THE SCALE EFFECT ON NOMINAL WAKE FRACTION OF SINGLE-SCREW SHIPS

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Symbols

a, b, d, e, g, h, k		constants
w_N		nominal wake fraction
w_{NF}		frictional component of nominal wake fraction
A_0	m^2	propeller disc area
B	m	breadth of ship
C_S		fullness of wetted surface of ship
C_F		frictional resistance coefficient
D	m	diameter of propeller
L	m	length of waterline
L_p	m	length between perpendiculars
Re		Reynolds number of ship
S	m^2	wetted surface of ship hull
T	m	draught of ship
V	ms^{-1}	shipspeed
V_{AN}	ms^{-1}	nominal propeller advance speed
ϱ	$kpsm^{-4}$	denisity of water
ν	m^2s^{-1}	kinematic viscosity of water
λ		model scale

Usually there are two reasons for measuring the velocities behind the shipmodel in towing condition without propeller:

1. The knowledge of velocity distribution in the wake of any ship gives us a possibility to determine the viscous resistance component of the ship and in this way we can separate the different resistance components.

2. The knowledge of the mean value of the velocity in the place of the propeller gives us a possibility to determine the resistance coefficient between the ship hull and the water going through the propeller disc area.

This paper deals with the investigation of the latter.

The mean velocity in the place of a propeller of a towed ship without any acting propeller is characterized by the "nominal wake fraction":

$$w_N = \frac{V - V_{AN}}{V}$$

where V is the shipspeed. The "nominal propeller advance speed" (V_{AN}) is the mean value of the axial components of the measured relative velocities in the place of propeller near to the hull of the ship or her model towed without propeller.

The local nominal wake fraction and the nominal wake fraction of any propeller radius can be spoken of. In the former case the velocity measured at a point is used instead of V_{AN}, and in the latter one the mean value of the velocities measured on a circle is in the formula of the nominal wake fraction.

The value of the nominal wake fraction of a ship can be determined only with the aid of a model experiment. Up to now it was assumed that the nominal wake fraction of any ship is equal or roughly equal to the nominal wake fraction of her model. But according to the investigations of different model families (geosims) it seems that the models made in different sizes give us different values of the nominal wake fraction for the same shipspeed. E.g. the values of nominal wake fraction are the following at 15 knots shipspeed in the case of Victory geosim [1]:

model scale	50	40	36	30	25	23	18	6
$10^3 \cdot w_N$	434	404	402	376	380	362	371	317

The values greatly differ in the cases of the smallest and biggest models. Apart from the jumping values of the models made at a model scale of 18 and 25, we can say that the nominal wake fraction of models are changing with the model scale and so we can assume a sort of scale effect. In the practice we must recalculate the measured model data to the actual ship, therefore, it is of importance to clarify the scale effect on the nominal wake fraction.

There are three reasons for the difference between the nominal advance speed (V_{AN}) and the ship speed (V):

1. The potential flow around the ship in a perfect fluid gives a relative velocity in the place of the propeller which is different from the shipspeed, also in deeply submerged conditions of ship's body. Usually the lines of water-lines are convergent at the place of propeller, and so this relative velocity is lower than the shipspeed.

2. The local velocity of water in the stern wave system of the ship moving on the surface of fluid gives a second component, which changes the velocity in the place of the propeller, too.

3. In the case of real fluid, there is a boundary layer near to the ship hull. The velocities are lower in the boundary layer, and therefore in a real fluid the ship has lower velocities near to her hull in the place of propeller than in the case of a perfect fluid.

Thus, we can resolve the nominal wake fraction into three components: potential component, wave component and frictional or viscous component.

In an ideal stream the flow pattern near to the ship is defined by the ship form only. Therefore, the potential component of the nominal wake fraction is the same for ship and her model, or in the case of a geosim. In the real fluid the boundary layer has different thicknesses depending on local Reynolds number and the relative roughness of the surface. Thus the thickness of the boundary layer is different at the ship and her models made in different sizes. According to the law of continuity when the boundary layer is thicker than the velocities outside the boundary layer must be higher and so the potential component of the nominal wake fraction changes little in the real fluid at the geometrically similar ship having different sizes.

