\left[\left(\frac{\lambda+\beta}{2}\right)^2 + \frac{\varkappa-1}{2}\left(\frac{\lambda-\beta}{2}\right)^2\right]\right\} \tag{11}$$

$$\frac{d\tau}{dZ} = \frac{\varkappa-1}{\varkappa}\,\frac{\frac{dP}{dZ}}{P}\,\tau + \frac{\frac{d\delta_{\text{in}}}{dZ}}{\delta}\left[\left(\frac{\lambda+\beta}{2}\right)^2 + \frac{\varkappa-1}{2}\left(\frac{\lambda-\beta}{2}\right)^2 - \tau\right]. \tag{12}$$

To define the variations of the charge coefficient in time, the following relationship is available:

$$\frac{d\delta_{\text{in}}}{dZ} = D_4\left(\frac{\lambda+\beta}{2}\right)^{\frac{2}{\varkappa-1}}\left(\frac{\lambda-\beta}{\varkappa-1}\right). \tag{13}$$

while the momentary charge coefficient is obtained by the integration of Eq. (13):

$$\delta = \delta_{\text{in}} + \delta_{\text{out}} = D_4\int_{Z_{\text{in1}}}^{Z_{\text{in2}}}\left(\frac{\lambda+\beta}{\varkappa-1}\right)^{\frac{2}{\varkappa-1}} dZ + \delta_{\text{out}} \tag{14}$$

where

Z_{in1} and Z_{in2} denote the beginning and end of inflow, respectively.

The momentary cylinder capacity is defined from:

$$W = D_6 + \frac{1}{2}\left[1 - \cos(2D_5 Z)\right]. \tag{15}$$

Eqs (10) to (15) and the system consisting of the equations are used for the calculation of the relative valve cross-section. φ enables the determination of β.

b) In the case of blowing out from the cylinder $\left(\lambda < P^{\frac{\varkappa - 1}{2\varkappa}} \right)$

$$\beta = \lambda \frac{3-\varkappa}{\varkappa+1} + 2 \sqrt{\frac{\varkappa-1}{\varkappa+1} \left[\tau - \frac{2}{\varkappa+1} \lambda^2 \right]^{1/2}} \tag{16}$$

The following equations are available to define the pressure and temperature in the cylinder:

$$\frac{dP}{dZ} = -\frac{\varkappa}{W} \left[D_5 P \sin(2D_5 Z) + \tau \left| \frac{d\delta_{\text{out}}}{dZ} \right| \right] \tag{17}$$

$$\frac{d\tau}{dZ} = \frac{\varkappa - 1}{\varkappa} \frac{\frac{dP}{dZ}}{P} \tau \tag{18}$$

The charge coefficient is calculated from the following equations:

$$\frac{d\delta_{\text{out}}}{dZ} = D_4 \frac{UP}{A^2} \left\{ \frac{\sqrt{1 + 2\tau f(\lambda, \beta, \varphi)} - 1}{\tau f(\lambda, \beta, \varphi)} \right\}^{\frac{\varkappa}{\varkappa-1}}, \tag{19}$$

where

$$f(\lambda, \beta, \varphi) = \frac{\frac{(\lambda - \beta)^2}{\varkappa - 1}}{D_8 \varphi^2 \left(\frac{\lambda + \beta}{2} \right)^4}.$$

The momentary charge is

$$\delta = \int_{Z_{\text{out1}}}^{Z_{\text{out2}}} \frac{d\delta_{\text{out}}}{dZ} dZ + \delta_{\text{in}},$$

where

Z_{out1}, Z_{out2} denote the instants of the beginning and end of the outflow, respectively.

c) In the case of closed valve: $\lambda = \beta$

Provided in the instant of closure cylinder capacity is W_z and pressure P_z, the pressure and temperature prevailing in the cylinder can be calculated in the following way:

$$P = P_z \left(\frac{W_z}{W} \right)^{\varkappa}$$

$$\tau = \tau_z \left(\frac{P}{P_z} \right)^{\frac{\varkappa - 1}{\varkappa}}.$$

3. *The motion of the automatic valve*

In consideration of the forces acting upon the valve disk, the following equation can be written:

$$m \frac{d^2 s}{dt^2} = \mu_1 f_t (p_t - p) - R_0 - ks$$

After rendering the equation dimensionless and proper rearrangement, we obtain the differential equation which determines the open valve cross-section:

$$\frac{d^2 \varphi}{dZ^2} + D_1 \varphi = D_2 [A_t^{\frac{\varkappa}{\varkappa - 1}} - P] - D_3 = g(Z) \tag{20}$$

where

$$A_t^2 = \left(\frac{\lambda + \beta}{2} \right)^2 + \frac{(\lambda - \beta)^2}{2(\varkappa - 1)}$$

The combined disturbing function in the right side of the differential equation $g(Z)$ at and close to the instant Z_i is substituted by a linear function.

