CONTRIBUTION TO THE RELATION BETWEEN THRUST DEDUCTION AND FRICTION

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(Received December 28, 1965)

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Symbols

a model scale
A wetted surface of ship (m^2)
A_0 propeller disk area (m^2) $A_0 = \frac{D^2 \pi}{4}$
c_F frictional resistance coefficient
k coefficient for determining the nominal speed causing the resistance $k = c_F(v_x/v_0)^2$
L length (m)
R resistance (kp)
R_e Reynolds' number $R_e = \frac{v L}{\nu}$
R_F component of the viscous resistance (kp)

$$R_F = c_f \frac{\varrho}{2} v^2 A$$

t thrust deduction coefficient
T thrust (kp)
v ship speed or model speed ($m\ s^{-1}$)
v_A propeller advance speed ($m\ s^{-1}$)
v_0 nominal speed of the water flowing through the propeller disk area ($m\ s^{-1}$)
v_x nominal speed causing the viscous resistance ($m\ s^{-1}$)
ν kinematic viscosity ($m^2\ s^{-1}$)
ϱ water density ($kp\ m^{-4}\ s^2$)

a and b constants calculated for a given model

For the usual design method of a ship propeller we must know the value of the thrust deduction coefficient which gives us information on the mutual influence of ship and propeller. The thrust deduction coefficient, as is known is

$$t = \frac{T - R}{T}$$

where T is the thrust of the propeller, R is the resistance of ship without any acting propeller, both at the same ship speed. The value of t is determinable only with the aid of model experiment, apart from simple approximative relations used for its precalculation.

We usually made two kinds of measurements with the model of the ship and of the propeller. The resistance of ship body without propeller is measured and the thrust of the self-propelled ship model, both at different ship speeds.

The thrust deduction fraction can be calculated with the aid of these measured values.

However, in case we make our ship model according to different scales, we get different values of t for the same ship speed. E.g.: The Victory model family has been investigated by the NSMB. This investigation gave the following values for 11 knots ship speed with the measuring of models with different scales (a) [1].

a =	50	40	30	23	18	6_{smooth}	6_{rough}
t =	0.209	0.175	0.183	0.210	0.214	0.304	0.212

[The largest model was a motorboat. Its surface was the same as the surface of the paraffin-wax model in one series of measures (smooth), and it

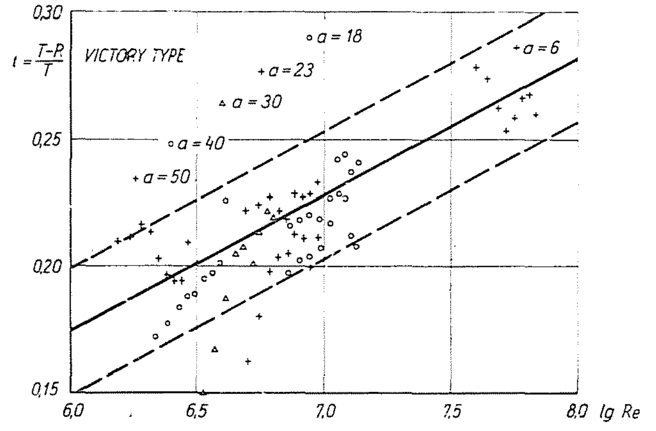

Fig. 1. Figures 1 and 2 are copies from [1] and [2]

was the same as the usual ship surface in service condition (rough), in the other series of measurements. The other models (a = 18—50) were the usual paraffin models.]

This means, that we ought to calculate with a scale effect when using the thrust deduction fraction determined by model experiment. Therefore, the thrust deduction fraction was investigated in connection with several quantities.

In *Fig. 1* values of t are plotted of the Victory model family as the function of Reynolds' number [1]. In *Fig. 2* the values of

$$c_D' = \frac{T-R}{\varrho/2 \cdot v_A^2 A}$$

are plotted against

$$c_T = \frac{T}{\varrho/2 \cdot v_A^2 A}$$

where v_A is the intake velocity of the propeller, A is the wetted surface of the ship body, ϱ is the density of water [2].