The second component, the wave component is the function of the Froude number. Therefore, in the perfect fluid this component is the same by the different models as when we use the Froude's law in our experiments. The changes of the thickness of the boundary layer at the different models influence this component a little too.

The third component, the frictional component is very different by the ship and her model, made in different sizes because the velocities in the boundary layer are different in these cases.

The purpose of our investigation is to determine how much the nominal wake fraction of a ship differs from the nominal wake fraction measured with a model. Similarly to the investigation of the scale effect on the other self-propulsion factors the results of the experiments with model families can be made use of.

At first approach the measured results show that the value of the nominal wake fraction is bigger by models having smaller sizes. The model surfaces can be regarded hydrodynamically as smooth ones. The frictional resistance coefficient of a smooth surface is the function of the Reynolds number only. Therefore, the frictional coefficients of the smaller models are greater i.e. the increase of the nominal wake fraction with the decrease of the model length is justified.

The water going through the disc area of the propeller has a velocity decrease from V (velocity at the bow) to V_{AN} (velocity in the place of propeller). According to the theorem of momentum:

$$\varrho \cdot A_0 \cdot V_{AN} \cdot V = \varrho \cdot A_0 \cdot V_{AN} \cdot V_{AN} + \left(\frac{1}{2} \cdot \varrho \cdot C_F \cdot V^2 \cdot S\right) \cdot k \tag{1}$$

where ϱ is the density of water, A_0 is the propeller disc area, C_F the frictional resistance coefficient of the hull surface, and S is the wetted surface of the ship. The parenthetical part of the last member is the frictional resistance of the ship-hull (R_F). The resistance of the water going through the place of propeller ($A_0\ V_{AN}$) is only a part of the above mentioned frictional resistance ($k \cdot R_F$).

Divided by $\varrho \cdot A_0 \cdot V^2$

$$\frac{V_{AN}}{V} = \left(\frac{V_{AN}}{V}\right)^2 + \frac{1}{2} C_F \cdot k \cdot \frac{S}{A_0},$$

taken from the expression of the nominal wake fraction

$$\frac{V_{AN}}{V} = 1 - w_{NF}$$

and substituting it into the above relation we obtain

$$(1 - w_{NF})\, w_{NF} = \frac{1}{2} \cdot C_F \cdot k \cdot \frac{S}{A_0}. \tag{2}$$

A similar expression can be obtained when writing the kinetic energy loss [2], [3], [4].

The value of k depends on the shipform, the relative location of propeller, the ratio of the wetted surface to the propeller disc area, and mainly on the frictional resistance coefficient. The geometrically similar models have the same form, relative location and ratio. Thus it can be stated that the value of the frictional component of the nominal wake fraction (w_{NF}) is merely the function of the frictional resistance coefficient.

The other two components of the nominal wake fraction (the potential and the wave components), as we have seen, differ only slightly by the ship and her models made on various model scales. This slight difference occurs because the thicknesses of the boundary layer of the models made on different scales are not geometrically similar. The thickness of the boundary layer changes with the Reynolds number, just as the frictional coefficient (C_F) in the cases of hydrodynamically smooth surfaces. So the boundary layer thickness of the different models of any ship can be regarded as the function of the frictional coefficient.

Consequently, the total nominal wake fraction can be divided into two parts. One varies with the frictional coefficient, the other is constant in the case of the same shipform:

$$(1 - w_N)\, w_N = f(C_F) + \text{const}.$$

According to the literature of this problem up to now three geosims (Victory, Strinda and Meteor) have measured data of the nominal wake fractions. The Victory geosim was investigated by the NSMB Wageningen [1], the other two geosims by Die Versuchsanstalt für Wasserbau und Schiffbau Berlin [5], [6], [7].