Introducing the variable $Z^* = Z - Z_i$ within the time period

$$Z_i - \Delta Z_{-1} > Z_i < Z_i + \Delta Z_{+1}$$

the disturbing function can be written in the following form:

$$g(Z) = g(Z^*) = K_1 Z^* + K_2 .$$

While K_1 and K_2 are constant within one definite interval, they vary in each instant Z_i. Their value can be defined on the assumption that in each instant Z, Z^* is equal to 0, and consequently (Fig. 3),

$$K_1 = \frac{g(Z_i) - g(Z_{i-1})}{\Delta Z_{-1}}$$

$$K_2 = g(Z_i),$$

where

$$g(Z_i) = D_2 \left\{ \left[\left(\frac{\lambda_i + \beta_i}{2} \right)^2 + \frac{(\lambda_i - \beta_i)^2}{2(\varkappa - 1)} \right]^{\frac{\varkappa}{\varkappa - 1}} - P_i \right\} - D_3 .$$

Eq. (20) thus can be written in the following form:

$$\frac{d^2\varphi}{dZ^{*2}} = D_1\varphi = K_1 Z^* + K_2 . \tag{21}$$

Its solution is given by the equation

$$\varphi = K_3 \sin (Z^* \sqrt{D_1}) + K_4 \cos (Z^* \sqrt{D_1}) + \frac{K_1}{D_1} Z^* + \frac{K_2}{D_1} . \tag{22}$$

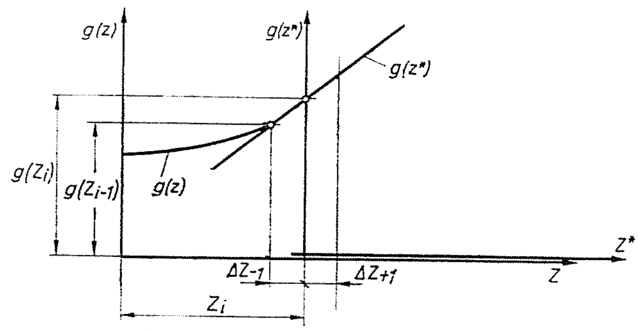

Fig. 3

The constants K_3 and K_4 can be determined from the starting conditions seen in the system of Z^* where

$$Z^* = 0$$

$$\varphi = \varphi_{Z_i}$$

$$\dot{\varphi} = \dot{\varphi}_{Z_i} ,$$

thus

$$K_3 = \frac{1}{\sqrt{D_1}} \left(\dot{\varphi}_{Z_i} - \frac{K_1}{D_1} \right)$$

$$K_4 = \varphi_{Z_i} - \frac{K_2}{D_1}$$

Under the given conditions the open valve cross-section can be defined from equation (22).

Calculation of the cylinder charge-up

Calculation starts at the instant $Z = 0$ with the assumption that the piston is in its upper dead centre position, the suction valve closed, atmospheric pressure prevails in both the suction system and the cylinder, and speed is zero.

The valve will open as soon as the pressure in the cylinder, on the effect of piston displacement, assumes the value of

$$P \leqq 1 - D_{10}.$$

The instant when equilibrium prevails in the valve can be calculated from the relationship:

$$Z = \frac{1}{2D_5} \arccos \left\{1 - 2D_6 \left[\frac{1}{(1 - D_{10})^{1/\varkappa}} - 1\right]\right\}$$

After a time ΔZ the valve will open by all certainty. To determine the degree of opening, the P and τ values are to be known. They can be calculated in the following manner:

$$P = \left\{\frac{D_6}{\frac{1}{2}[1 - \cos 2D_5 (Z + \Delta Z)] + D_6}\right\}^{\varkappa}$$

$$\tau = P^{\frac{\varkappa - 1}{\varkappa}}$$

The examination of the suction process can start with the assumption that in this time instant $\lambda = \beta = 1$.

Valve movement can be calculated from (22), cylinder pressure and temperature from (11) and (12), resp., the variations in the charge coefficient from (13). In the course of suction, the valve first opens entirely then begins to close. The instant when closure starts, is determined by the criterion

$$A_t^{\frac{2\varkappa}{\varkappa-1}} - P \leqq D_{10} + D_{11}.$$

Towards the termination of valve closure there is a backflow from the cylinder to the suction pipe. Under such conditions, cylinder pressure and temperature can be calculated from (17) and (18) resp., the variations in the charge coefficient from (19).

The physical characteristics in the suction pipe during compression vary according to the conditions valid for closed valves.

The beginning of the new suction stroke is determined by the boundary condition

$$A_t^{\frac{2\varkappa}{\varkappa-1}} - D_9 \left[\frac{D_6}{\frac{1}{2}(1 - \cos 2D_5 Z) + D_6}\right]^{1/\varkappa} \leqq D_{10}.$$

Since experience has shown that the results are very little influenced by consideration of the third or further suction strokes, calculations may be finished at the end of the second suction stroke.

Examinations of the factors affecting the charge coefficient

As stated, the objective of the calculations may be to determine the charge coefficient, or better, to see how the variations in the parameters of the compressor and the associated suction system influence the value of the charge coefficient. To reduce the great number of variables, they may be collected in dimensionless groups, producing so-called π numbers.

The constants D obtained by writing down the equations in dimensionless form include the dimensionless π numbers.

