BASIC RESEARCH ON VIBRATIONS AND ITS RESULTS*

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(Received February 20, 1967)

On the recommendation of the Engineering Committee of the Hungarian Academy of Sciences I was invited by the Department of Technical Sciences of the Academy to report on the results of basic research work on vibration phenomena. I accepted the invitation with great pleasure and give my report herebelow — abstaining, however, from giving a comprehensive survey on the available literature.

The better part of my work was devoted to the systems with a finite degree of freedom. It is common knowledge that in many cases the movement of real systems can be followed well on discrete models. If we start out from a model which can be described by a system of linear differential equations with constant coefficients and no velocities, we may gather a considerable amount of information. Perhaps the most important problems are the determination of the natural angular frequencies and the amplitude distribution of steady-state forced vibrations.

I shall first deal with the problem of the determination of natural angular frequencies.

When I set to work, I knew several methods to solve the problem. My aim was to select a method which permits analytical treatment as well as direct graphical illustration and which — at least in principle — is capable of implementing its function along a kinematic mechanism.

I found that the only model which meets all these criteria is the one which allows arrangement in the so-called single-connected chain.

The question arose whether the system of general configuration and a finite degree of freedom could be converted into such a single-connected chain whose natural frequencies are in simple, unequivocal connection with the original frequencies or, at least, show a fair agreement with them. In my examinations I started out from a process published by Lánczos in 1953 (it had been elaborated for a different purpose). Using the matrix calculation method I came to the conclusion that the problem could be solved by a so-called finite

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* Paper read on the 24th of January 1967 at the Engineering Committee of the Hungarian Academy of Sciences.

iteration process (which after a finite number of steps yields a theoretically exact result). It was later on found that this process coincides with the algorhythm derived independently by Falk, to which he had arrived by quite different methods.

According to the above, the sphere of application of the methods which are suitable for the single-connected chain can well be extended to systems of more complex configuration and having a finite degree of freedom.

When examining the models of single-connected chains from the aspects mentioned in the introduction my attention was drawn particularly to the well-known Kutzbach and Föppl — Kohn processes, the first of which is suited exclusively for the graphical determination of the highest natural frequency and the latter — theoretically — for that of all natural frequencies. The Kutzbach method has a drawback: the need for the complicated trial and error procedure which, at higher degrees of freedom, renders its application rather cumbersome — sometimes even impossible. The trial and error method is used also in the Föppl — Kohn process, but here it is not complicated.

Nevertheless, if the set objective is not the determination of the greatest natural frequency, even the Föppl — Kohn construction becomes complicated and difficult to survey.

On the basis of what has been said above, the Föppl — Kohn process would seem the most favourable, in case the same easy-to-survey procedure would be required for the determination of all natural angular frequencies as for the highest.

To put this idea into practice, a process was needed to diminish the order. Such processes have the common characteristic that in possession of any one of the natural frequencies, they enable the transposition of the original problem into one of a lower order, in which the natural frequencies equal the original's still undetermined natural frequencies. Similar processes were known already then. For its applicability in chain-like systems and for its easy interpretation, I chose the Baranov process as the one best suited for the purpose in hand.

I found that the combination of the Föppl — Kohn and the Baranov processes may be advantageous: this idea has never before been published in literature.

Through the combination of the said two processes a method was obtained for the determination of all natural frequencies which features the following principal properties:

a) it lends itself readily for graphical illustration and, even in this form, yields sufficiently exact values;

b) it is simple analytically and easy to programme;

c) in theory it enables the design of a mechanism which automatically yields all natural frequencies;

d) it provides facilities for the evolution of such simple passive electric connections composed exclusively of ohmic resistors, which yields all natural frequencies semiautomatically.

Combined with the finite iteration process outlined in the introduction I elaborated a process for the determination of critical speeds resp. natural frequencies, associated with flexural vibrations, in explicit form. It proved to be suitable for systems even with multiple natural frequencies.

As to point *d)* I wish to add that on the principle and arrangement made available to Mr. Gy. Ludvig, he constructed the apparatus in question.

The unequalities exposed by Prof. Dr. József Barta, one of the opponents of my candidate's thesis — himself engaging in these problems — may in my opinion be regarded a further development of the above described process for the determination of natural frequencies.

Furthermore it was possible to reverse the process of the determination of natural frequencies and evolve one for the synthesis of the systems having predetermined natural frequencies. Since at each successive step (the "incorporation" of each natural frequency) one parameter can partly be selected at will, it meets further requirements.

And now about the amplitude distribution of stationary-state forced vibrations.

With proper modifications, I succeeded in making the above method, used for the determination of the natural frequencies, suitable for the determination of the amplitude distribution of stationary-state forced vibrations — naturally with the exclusion of the case of resonance. The modification essentially consisted in determining the stationary-state amplitudes produced by nonresonance-frequency sinusoidal excitation, by juxtaposing to the original problem a suitable natural vibration pattern (eigenvector) determination.

It was most interesting to note that the method enables the direct illustration of theorems which had so far been verified only analytically, by the determinant theory or by the matrix calculus. Such are, for instance, the phenomena of apparent resonance, dynamic vibration damping, and partial vibration.