The measured data of the nominal wake fractions (w_N), the values of $(1 - w_N) \cdot w_N$ and the frictional resistance coefficients (C_F) are to be found

Table 1

Victory shipspeed 15 knots								
Scale	6	18	23	25	30	36	40	50
$10^3 \cdot C_F$	2.15	2.92	3.10	3.20	3.33	3.51	3.63	3.84
$10^3 \cdot w_N$	317	371	362	380	376	402	404	434
$(1-w_N)w_N$	0.216	0.233	0.231	0.236	0.234	0.240	0.240	0.246

Strinda		Shipspeed in knots				
Scale		12	14	16	17	18
7.5	$10^3 \cdot C_F$	2.37	2.30	2.26	2.24	—
	$10^3 \cdot w_N$	346	334	322	319	—
	$(1-w_N)w_N$	0.226	0.222	0.218	0.217	—
25	$10^3 \cdot C_F$	3.12	3.05	2.99	—	2.92
	$10^3 \cdot w_N$	400	393	383	—	374
	$(1-w_N)w_N$	0.240	0.238	0.236	—	0.234
35	$10^3 \cdot C_F$	3.44	3.35	3.26	—	3.19
	$10^3 \cdot w_N$	415	407	400	—	396
	$(1-w_N)w_N$	0.242	0.241	0.240	—	0.239
45	$10^3 \cdot C_F$	3.71	3.61	3.50	—	3.44
	$10^3 \cdot w_N$	420	414	408	—	403
	$(1-w_N)w_N$	0.244	0.242	0.241	—	0.240
55	$10^3 \cdot C_F$	—	3.83	3.72	—	3.64
	$10^3 \cdot w_N$	—	420	414	—	410
	$(1-w_N)w_N$	—	0.244	0.242	—	0.242

Meteor		Shipspeed in knots			
Scale		8	10	12	14
13.75	$10^3 \cdot C_F$	3.32	3.19	3.09	3.00
	$10^3 \cdot w_N$	162	152	143	138
	$(1-w_N)w_N$	0.136	0.129	0.123	0.119
19	$10^3 \cdot C_F$	3.62	3.47	3.36	3.26
	$10^3 \cdot w_N$	182	172	163	158
	$(1-w_N)w_N$	0.149	0.143	0.136	0.133
25	$10^3 \cdot C_F$	3.90	3.74	3.60	3.50
	$10^3 \cdot w_N$	206	192	176	164
	$(1-w_N)w_N$	0.164	0.155	0.149	0.137

in Table 1. The frictional resistance coefficients are calculated with the ITTC formula

$$C_F = 0.075 \cdot (\lg \mathrm{Re} - 2)^{-2}$$

where the Reynolds number is

$$Re = \frac{V \cdot L}{\nu}$$

(V is the shipspeed, L the length of ship, ν the kinematic viscosity of water).

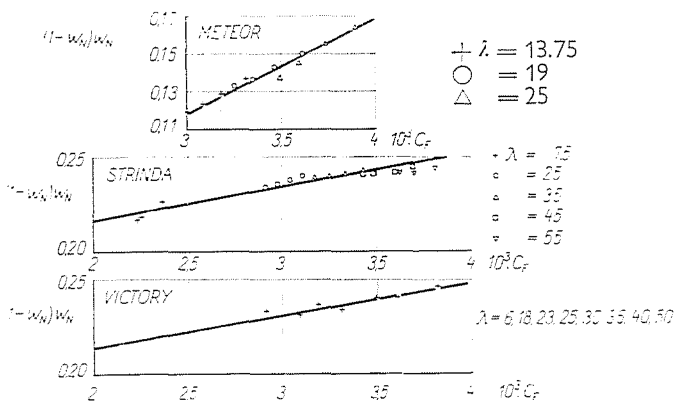

Fig. 1

In Fig. 1 the values of $(1 - w_N)w_N$ are plotted against C_F. We can draw straight lines through the figured points in the cases of all three geosims. These lines can be written with the following equation

$$10^3 \cdot (1 - w_N)\, w_N = d \cdot 10^3 \cdot C_F + e \tag{3}$$

where the values of d and e are:

	d	e
Victory	17	180
Strinda	18	180
Meteor	51	− 36

In Fig. 2 the values of $(1 - w)w$ are shown on the basis of w (the continuous line). The difference between the nominal wake fraction of the smallest model and that of the actual ship is about 0.1 for a geosim. From Fig. 2 we can see that in this narrow range of the curve $(1 - w)w$, it can be repalced by a straight line (with dotted lines). Therefore, the following approximation can be written:

