# ON THE ACCURACY AND RAPIDITY OF EVALUATION OF ONE-NODE NATURAL FREQUENCY AND ANGULAR AMPLITUDES OF UNDAMPED MULTICYLINDER ENGINE SYSTEMS

By

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## 1. Introduction

Several methods have been developed to find the one-node frequencies of undamped engine systems. Some of them are very accurate but more time consuming. Other methods have improved quickness of evaluation but mostly approximate. It is always the aim of an engineer to combine both these diverse aspects and thus achieve rapidity as well as accuracy of assessment. True that some of these methods are not for the benefit of a newcomer but for the use of an experienced engineer who is very much pressed for time. The relative merits and demerits of different methods are discussed in the present paper and suggestions are given how to improve the accuracy of evaluation with reasonable rapidity.

The different procedures to evaluate the one-node natural frequency of a given system broadly fall into two categories.

(i) Methods which require the frequency equation and

(ii) Those which do not.

In the former group it is required just to solve an algebraic equation of higher order for its roots and this goes into the area of algebra. Forming the higher order algebraic equation itself is an interesting problem for an engineer also. One of the methods to form them from a determinant of specific form which occurs in the case of a multicylinder inline engine is given by CROSSLEY and GERMEN [1].

## 2. The polynomial equation

Considering an $n$-mass system shown in Fig. 1 the equations of motion could be written as

$$J_1 \ddot{\Theta}_1 + k_1 (\Theta_1 - \Theta_2) = 0 \tag{1}$$

$$J_2 \ddot{\Theta}_2 + k_1 (\Theta_2 - \Theta_1) + k_2 (\Theta_2 - \Theta_3) = 0 \tag{2}$$

$$J_3 \ddot{\Theta}_3 + k_2 (\Theta_3 - \Theta_2) + k_3 (\Theta_3 - \Theta_4) = 0 \tag{3}$$

etc. until

$$J_n \ddot{\Theta}_n + k_{n-1} (\Theta_n - \Theta_{n-1}) = 0 \tag{4}$$

where $k_1, k_2, k_3, \ldots, k_{n-1}$ are the spring constants of the connecting shafts, $J_1, J_2, J_3, \ldots, J_n$ the polar moments of inertia, $\Theta_1, \Theta_2, \Theta_3, \ldots, \Theta_n$ the angular amplitudes of different masses.

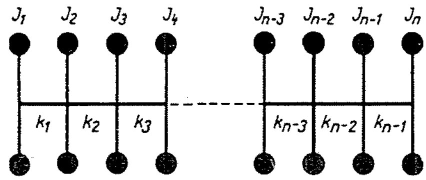


*Fig. 1*

If the system is assumed to be vibrating simple harmonically with a natural frequency $\omega$ then

$$\ddot{\Theta} = -\omega^2 \Theta \tag{5}$$

Substituting 5 in equations 1, 2, 3, and 4

$$(k_1 - J_1 \omega^2) \Theta_1 - k_1 \Theta_2 = 0 \tag{6}$$

$$-k_1 \Theta_1 + (k_1 + k_2 - J_2 \omega^2) \Theta_2 - k_2 \Theta_3 = 0 \tag{7}$$

$$-k_2 \Theta_2 + (k_2 + k_3 - J_3 \omega^2) \Theta_3 - k_3 \Theta_4 = 0 \tag{8}$$

etc. until

$$-k_{n-1} \Theta_{n-1} (k_{n-1} - J_n \omega^2) \Theta_n = 0 \tag{9}$$

Equations 6 to 9 can be written in the form of a determinant

$$\Delta = \begin{vmatrix} \Theta_1 & \Theta_2 & \Theta_3 \ldots & \Theta_n \\ a_{11} & a_{12} & 0 \ldots & 0 \\ a_{21} & a_{22} & a_{23} \ldots & 0 \\ 0 & a_{32} & a_{33} \ldots & 0 \\ \text{etc. until} & & & \\ 0 & 0 & 0 \ldots & a_{nn} \end{vmatrix} = 0 \tag{10}$$