Even the highly interesting Routh phenomenon to which Professor J. Egerváry had attracted my attention in one of his papers, has become accessible for direct study, besides phenomena having a bearing on the theory of mechanical and electrical filter chains.

The method is suitable not only for illustration and analysis but also for synthesis: it helps design systems with predetermined distribution of forced vibration amplitude, or a chain with predetermined "filtering" characteristics.

Professor I. Sályi has further developed the method of determining forced vibration amplitudes, in consideration of damping. Later on, with the consideration of damping from another aspect, author also applied the process for the calculation of torsional vibrations in internal combustion engines.

Furthermore the method was used to verify a premise in the general theory of the forced vibrations of systems with artificial damping.

At the time when I elaborated the above methods, the so-called sectional matrix process had not yet been elaborated to such a degree as it is now. Today we can see that the two processes show many affinities and this recognition opens up the way to further development which, however, are beyond the scope of this paper.

In the theory of vibration damping, the damper is usually dimensioned on the basis of the following general principle: the damper should be such as to keep at the possibly lowest level the highest value of the stress amplitude which is the most harmful in the given frequency range. This principle is relatively easy to apply if the system to be damped may be regarded as one having a single degree of freedom, but it would be considerably more complex or quite impracticable, if higher freedom degrees were concerned.

To circumwent this difficulty, I proposed a different principle as the basis of dimensioning in which the energy absorbed by the damper during one period must be increased to the possibly highest degree, instead of minimizing the maximum amplitude. Could we verify that the application of this principle leads to, by and large, the same result, the latter would be the more advantageous since it does not require the location of the point of most harmful amplitude and its suppression, only the minimization of the parameters (absorbed energy) prevailing at certain given points.

These examinations have so far yielded but tentative results; the study gives very considerable calculation work. (In the calculations so far performed the sectional matrix method played a substantial role.)

In relation with systems having finite degrees of freedom, I may report on another result which directly concerns the chain performing flexural vibrations, in a direct way, viz. without the previous application of the above mentioned finite iteration.

It was easy to realize that in the process we have a means for determining the support moments of continuous straight beams. This fact, in addition to its interesting application in the field of structural engineering, enables the determination of the influence factors in the simplest model of flexural vibrations and through it, critical velocity. This may considerably accelerate the calculation of the said influence factors.

I have examined the vibration produced in stationary (as the first step) vehicle bodies in piston-type internal combustion engines. Here practice poses the following problems:

The connecting rods — although theoretically fully equal — differ in mass, a certain degree of deviation being permissible. The problem is whether it is possible to determine the sequence of their mounting (to define which rod should be fitted to the first, which to the second etc. cylinder), to optimize the vibration of the body — and if so, what this sequence should be?

Another problem: what characteristics should the rubber springs have which support the engine and at which points should they be applied so as to optimize vibrations?

Having constructed an appropriate model for the problem, assuming small vibrations, I have arrived at an inhomogeneous system of linear differential equations with varying coefficients. Preparations for the numerical analysis of the problem are in progress.

I have studied the operation of the so-called Stockbridge vibration damper used for the damping of mechanical vibrations in electric transmission lines, partly by a continuum and partly by a discrete model, taking the conducting cable as the continuum and the damper as discrete. In this connection two principal problems have emerged: 1. what dynamic properties should be aimed at in the dimensioning of the device, and 2. where to fit the appropriately dimensioned device, to ensure optimum mechanical protection of the line?

With regard to the location and dimensioning of the device, various theoretical considerations and principles — base partly on the latter and partly on experience — are known. However, the mddel they use is not refined enough and neither is the interaction between theoconductor and the device considered with sufficient consistency. In my examinations I aimed at remedying this deficiency. Detailed numerical calculation work is in progress.

As a matter organically relating to the present report I wish to note that I have had the opportunity to function as opponent and draw up papers for the discussion of numerous candidate's theses dealing with the subject matter of my paper and that I have written a number of critical essays and surveys on articles in this field. They all emphasized the need for thorough analysis, to the details of which I shall not extend in this paper.

Concluding this rather sketchy report on the main results I have achieved in my work, I should like to outline my future plans.

I shall not go into details of the proposal elaborated in cooperation with Prof. Dr. Pál Csonka, Dr. Imre Kozák and Prof. Dr. János Szabó on research as going on in Hungary in dynamics and kinematics and vibrations, since it has recently been submitted to the Department of Technical Sciences of the Hungarian Academy of Sciences.

My intention is to concentrate primarily on elastic bodies — in the first place on bars — and examine their kinetic stability. This domain still lacks the formulation of certain concepts, theories and methods. These, to solve the existing problems, are of great theoretical and practical significance and claim international attention.

At this stage it would be premature to report on further details.

Summary

Author's report gave an account of some of his results in connection with vibration theory and application. The principal stations of the research work are: construction of a special method and a calculating machine for the determination of eigenfrequencies and amplitudes of the steady state forced vibration, synthesis of a vibrating system with prescribed properties, addition to the theory of vibration damping and continuous girders; optimization of the vibration of a vehicle with piston-type internal combustion engine; new theory for the dimensioning of the Stockbridge type damper of overhead transmission lines.

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