$$(1 - w_N)\, w_N = g \cdot w_N + h$$

From this

$$w_N = \frac{(1 - w_N) \cdot w_N - h}{g} = \frac{d \cdot C_F + e - h}{g}$$

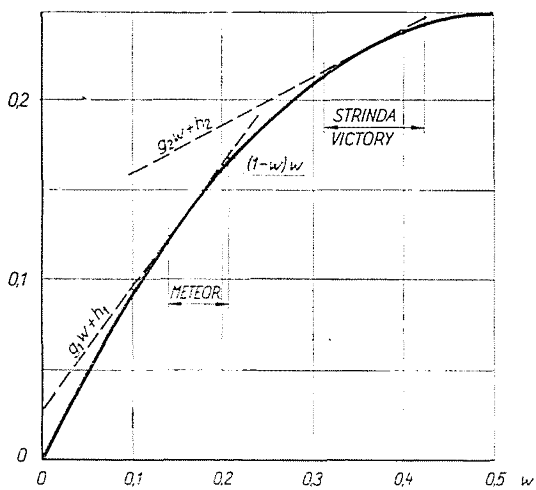

Fig. 2

Using the following substitution

$$a = \frac{d}{g} \qquad\qquad b = \frac{e - h}{g} \tag{4}$$

the nominal wake fraction can be written as follows

$$w_N = a \cdot C_F + b\,.$$

In Fig. 3 the values of the nominal wake fractions (w_N) are plotted against C_F. As it is to be seen we can really draw straight lines in all three cases described by the following equation:

$$10^3 \cdot w_N = a \cdot 10^3 \cdot C_F + b\,. \tag{5}$$

The values of a and b are:

	a	b
Victory	64	174
Strinda	65	180
Meteor	76	—92

According to equations No. 4. the values of the constants g and h are:

	g	h
Victory	0.266	0.134
Strinda	0.277	0.130
Meteor	0.670	0.026

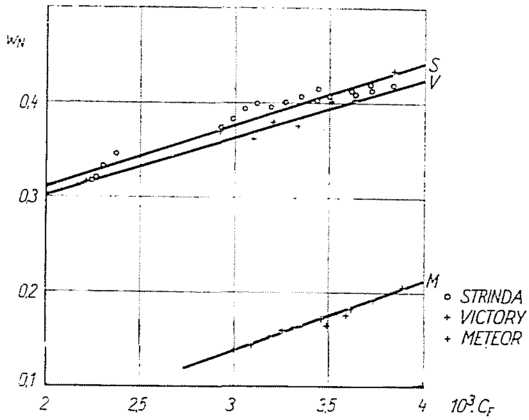

Fig. 3

The lines calculated with these constants ($g \cdot w_N + h$) are also drawn in Fig. 2. We can see that the dotted straight lines approximate very well the curve of $(1 - w)w$. According to the equation (2) the value of a in the formula (5) must be proportional to the value of

$$\frac{1}{2} k \frac{S}{A_0} .$$

Factor k also depends on the geometrical data. In order to take the shipform into consideration the length and breadth ratio can be used according to the results of the theoretical investigations of the wake fraction [8]. If the ship is narrow, her wake fractions are smaller, therefore we can assume that k is proportional to B/L_p ratio. Instead of the wetted surface we use the dimensionless value of wetted surface

$$C_S = \frac{S}{(2 \cdot T + B) \cdot L} .$$

The principal dimensions of the investigated ships and a few of their dimensionless ratios are shown in Table 2. The drafts of the aft end of these ships made on the same scale are to be seen in Fig. 4. The diameter of screws relative to the shiphull are very similar in all three cases. The ratios D/L_p and D/T are very near together. Therefore, there is no possibility of investigating the influence of the value of propeller disc area (A_0).

Table 2

		Victory	Strina	Meteor
Length of waterline	L m	135.562	172.0	77.30
Length between perpendiculars	L_p m	133.045	168.0	72.80
Breadth	B m	18.898	22.7	13.50
Draught	T m	8.687	9.42	4.80
Diameter of propeller	D m	5.3	5.7	2.9
Wetted surface of ship	S m	3 687	5 665	1200
Deplacement	∇ m³	15 019	26 769	2650
Fullness of wetted surface	C_S	0.750	0.792	0.673
Ratios:	L_p/B	7.04	7.40	5.40
	D/L_p	0.0391	0.0331	0.0375
	D/T	0.610	0.605	0.605