Whereas the first plotting gives a very scattered set of values of t, in the second case it was possible to make two linear functions for the mentioned

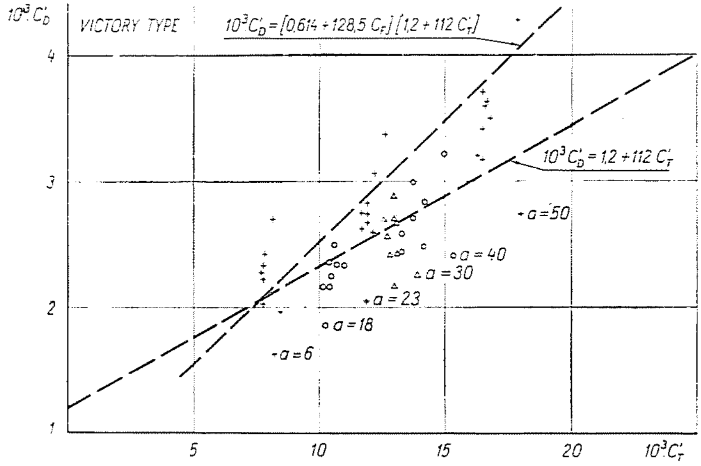

Fig. 2

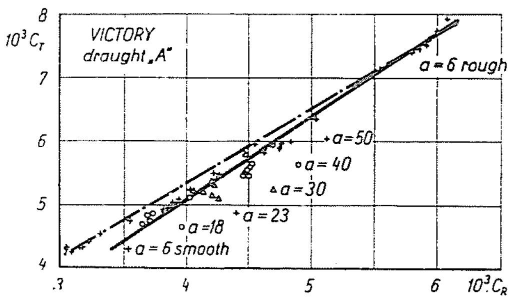

Fig. 3

coefficients by analysing the values determined for the same ship speed. One of these relations was made for the smooth models, and the other for the rough ship. In the second formula c_F is the frictional resistance coefficient.

The results of the Victory model family are also seen in *Fig. 3*. Thrust coefficient

$$c_T = \frac{T}{\varrho/2 \cdot v^2 A}$$

(where v is the ship speed) is given as the function of resistance coefficient

$$c_R = \frac{R}{\varrho/2\, v^2 A}$$

according to TELFER's method [3].

We can draw two straight lines in this case. One of them contains the measured values of the two motorboats ($a = 6$) and the other goes through the points of the paraffin models and the rough motorboat. The motorboat was investigated in open water while the other models were investigated in a model tank. Consequently with the wall effect we can account for the increase of the difference between the two lines in the direction of the larger model.

The model family of the Victory ship was investigated with the aim to give some method for the extrapolation of the measured data of an individual shipmodel to the ship. Thus, it is necessary to continue this work.

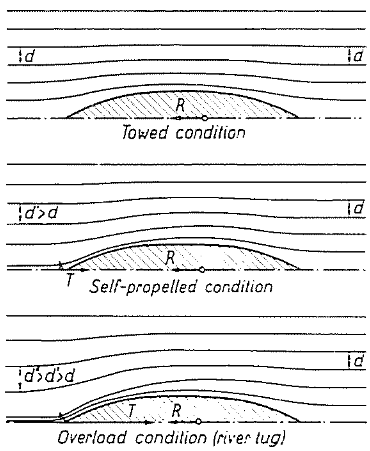

Fig. 4

The real thrust of a ship's propeller, acting on a ship, must be equal to the resultant force of all other active forces. Giving the results of resistances of the self-propelled ship body, it seems according to our experiments, that the thrust is not equal to the ship-resistance which we can measure in the towed condition of the ship, without active propeller. Thus, we must assume that the resistances of the ships are different in the self-propelled and in the towed conditions. This difference is the result of the fact that the streamlines are different in the neighbourhood of the ship at the two mentioned conditions.