where

$$a_{11} = k_1 - J_1 \omega^2 \qquad a_{12} = -k_1$$

$$a_{21} = -k_1 \qquad a_{22} = k_1 + k_2 - J_2 \omega^2 \qquad a_{23} = -k_2$$

$$a_{32} = -k_2 \qquad a_{33} = k_2 + k_3 - J_3 \omega^2 \qquad a_{34} = -k_3$$

Expanding the determinant we get a polynomial equation of $n$th degree in $\omega^2$

$$i.e.\quad \omega^2 [\omega^{2(n-1)} - A_1 \omega^{2(n-2)} + A_2 \omega^{2(n-3)} - \ldots + (-1)^{n-1} A_{n-1}] = 0 \quad (11)$$

where $A_1, A_2, A_3, \ldots, A_{n-1}$ depend on the physical constants of the system, namely the masses and springs. $\omega^2 = 0$ represents a rigid body rotation (not a vibratory motion) and hence can be omitted. There are $(n - 1)$ real values of $\omega^2$. To evaluate for $A_1, A_2, A_3, \ldots, A_{n-1}$ CROSSLEY and GERMEN have developed the following numerical method [1].

Considering a determinant of the form

$$\Delta \equiv \begin{vmatrix} (a_1 + b_1 - \omega^2) & -b_1 & 0 & 0 \ldots 0 \\ -a_2 & (a_2 + b_2 - \omega^2) & -b_2 & 0 \ldots 0 \\ 0 & -a_3 & (a_3 + b_3 - \omega^2) & -b_3 \ldots 0 \\ \text{etc.} & & & \end{vmatrix} = 0 \quad (12)$$

where

$$a_n = \frac{k_{n-1}}{J_n} \qquad \text{and} \qquad b_n = \frac{k_n}{J_n},$$

the solution for the above determinant can be illustrated by the following numerical example of an eight cylinder engine where

$J_1, J_2, \ldots, J_8 = 150$ lb in. sec$^2$,
$J_9 = 22.500$ lb in. sec$^2$,
$k_1, k_2, \ldots, k_7 = 20 \times 10^7$ lb in. sec$^2$,
$k_8 = 18 \times 10^7$ lb in./rad

The results are given in Table 1.

Col 2. Add the numbers in column 1 from the bottom up, putting the successive two rows higher.
Col 3. Multiply the numbers in Column 2 by those in column 1, row by row.
Col 4. Add the numbers of Column 3 from the bottom up and set two rows higher, such as for column 2.
Col 5. and all odd number columns: Multiply the numbers of the preceding column by those in Column 1 in the same row.
Col 6. and all even number columns: as with Columns 2 and 4.

Lastly, add the total numbers in each odd-number column. These will be the coefficients of the polynomial equation and the equation is

$$\omega^2 [\omega^{16} - 19.8747 \times 10^6 \omega^{14} + 159.5885 \times 10^{12} \omega^{12} - 663.3197 \times 10^{18} \omega^{10} +$$
$$+ 1516.3387 \times 10^{24} \omega^8 - 1866.5223 \times 10^{30} \omega^6 + 1117.4365 \times 10^{36} \omega^4 -$$
$$- 252.4550 \times 10^{42} \omega^2 + 9.4656 \times 10^{48}] = 0 \quad (13$$