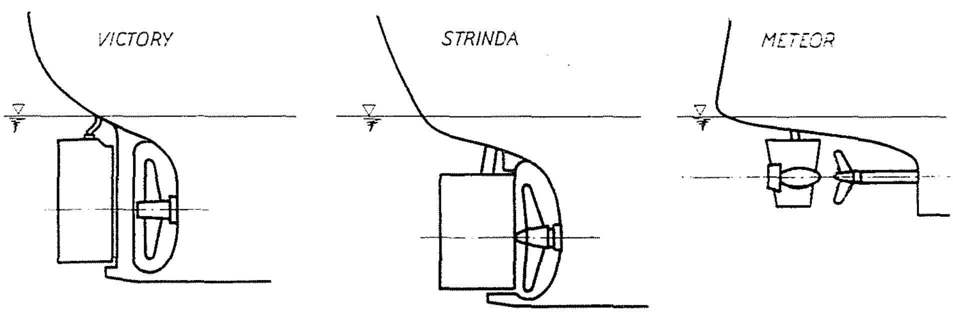

Fig. 4

Thus we can assume that the value of a is determinable in the following way

$$a = \text{constant} \cdot C_S \cdot \frac{B}{L_p} .$$

According to the results of the above mentioned investigations, the value of the constant is 605. When we calculate with the equation

$$a = 605 \cdot C_S \cdot \frac{B}{L_p}$$

we obtain the following values:
for the Victory family

$$a = 605 \cdot 0.750 \cdot \frac{19.898}{133.045} = 64.4 \cong 64$$

for the Strinda family

$$a = 605 \cdot 0.792 \cdot \frac{22.7}{168.0} = 64.8 \cong 65$$

for the Meteor family

$$a = 605 \cdot 0.6725 \cdot \frac{13.50}{72.80} = 75.5 \cong 76$$

The constant b in formula (5) represents that component of the nominal wake fraction which is independent of the viscosity of the water, therefore, it is approximately the potential component of the nominal wake fraction, i.e. the wave component is very small because in the figures we cannot see any effect of the Froude's number; at different shipspeeds the values of the nominal wake fraction change only with frictional coefficient.

The potential component of nominal wake fraction is the function of the form of shipbody and the relative location of the propeller. The small number of investigated model families did not give possibility to study this problem more deeply.

Conclusion

The measured value of the nominal wake fraction of a geometrically similar shiphull made on different model scales merely depends on the frictional resistance coefficient of the shiphull. This function is a linear one in the cases of the investigated single screw ships.

The wave component of the nominal wake fraction is negligible because at different shipspeeds (at different wave systems) the value of nominal wake fraction is only the function of frictional resistance coefficient.

According to the results of the investigated three model families we can make the following approximative formula

$$w_N = 605 \cdot C_F \cdot C_S \cdot \frac{B}{L_p} + b$$

where the value of b is constant for a model family.

When the results of the subsequent investigations of model families justify the formula, or these results give a more common relation, then the measured nominal wake fraction of a single model will be enough for the exact determination of the nominal wake fraction of a ship having different roughnesses.

At present we can take the results of two different models. (Two models made in various model scales or one model investigated with two different roughnesses of its surface.) The measured nominal wake fractions of these two models plotted against the frictional coefficient define a straight line. According to this line we can get the wake fraction of the ship.

The results of the investigations show that the frictional component of the nominal wake fraction is independent of the location of the propeller in the direction of the propeller axis. We obtained the same constant (605) for all three model families, while the distance between the propeller and the shiphull was very different.

The ratios of the dimension of the diameter of the screw propeller and the shiphull influence the value of nominal wake fraction. In the investigated cases the sizes of the propellers were very similar in relation to the shiphull. Therefore, the investigation of the influence of propeller size will be possible by means of other investigations where ratios D/L_p and D/T will have other values than the model families investigated up to now.

Summary

According to the results of the investigations of several model families the nominal wake fraction of a ship is not equal to the nominal wake fraction of her model. The viscous component of this fraction depends on the frictional resistance coefficient of shiphull. Therefore, the difference between the nominal wake fractions of geometrically similar ships (of a ship and her models) can be determined by knowing the function $w_N = f(C_F)$. In the cases of the investigated single-screw ships this function is approximately a linear one.

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