A total of twelve π numbers seem to be necessary. The first three contain the main dimensions and characteristics of the compressor, and since their effect is expected to be considerable, they should be pointed out specifically:

$$\pi_1 = \frac{nL}{a_0}$$

can be derived from the constant D_5 and its character conforms to that of the Strouhal number.

$$\pi_2 = \frac{f_{cs}\, a_0}{V_l \cdot n}$$

can be derived from D_4 and its character is the same as that of the Mach number.

$$\pi_3 = \frac{p_e}{p_0}$$

is identical with the constant D_9.

The four dimensionless groups herebelow show the main characteristics of the suction valve. Their variations affect not only the charge coefficient but also the conditions of the movement of the valve disk.

The numbers

$$\pi_4 = \frac{k}{m \cdot n^2},$$

$$\pi_5 = \frac{s_{\max} \cdot k}{p_0 f_l}$$

$$\pi_6 = \frac{R_0}{p_0 f_t}$$

$$\pi_7 = \frac{f_{sz\,\max}}{f_{cs}}$$

follow from the constants D_1, D_{11}, D_{10}, and D_7.

Further five π numbers, of slighter influence, are the following:

$$\pi_8 = \frac{V_K}{V_l}$$

$$\pi_9 = \mu_1$$

$$\pi_{10} = \mu_2$$

$$\pi_{11} = \mu_3$$

$$\pi_{12} = \varkappa$$

What has gone before shows that even with fixed characteristics of the compressor, the charge coefficient can be optimized by varying certain parameters (for instance the length of the suction pipe L).

Summary

The calculation process outlined in the paper, setting out from the methods applied for similar reciprocating machines, enables the determination of the unsteady gas flow arising in the suction system of machines operating with automatic valve, and the examination of the process of the charging up of the compressor cylinder. By forming dimensionless numbers from the parameters of the compressor and the suction system, author discloses the shortest way of examining the conditions which affect the charge coefficient.

Symbols

u	[m/s]	the velocity of the medium in flow
a	[m/s]	local sound velocity
a_0	[m/s]	sound velocity under atmospheric conditions
x	[m]	length along the suction pipe
L	[m]	total length of suction pipe
t	[s]	time
$\varkappa$		exponent of the isentropic change of state
λ, β		Riemann variables
p	$\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$	pressure in the cylinder space
p_0	$\left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$	atmospheric pressure
T	[°K]	temperature in the cylinder space

T_0	[°K]	atmospheric temperature
V_h	[m³]	momentary cylinder capacity
V_l	[m³]	displacement
V_k	[m³]	idle space
ϱ_0	$\left[\frac{\text{kg}}{\text{m}^3}\right]$	density of medium under atmospheric conditions
M	[kg]	momentary gas volume in the cylinder space
μ_2		contraction factor during inflow through the cylinder
p_e	$\left[\frac{\text{N}}{\text{m}^2}\right]$	backpressure
μ_3		contraction factor with backward blow
$f_{sz\,\max} = s_{\max} \cdot k_{cs}$	[m²]	maximum free valve cross-section
$f_{sz} = s \cdot k_{sz}$	[m²]	momentary free valve cross-section
f_t	[m²]	surface of valve disk
R_0	[N]	prestress of valve spring
k	[N/m]	spring constant
s	[m]	momentary value of valve disk displacement
$s_{\max}$	[m]	maximum valve shift
m	[kg]	the swinging mass of the valve
μ_1		proportionality coefficient
k_{sz}	[m]	inflow circumference of valve disk
f_{cs}	[m²]	cross-section of suction pipe
n	[1/p]	r.p.m. of the compressor
U	$\frac{u}{a_0}$	
A	$\frac{a}{a_0}$	
X	$\frac{x}{L}$	
Z	$\frac{a_0 t}{L}$	
P	$\frac{p}{p_0}$	
τ	$\frac{T}{T_0}$	
W	$\frac{V_h}{V_l}$	

$\delta = \frac{M}{\varrho_0 V_l}$ momentary charge coefficient

$\varphi = \frac{f_{sz}}{f_{sz\max}}$ momentary relative valve opening

$$D_1 = \frac{k}{m} \frac{L^2}{a_0^2}$$

$$D_2 = \frac{\mu_1 k_{sz} f_t L^2 p_0}{m f_{sz\max} a_0^2}$$

$$D_3 = \frac{k_{sz} L^2 R_0}{m f_{sz\max} a_0^2}$$

$$D_4 = \frac{f_{cs} L}{V_l}$$

$$D_5 = \frac{\pi}{60} \frac{nL}{a_0}$$

$$D_6 = \frac{V_k}{V_l}$$

$$D_7 = \left[\frac{f_{sz\max}\,\mu_2}{f_{cs}}\right]^2$$

$$D_8 = \left[\frac{f_{sz\max}\,\mu_3}{f_{cs}}\right]^2$$

$$D_9 = \frac{p_e}{p_0} \text{ pressure ratio}$$

$$D_{10} = \frac{R_0}{f_t \cdot p_0}$$

$$D_{11} = \frac{S_{\max} \cdot k}{p_0 f_t}$$

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