**Table 1**

CROSSLEY's and GERMEN's

| Col. $1/10^6$ | Col. $2/10^6$ | Col. $3/10^{12}$ | Col. $4/10^{12}$ | Col. $5/10^{18}$ | Col. $6/10^{18}$ | Col. $7/10^{24}$ | Col. $8/10^{24}$ |
|---|---|---|---|---|---|---|---|
| $a_1 = \frac{k_0}{J_1} = 0$ | | | | | | | |
| $b_1 = \frac{k_1}{J_1} = 1.3333$ | 17.2076 | 22.9429 | 115.4804 | 153.9701 | 381.2022 | 508.2569 | 639.3792 |
| $a_2 = \frac{k_1}{J_2} = 1.3333$ | 15.8743 | 21.1652 | 96.1129 | 128.1474 | 276.5339 | 368.7026 | 382.0863 |
| $b_2 = \frac{k_2}{J_2} = 1.3333$ | 14.5410 | 19.3675 | 78.5031 | 104.6683 | 192.9745 | 257.2929 | 211.2186 |
| $a_3 = \frac{k_2}{J_3} = 1.3333$ | 13.2077 | 17.6098 | 62.6710 | 83.5593 | 128.1540 | 170.8677 | 104.9531 |
| $b_3 = \frac{k_3}{J_3} = 1.3333$ | 11.8744 | 15.8321 | 48.6166 | 64.8206 | 79.7019 | 106.2655 | 44.6236 |
| $a_4 = \frac{k_3}{J_4} = 1.3333$ | 10.5411 | 14.0544 | 36.3399 | 48.4521 | 45.2483 | 60.3295 | 14.7271 |
| $b_4 = \frac{k_4}{J_4} = 1.3333$ | 9.2078 | 12.2767 | 25.8408 | 34.4536 | 22.4229 | 29.8965 | 2.9199 |
| $a_5 = \frac{k_4}{J_5} = 1.3333$ | 7.8745 | 10.4991 | 17.1194 | 22.8254 | 8.8556 | 11.8072 | 0.0189 |
| $b_5 = \frac{k_5}{J_5} = 1.3333$ | 6.5412 | 8.7214 | 10.1757 | 13.5673 | 2.1758 | 2.9010 | |
| $a_6 = \frac{k_5}{J_6} = 1.3333$ | 5.2079 | 6.9437 | 5.0099 | 6.6798 | 0.0142 | 0.0189 | |
| $b_6 = \frac{k_6}{J_6} = 1.3333$ | 3.8746 | 5.1660 | 1.6212 | 2.1616 | | | |
| $a_7 = \frac{k_6}{J_7} = 1.3333$ | 2.5413 | 3.3885 | 0.0106 | 0.0142 | | | |
| $b_7 = \frac{k_7}{J_7} = 1.3333$ | 1.2080 | 1.6106 | | | | | |
| $a_8 = \frac{k_7}{J_8} = 1.3333$ | 0.0080 | 0.0106 | | | | | |
| $b_8 = \frac{k_8}{J_8} = 1.2000$ | | | | | | | |
| $a_9 = \frac{k_8}{J_9} = 0.0080$ | | | | | | | |
| $b_9 = \frac{k_9}{J_9} = 0.0000$ | | | | | | | |
| 19.8747 | | 159.5885 | | 663.3197 | | 1516.3387 | |

method to evaluate a large determinant

| Col. $1/10^6$ | Col. $9/10^{30}$ | Col. $10/10^{30}$ | Col. $11/10^{36}$ | Col. $12/10^{36}$ | Col. $13/10^{42}$ | Col. $14/10^{42}$ | Col. $15/10^{48}$ |
|---|---|---|---|---|---|---|---|
| $a_1 = \frac{k_0}{J_1} = 0$ | | | | | | | |
| $b_1 = \frac{k_1}{J_1} = 1.3333$ | 852.4843 | 504.6023 | 672.7862 | 147.3931 | 196.5192 | 7.0546 | 9.4060 |
| $a_2 = \frac{k_1}{J_2} = 1.3333$ | 509.4357 | 222.9485 | 297.2572 | 36.6619 | 48.8813 | 0.0448 | 0.0596 |
| $b_2 = \frac{k_2}{J_2} = 1.3333$ | 281.6178 | 83.0505 | 110.7312 | 5.2575 | 7.0098 | | |
| $a_3 = \frac{k_2}{J_3} = 1.3333$ | 139.9340 | 23.5539 | 31.4044 | 0.0336 | 0.0448 | | |
| $b_3 = \frac{k_3}{J_3} = 1.3333$ | 59.4966 | 3.9183 | 5.2242 | | | | |
| $a_4 = \frac{k_3}{J_4} = 1.3333$ | 19.6356 | 0.0252 | 0.0336 | | | | |
| $b_4 = \frac{k_4}{J_4} = 1.3333$ | 3.8931 | | | | | | |
| $a_5 = \frac{k_4}{J_5} = 1.3333$ | 0.0252 | | | | | | |
| $b_5 = \frac{k_5}{J_5} = 1.3333$ | | | | | | | |
| $a_6 = \frac{k_5}{J_6} = 1.3333$ | | | | | | | |
| $b_6 = \frac{k_6}{J_6} = 1.3333$ | | | | | | | |
| $a_7 = \frac{k_6}{J_7} = 1.3333$ | | | | | | | |
| $b_7 = \frac{k_7}{J_7} = 1.3333$ | | | | | | | |
| $a_8 = \frac{k_7}{J_8} = 1.3333$ | | | | | | | |
| $b_8 = \frac{k_8}{J_8} = 1.2000$ | | | | | | | |
| $a_9 = \frac{k_8}{J_9} = 0.0080$ | | | | | | | |
| $b_9 = \frac{k_9}{J_9} = 0.0000$ | | | | | | | |
| | 1866.5223 | | 1117.4365 | | 252.4550 | | 9.4656 |