Let us assume that there is an ideal two-dimensional stream around the ship's hull in the horizontal plane of the propeller axis. We can draw the streamlines in both the towed and the self-propelled conditions *(Fig. 4)*. Behind the ship we can find higher velocity in the jet of the propeller, and consequently lower velocities out of it. Because of the action of the propeller, there are higher velocities immediately near the surface of the stern than in the above-mentioned towing condition. If the propeller load is not very high as in self-propelled conditions, the growth of velocities along the stern has greater importance than the decrease of velocities farther from the ship body.

But, the latter is also very important if the propeller load is very high and if the thrust is a multiple of the own resistance of the ship body. (In the case of a river tug.)

The difference of resistance has the same three components as the ship resistance in towed condition (R): frictional, pressure and wave making components. The frictional and the pressure components are dependent on the Reynolds' number, the roughness of surface and the form of the ship body. Thus both can be investigated in the same way. Together we call them "viscous" resistance components.

According to earlier research work, we have arrived at the conclusion, in the case of self-propelled ships at lower speed that the viscous component is the most important part of the above-mentioned resistance difference [2]. This is also shown by the picture of stream lines. The differences of the relative velocities are higher immediately by the ship hull and farther the picture of stream lines is the same in the towed and self-propelled conditions. Thus, approximately the same waves are made by the ship in the two cases, but there is a difference in the viscous resistance.

It is disputable whether the thrust measured on the propeller shaft is equal to the resistance of the self-propelled ship body. The screw, mainly when it is behind the stern in the case of a single-screwship, does not work in a homogeneous velocity distribution. Thus, the thrust is varying in time, and the thrust measured may be different from the real average thrust. Yet the difference is negligible as compared to the error of the other measured data in the investigation of ship models.

The difference of the measured thrust and ship resistance ($T - R$) is always proportional to the difference of the resistance in self-propelled and towed conditions. The difference of resistance is approximately equal to the difference of their viscous components. Thus, let us assume that

$$T - R = T_F - R_F,$$

where T_F is the viscous resistance in the self-propelled condition and R_F the viscous resistance in the towed condition.

In the towed condition the viscous resistance is

$$R_F = \varrho/2\, v^2 A c_F,$$

where ϱ is the density of water, v the ship speed, A the wetted surface of ship, c_F the viscous resistance coefficient. In this formula v is not the real velocity in the neighbourhood of the ship hull. The real velocities are very different along the ship's length. We can say that v is only a nominal velocity from the viewpoint of viscous resistance. But it is true that the real velocities are in proportion to the ship's velocity.

Let us write the viscous resistance of the self-propelled ship in the same way

$$T_F = \varrho/2\, v_X^2\, A c_F\,;$$

where v_X will be the nominal velocity, with which we can obtain the effective viscous resistance of the self-propelled ship. This v_X nominal velocity and with v_X, the real local velocities in the neighbourhood of the ship depend on the velocity of the ship (v) and the velocity of the water flowing through the propeller (v_0). It is also important to know on what size of the ship surface the local velocities are changed in consequence of the action of the propeller. Thus how many parts of surface of the ship have other local velocities in the self-propelled and in the towed conditions. It depends on the ship's form (afterbody), the position of the propeller relative to the ship, the proportion of the wetted surface and the mass of water flowed through the propeller ($A_0 \cdot v_0$) mainly depends on the local frictional coefficient between the water and surface.

When geometrically similar ships are investigated (in the case of a model family), the effect of ship-form and position of propeller can be omitted. We can observe the other components in this case.

Consequently, in the case of the investigation of a model family we can write:

$$v_X = f_1\,(v,\, v_0,\, A,\, A_0,\, c_F)$$

The velocity of the water flowing through the propeller (v_0) is proportional to the velocity in the jet behind an ideal propeller having the same thrust. Accordingly we can use this velocity for our investigation. The thrust of an ideal propeller is:

$$T = \varrho\, A_0\, \frac{v_0 + v}{2}\, (v_0 - v)$$

From this equation

$$v_0^2 = \frac{T}{\varrho/2\, A_0} + v^2$$

If we get the thrust coefficient

$$c_T = \frac{T}{\varrho/2\, v^2\, A}$$

we may write

$$v_0^2 = v^2 \left(1 + \frac{A}{A_0}\, c_T\right)$$

This formula contains the ship speed, the wetted surface and the propeller disk area. Our function for determining v_X then becomes simpler

$$v_X = f_2(v_0 c_F)\,.$$

Last, we can carry v_0 to the other side. So we can write:

$$\frac{v_x}{v_0} = f_3(c_F)$$

For the determination of this function, the values of $\frac{v_x}{v_0}$ from the measured results of the Victory model family were calculated. These could be determined from the measurements of the ship speed, thrust coefficients (c_T) and resistance coefficients (c_R). We can then calculate the viscous resistance coefficient c_F.

