CHARACTERIZATION OF THE RIGIDITY OF MACHINE-TOOLS

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Machining accuracy on a machine-tool and inclination to vibrations are greatly influenced by the rigidity of the machine-workpiece-tool (M-W-T) system.

Efforts have been made for a long time to formulate the concept of machine-tool rigidity in a generally valid form, permitting the unequivocal comparison and qualification of machine tools from the aspect of rigidity. Several research workers were engaged with the clarification of the concept of rigidity, but we must say that the number of proposed rigidity factors nearly equals the number of authors.

The concept of machine-tool rigidity was first defined by KRUG. The rigidity of the machine-tool element is equal to the unit if its elastic deformation is 1 μm in the direction of the load of 1 kp.

With the designations usual at present,

$$j = \frac{P}{f},$$

where j is the measure of rigidity (rigidity factor), and

f is the displacement in the direction of force P.

The physical phenomenon may naturally be characterized just as well by the quotient of the displacement in the direction of the force.

$$k = \frac{1}{j} = \frac{f}{P}.$$

The higher the k value, the higher is the deformation of the element and the weaker is the machine-tool. Let us name the value k the factor of "weakness".

The definition of rigidity as proposed by KRUG does not express the fact that in the M-W-T system the rigidity factor generally depends on the direction of the force.

Rigidity as defined by SOKOLOVSKI can be written in the form

$$j = \frac{P_m}{y} \left(kp\,(\mu\, m)\right),$$

where P_m is the component of the cutting force in the direction y, and

y that component of the displacement, in the direction of which accuracy is influenced to the highest degree by the relative displacement of tool and workpiece. Considering cylinder turning as a basis, P_m is the force in the direction of cut. Since the rigidity factor of SOKOLOVSKI depends on the ratio P_f/P_m too, this should be indicated in each case.

REUTLINGER defines the rigidity factor as the reciprocal value of the spring constant c

$$C = \frac{1}{c} \left(\frac{cm}{kp}\right).$$

SCHENK has given an empirical formula for the rigidity of machine-tool spindles which is very widely used in the United States.

$$R_b \left(\frac{kp}{\mu\, m}\right) = 530\, \frac{D^4 - d^4}{l^3}$$

where D is the average outside diameter of the spindle, d the diameter of the spindle bore and l the distance between the radially loaded bearings. (Dimensions should be substituted in cms.)

The characteristic definitions as enumerated above indicate that different research workers interpret the concept of rigidity in different ways. A similar confusion is to be found in the field of denominations, since many expressions are used in the literature for the same concept such as, to mention only a few, rigidity, tightness, spring tightness, spring constant, spring number, kinetic influence number, weakness factor, relaxation coefficient, etc.

It is conceivable on the basis of the above discussion that no generally valid formulation for rigidity exists. The definitions of rigidity are incomplete and can be interpreted arbitrarily.

In the following the values characterizing the elastic behaviour of a certain point of the machine-tool are defined, as an addition to the paper of E. Heydrich read at the Conference of the II. Hungarian Machine Construction Week at Esztergom, on May 11, 1960, under the title "The Rigidity Tensor, a Characteristic of Machine-Tools."

As machine-tools cannot be regarded as unequivocally linear systems, an idealization is employed with the aim of facilitating calculations. The M-W-T system is regarded as a linear system.

For the purpose of linearization, what is valid with gccd approximation in a determined force interval, let us assume that the elements of the system are fitted without gaps and the behaviour of the fits is similar to that of a continuous material, and that the members of the system are ideally elastic.

The elastic behaviour of the machine tool is examined in that characteristic point where the cutting operation is taking place. In this case the deforming force acting on the M-W-T system is the cutting force.

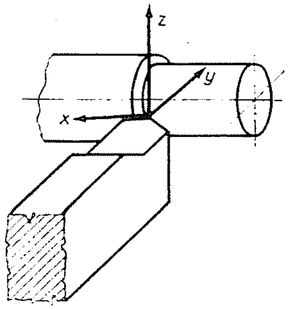

Fig. 1

The relative displacement between the tool and the workpiece, brought by the cutting force $\overline{P}$, can be expressed by the following system of equations:

$$\begin{aligned} f_x &= k_{xx} P_x + k_{xy} P_y + k_{xz} \cdot P_z \\ f_y &= k_{yx} P_x + k_{yy} P_y + k_{yz} P_z \\ f_z &= k_{zx} P_x + k_{zy} P_y + k_{zz} P_z, \end{aligned} \qquad (1)$$

where x, y and z are the axes of our system of rectangular coordinates (Fig. 1), P_x, P_y, P_z the respective force components, f_x, f_y, f_z the displacements in the respective coordinate directions.

The constants k_{xx}, k_{xy}, . . . are the so-called weakness coefficients, where the first index designates the direction of the displacement, while the second index that of the force. E.g. the weakness coefficient

$$k_{xy} = \left(\frac{f_x}{P_y}\right)_{P_x = P_z = 0}$$

indicates the ratio of the displacement in direction x in consequence of a force in direction y, and at that force.

It is apparent from the structure of relationship (1) that we have a homogeneous linear vector-vector function,

$$\bar{f} = \overline{\overline{K}} . \overline{P}$$

where we may denominate $\overline{\overline{K}}$, referring to the elements it contains, as the weakness tensor.