### 3. Evaluation of the polynomial equation for its roots

Equation (13) has eight real roots for $\omega^2$ leaving $\omega^2 = 0$ which represents the rigid body motion as already mentioned. This equation can be solved by

(i) using a computer, or

(ii) Graeffe's root squaring method, or

(iii) Newton's approximations.

Using a computer a series of calculations are to be carried out by increasing $\omega^2$ automatically by equal amounts until there is a change of sign. This increment would repeatedly be halved until the required value of $\omega^2$ is obtained with sufficient accuracy.

NEWTON's method of solving the polynomial equation would be illustrated below for Equation (13).

Let

$$\begin{aligned} f(x) &= x^8 - 19.8747\times 10^6 x^7 + 159.5885\times 10^{12} x^6 - \\ &- 663.3197\times 10^{18} x^5 + 1516.3387\times 10^{24} x^4 - 1866.5223\times 10^{30} x^3 + \\ &+ 1117.4365\times 10^{36} x^2 - 252.4550\times 10^{42} x + 9.4656\times 10^{48} = 0 \end{aligned}$$

where $\omega^2 = x$

Then for a first approximation the lowest root would be

$$x_1 = \frac{9.4656\times 10^{48}}{252.455\times 10^{42}} = 3.7494\times 10^4$$

$$f(x_1) \simeq 1.4756 \times 10^{48}$$

If $f'(x)$ is the derivative of Equation (13) with respect to $x$ then $x_2$ the second nearest approximation for the root is equal to

$$x_1 - \frac{f(x_1)}{f'(x_1)} \tag{14}$$

$$\therefore\ x_2 \simeq 3.7494\times 10^4 + \frac{1.4756\times 10^{48}}{1.762199\times 10^{44}} = 4.5868\times 10^4$$

where $f'(x_1) \simeq -176.2199\times 10^{42}$

This process should be continued until two successive values of the root are the same.

GRAEFFE's root squaring method is dealt with in references [2] and [3].

The process of forming the algebraic equation is, in itself, so time consuming that these methods cannot be considered quick enough even though they are accurate.

Among those which do not require the frequency equation some are accurate and some approximate. HOLZER's, matrix, continental, impedance, admittance, mobility and analogous methods can be termed as accurate methods whereas BRADBURY's, B.I.C.E.R.A.'s, LEWIS', graphical, semigraphical and GUPTA's methods as approximate ones.

Out of the accurate methods HOLZER's is the simplest and most direct one. But its disadvantage is that the frequency is to be first assumed and then verified whether it is true or not; in other words, it can be said that it is a method of successive approximations. So if the first assessment can be made quickly and accurately then HOLZER's calculations become very easy and hence the approximate methods should be resorted to in finding the nearest one-node natural frequency.