The value of c_F is calculated here with TELFER's formula

$$c_F = 1 \cdot 2 + 3 \cdot 65 \frac{100}{\sqrt[3]{R_e}}$$

The values of constants were determined by the analysis of the Victory model family, therefore it gives exactly the viscous part of the model resistance.

c_F is also calculated with the ITTC 1957

$$c_F = \frac{0.075}{(\lg R_e - 2)^2}$$

but the results of the following calculations are the same with both methods.

In the case of the "rough motorboat", instead of the c_F (for paraffin-wax models) was obtained,

$$c'_F = c_F + c_{R\,\text{rough}} - c_{R\,\text{smooth}}$$

where $c_{R\,\text{rough}}$ and $c_{R\,\text{smooth}}$ are the total resistance coefficients for the two different models.

As assumed formerly, the total difference of the thrust and the resistance is equal to the viscous section of the resistance difference:

$$T - R = T_F - R_F = \varrho/2\, c_F\, A(v_x^2 - v^2)$$

or divided with $\varrho/2\, v^2 A$

$$c_T - c_R = c_F \left(\frac{v_x^2}{v^2} - 1 \right)$$

But

$$\left(\frac{v_x}{v} \right)^2 = \left(\frac{v_x}{v_0} \right)^2 \cdot \left(\frac{v_0}{v} \right)^2 = \left(\frac{v_x}{v_0} \right)^2 \cdot \left(1 + \frac{A}{A_0} c_T \right)$$

and so

$$c_T - c_R = c_F \left(\frac{v_x}{v_0}\right)^2 \cdot \left(1 + \frac{A}{A_0} c_T\right) - c_F$$

Calculated the values of

$$k = c_F \left(\frac{v_x}{v_0}\right)^2 = \frac{c_T - c_R + c_F}{1 + \frac{A}{A_0} c_T}$$