The matrix of the weakness tensor $\overline{\overline{K}}$ is found to be:

$$\overline{K} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \tag{2}$$

The weakness of the machine can evidently be characterized by values suitably formed from the components of the matrix $\overline{K}$. Let us therefore decompose $\overline{K}$ to a symmetrical and an asymmetrical part:

$$\overline{K} = \overline{K}_s + \overline{K}_{AS} \tag{3}$$

This decomposition results in a suitable decomposition in the displacement $\bar{f}$ as well.

$$\bar{f} = \overline{K}\overline{P} = \overline{K}_s \overline{P} + \overline{K}_{AS} \overline{P} = \lambda_1 \overline{P}_1 + \lambda_2 \overline{P}_2 + \lambda_3 \overline{P}_3 + \bar{v}x\overline{P} \tag{4}$$

where $\overline{P}_1$, $\overline{P}_2$, $\overline{P}_3$ are the components of the force vector $\overline{P}$ in the system composed of the characteristic vectors of $\overline{K}_3$,

λ_1, λ_2, λ_3 are the characteristic values belonging to the corresponding characteristic vectors, and $\bar{v}$ is the vector invariant of the asymmetrical matrix $\overline{K}_{AS}$.

Taking decomposition (4) into consideration, the following two quantities are chosen for characterizing machine-tool weakness:

a) The scalar invariant $\Lambda = \sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}$
and

b) the vector invariant $\bar{v}$.

Namely, if any direction is allowed for the cutting force $\overline{P}$, then evidently the magnitude of the scalars λ_i $(i = 1, 2, 3)$ and the vector $\bar{v}$ will generally have a role in displacement $\bar{f}$.

Information on the displacement described by the symmetrical tensor $\overline{K}_s$ is supplied by the expression $\Lambda = \sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}$, while that on the displacement originating from the asymmetrical part $\overline{K}_{AS}$ by the vector invariant $\bar{v}$. It is immediately apparent from the structure of the expression $\bar{v}x\overline{P}$ that it describes a pure rotation where the axis of rotation is represented just by the vector $\bar{v}$.

It may seem at first sight that it would be more advisable to choose the so-called first scalar invariant, $\lambda_1 + \lambda_2 + \lambda_3 = I$, for characterizing the machine — in place of Λ —, especially since it is more easy to connect some physical meaning to it. In displacement $\lambda_1\overline{P}_1 + \lambda_2\overline{P}_2 + \lambda_3\overline{P}_0$ originating from the symmetrical tensor $\overline{K}_S$, however, evidently only the magnitude of the characteristic values $\lambda_i (i = 1, 2, 3)$ will have a role, since the characteristic vectors, — $\overline{K}_s$ being a symmetrical tensor —, hence the force components $\overline{P}_i$ also in the direction of the characteristic vectors, are perpendicular between themselves, consequently the displacement components $\lambda_1 \overline{P}_1$, $\lambda_2 \overline{P}_2$, $\lambda_3 \overline{P}_3$ are also perpendicular. Thus, the first scalar invariant may even be 0, nevertheless it may happen, that the displacement $\lambda_1 \overline{F}_1 + \lambda_2 \overline{P}_2 + \lambda_3 \overline{P}_3$ is at the same time considerable. So far as the "physical content" is concerned, the descriptive meaning, as obtained in the case of some velocity vector field $\overline{v} = \overline{v}(\overline{r})$ for div $\overline{v}$, the scalar invariant of the derivate tensor of the vector-vector function $\overline{v} = \overline{v}(\overline{r})$, can obviously not be simply employed for a vector field having another physical meaning. Then — as in our case where the displacement vector field is examined in function of forces — apparently further investigations are required.

It should be emphasized, however, that the two values as interpreted in the above way supply information on the degree of machine-tool weakness only *in general.* It may happen, namely, that in the case of forces in a certain direction, though both A and $|\overline{v}|$ are high values, the resultant displacement is nevertheless insignificant. Hence, in that case when the direction of forces acting on the machine-tool at a given point is limited to a certain solid angle, we can eventually find a value, in the knowledge of the characteristic values, characteristic vectors, of the vector invariant, and of the solid angle in question, which is better suited for characterizing machine-tool weakness.

During the examination of the solution of this problem a measuring method was elaborated, permitting the determination of the elements of the matrix belonging to the weakness tensor $\overline{K}$. During measurements the force components $P_f = P_z = 30$ kp, $P_m = 0.5\, P_f = P_y$ and $P_e = 0.2\, P_f = P_x$ were actuated separately and the relative (workpiece-tool) displacements in the directions f_x, f_y, and f_z caused by these forces were determined.

In the case of a high precision A type lathe, by choosing the system of coordinates in the usual way (Fig. 1), the elements of the matrix were obtained as follows

$$\overline{K} = \begin{bmatrix} \frac{1}{6} & 0 & 0 \\ 0 & \frac{14}{15} & -\frac{24}{15} \\ 0 & \frac{11}{15} & \frac{16}{15} \end{bmatrix}$$

From this we obtain by simple calculation that

$$A = 1.6 \frac{\mu m}{kp} \text{ and } \overline{v} = \frac{7}{6} \overline{i} \frac{\mu m}{kp} \cdot$$

This means that in the case of the examined high precision F type lathe both the symmetrical and asymmetrical parts have an influence on the displacement brought about by the cutting force.

As a matter of interest, we note that the vector $\bar{v}$ providing the direction of pure rotations is parallel with the axis of rotation of the main spindle.

Summary

For describing the elastic behaviour of a machine-tool the authors give a tensor (rigidity tensor) in place of a scalar quantity. At the same time a scalar and a vector quantity are created from the elements of this tensor, for the quantitative characterization of the rigidity of the machine-tool. Finally an A type lathe is characterized by concrete measurement results.

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