## 4. Lewis' method [4]

When the approximate position of the node of the system under consideration is known, then this method can be used and is less time consuming.

Accordingly

$$\omega = \frac{\pi}{2}\left[\frac{k_E}{J_E}\right]^{1/2} \text{rad/sec} \tag{15}$$

where $J_E = \Sigma J_{\text{cyl}}$,

$$k_E = k_{\text{cyl}}/(N_{\text{cyl}} + 1/2)\,,$$

$J_{\text{cyl}}$ = moment of inertia per line,

$k_{\text{cyl}}$ = crankthrow stiffness (between cylinder centers)

and the node is situated close to the flywheel.

$$\begin{aligned}\omega &= \frac{\pi}{2}\left[\frac{20\times 10^7}{8.5\times 1200}\right]^{1/2} \quad \text{rad/sec,}\\ &= 35 \ \text{cy/sec}\end{aligned}$$

This formula always gives a higher value compared to most of the other methods because of the assumption that one node is at the flywheel. If a more accurate value could be assessed by using any other approximate method and without needing more time then it would be more practicable.

## 5. B.I.C.E.R.A. formula [4]

According to this

$$F = 9.55\left[\frac{k_{\text{cyl}}}{J_{\text{cyl}}}\right]^{1/2}\left[\frac{A+B}{AB}\right]^{1/2} \quad \text{cy/min} \tag{18}$$

where $k_{cyl}$: crankthrow stiffness lb in./rad,

$J_{cyl}$: moment of inertia per cylinder lb in. sec$^2$,

$B$: ratio of flywheel inertia to moment of inertia per line,

$A$ : $N(N+1)/2$ where $N$ is the number of cylinders.

$$F = \frac{9.55\times 10^3}{900\times 60}(200)^{1/2}(186)^{1/2} = 34.11 \quad \text{cy/sec}$$

Assumptions made in the derivation of the formula are that the node position is at the flywheel and the relative amplitudes decrease linearly from the cylinder end to the flywheel end. Both assumptions are not true (even though assumed to be true for a first approximation) and hence the discrepancy in the value obtained for the one-node natural frequency.

## 6. Bradbury's diagram for estimating the one-node natural frequencies [5]

BRADBURY's diagram to find one node natural frequencies is plotted between the ratio $(k_{cyl}/J_{cyl}\times 10^6)$ on $X$-axis (frequency in vib/sec), on $Y$-axis (number of cylinders as parameter). $k_{cyl}$ represents the stiffness per crankthrow and $J_{cyl}$ the moment of inertia per line. The assumptions made while plotting the graph are

(i) all the moments of inertia per line are equal,

(ii) the ratio $J_F/J_{cyl}$ is not less than 40 ($J_F$ = moment of inertia of the flywheel),

(iii) all crankthrow stiffnesses are equal and the stiffness of the shaft portion $k_F$ between the end cylinder center and the flywheel is also equal to $k_{cyl}$.

The frequency value could also be calculated from the formula

$$F = \frac{uv(k_{cyl})^{1/2}}{10^3\,(J_{cyl})^{1/2}} \text{ cy/sec}, \tag{17}$$

where u = 56.9, 46.9, 40, 34.9, 31.05, 27.9 and 25.45 for 4, 5, 6, 7, 8, 9 and 10 cylinders respectively, and $v$ the correction factor, has been given by BRADBURY in the form of another graph; number of cylinders on $X$-axis, $v$ on $Y$-axis with $k_{cyl}/k_F$ as parameter ranging from 0.8 to 1.2.

Results obtained are not true to a reasonable extent (as would be shown later) because of the inaccuracy in the second assumption above.

$$F = \frac{31.05\times 0.989\times 10^3}{10^3}\cdot[4/3]^{1/2} = 35.459 \text{ cy/sec}$$

## 7. Gupta's method [6]

Charts have been prepared by GUPTA by matrix evaluation to find the fundamental natural frequency and relative amplitudes. Understanding the procedure requires defining some terms as

$$\alpha = \frac{I_{cyl}}{I_F}, \qquad \beta = \frac{k_{cyl}}{k_F} \tag{18}$$

and

$$x^2 = \omega^2/(k_{cyl}/I_{cyl}) \tag{19}$$

with the usual notation.

For the problem for which the solution is required

$\alpha = 0.0066$,
$\beta = 1.1111$.

Referring to Fig. 6, page 163 [Ref. 6],
$x^2 = 0.0345$,

$$F = \frac{1}{2\pi}\left(\frac{k_{cyl}}{I_{cyl}}\, x^x\right)^{1/2} \text{ cy/sec}, \tag{20}$$

$= 34.139$ cy/sec,

and the relative amplitudes can be calculated from Fig. 9, page 164 [Ref. 6].

According to BRADBURY, $\omega$ is the same for all engine systems if the value of $\omega$ is less than $1/40$.

But following GUPTA's charts,
$x^2 = 0.0335$ for $\alpha = 0.001$,
$= 0.0388$ for $\alpha = 0.025$, and
$F = 33.637$ for $\alpha = 0.001$,
$= 36.203$ for $\alpha = 0.025$.

So BRADBURY's diagram does not give correct values of $\omega$ even for values of $I_F/I_{cyl}$ more than 40.

BRADBURY [7] is also the first to give data facilitating the construction of GORFINKEL tables, for engines having five to eight cylinders, with equal moment of inertia per cylinder line and stiffness equal between cylinders. The variation of flywheel inertia also is taken into consideration. The values of relative amplitudes and $x$ (as indicated in GUPTA's charts) can be read off for any orthodox system and the natural frequency can be calculated by inserting stiffness and moment of inertia values. GORFINKEL's method [8] is in itself a simplified modification of HOLZER's, which reduces the arithmetical work for engine systems comprising a large number of cylinder inertias $J_{cyl}$ and crankthrow stiffness $k_{cyl}$, with identical values.

It is not out of place to mention that in *Engineering* of 19th February, 1937 there is an article that gives a formula for the lowest natural frequency of any system with four to ten cylinders, values of $\alpha$ ranging from 0 to 0.04 and $\beta$ from 0.8 to 1.2.

## 8. Other methods

Matrix methods do not offer any advantage over HOLZER's method or solution of the determinant equation as far as simple systems are concerned, except for the fact that they are more useful for solution of complex systems and can be readily programmed for a digital computer.

Graphical and semi-graphical methods take comparatively more time and the accuracy depends mainly on the accuracy of the diagram.

Impedance, admittance, mobility and analogous methods are easy for simple systems but become complicated when multimass systems are dealt with.

With methods described so far the assumptions are that the equivalent system constitutes weightless springs and point polar mass moments of inertia. In the distributed mass method and, to some extent, in the continental method for frequency evaluation, the masses of springs are also considered. But they involve more time in calculating the one-node natural frequency.

From all these considerations it could be concluded that the natural frequency of the engine system under study could be obtained within the least amount of time and with maximum possible accuracy by using GUPTA's charts and HOLZER's method combined.

## 9. Comparison

Comparison of the one-node natural frequencies obtained by different methods shows that GUPTA's charts give the nearest approximate value to the correct one.

| Method | HOLZER | GUPTA | B.I.C.E.R.A. | LEWIS | BRADBURY |
|---|---|---|---|---|---|
| Frequency | 34.240 | 34.139 | 34.110 | 35.000 | 35.459 |

## 10. Conclusions

1. Different methods to evaluate the one-node natural frequencies are compared to arrive at the best possible way of determining the one-node

natural frequencies of multicylinder engine systems with improved rapidity and accuracy.

2. It has been found that the best approximate method for one-node natural frequencies and angular amplitudes is given by GUPTA.

3. A more accurate assessment can only be made by HOLZER's calculations.

## 11. Summary

Numerous methods for evaluating the one-node natural frequencies of multicylinder engine systems are available in technical literature. As such an attempt, this paper suggested the course to be taken in determining the one-node natural frequencies and angular amplitudes with improved rapidity and accuracy.

## 12. Acknowledgements

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