and plotted against c_F a linear function was obtained *(Figs 5 and 6)*

$$k = a + bc_F$$

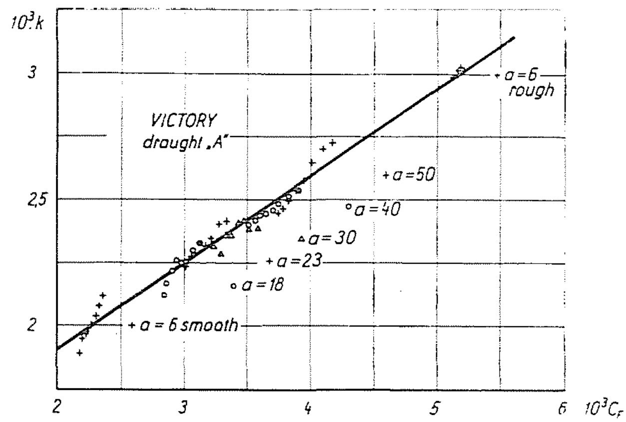

Fig. 5

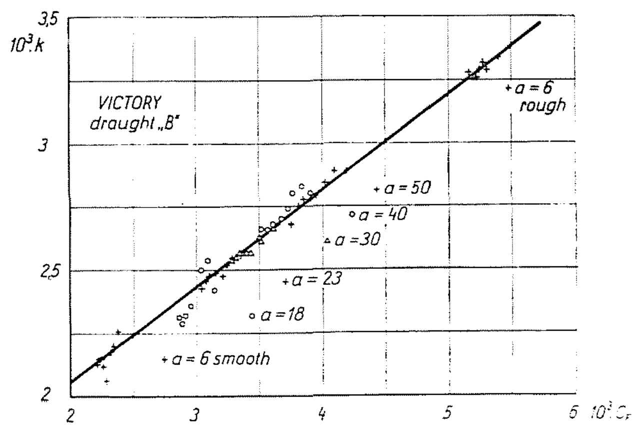

Fig. 6

Table I

Scale	v knot	Loaded condition			Light condition		
		$10^3\ (c_T - c_R)$		Difference per cent	$10^3\ (c_T - c_R)$		Difference per cent
		measured	calculated		measured	calculated	
6 smooth	10	1.30	1.13	−13.1	1.30	1.20	− 7.7
	11	1.23	1.12	− 9.	1.22	1.19	− 2.5
	12	1.17	1.12	− 4.3	1.21	1.20	− 0.8
	13	1.15	1.15	0	1.01	1.18	−16.8
	14	1.17	1.17	0	1.22	1.27	+ 4.1
	15	1.20	1.21	+ 0.8	1.29	1.29	0
	16	1.22	1.25	+ 2.5	1.35	1.32	− 2.2
	17	1.19	1.35	+11.7	1.38	1.39	+ 0.7
6 rough	9	1.61	1.53	− 5			
	10	1.61	1.57	− 2.5	1.75	1.70	− 2.8
	11	1.62	1.62	0	1.71	1.68	− 1.5
	12	1.68	1.65	− 1.8	1.69	1.70	+ 0.5
	13	1.75	1.73	− 1.1	1.70	1.73	+ 1.8
	14	1.87	1.84	− 1.6	1.73	1.76	+ 1.7
	15				1.78	1.81	+ 1.7
	16				1.97	1.96	− 0.5
18	10	1.03	0.96	− 6.8	0.94	1.05	−11.7
	11	1.03	0.97	− 5.8	1.27	1.14	−10.2
	12	0.99	0.99	0	1.00	1.07	+ 7
	13	1.06	1.04	− 1.9	0.78	—	—
	14	1.14	1.08	− 5.3	1.04	1.14	+ 9.6
	15	1.12	1.11	− 1.3	1.03	1.15	+11.6
	16	1.07	1.13	− 5.6	1.01	1.18	+16.8
	17	1.09	1.22	−12	1.14	1.26	+10.5
23	10	1.09	0.97	−11	1.09	1.09	0
	11	1.12	0.99	−11.6	1.06	1.05	− 1
	12	1.08	1.02	− 5.6	1.07	1.07	0
	13	1.08	1.05	− 2.8	1.06	1.10	+ 3.8
	14	1.15	1.08	− 6.1	1.13	1.14	+ 0.9
	15	1.15	1.12	− 2.6	1.19	1.17	− 1.7
	16	1.20	1.19	− 0.8	1.19	1.19	0
	17	1.29	1.30	+ 0.8	1.25	1.27	+ 1.6
30	10	0.84	0.96	+14.3	1.07	1.04	− 2.8
	11	0.94	1.00	+ 6.4	1.02	1.03	+ 1

(Table I cont.)

Scale	v knot	Loaded condition $10^3\ (c_T - c_R)$ measured	Loaded condition $10^3\ (c_T - c_R)$ calculated	Loaded condition Difference per cent	Light condition $10^3\ (c_T - c_R)$ measured	Light condition $10^3\ (c_T - c_R)$ calculated	Light condition Difference per cent
30	12	1.07	1.04	− 2.8	1.06	1.06	0
	13	1.05	1.01	− 3.8	1.06	1.11	+ 4.7
	14	1.08	1.10	+ 1.9	1.12	1.14	+ 1.8
	15	1.16	1.16	0	1.20	1.18	− 1.7
	16	1.20	1.20	0	1.21	1.21	0
	17	1.32	1.34	+ 1.5	1.25	1.25	0
40	10	0.96	0.99	+ 3.1	1.25	1.16	− 7.2
	11	1.01	1.02	+ 1	1.27	1.14	−10.2
	12	1.02	1.05	+ 2.9	1.31	1.18	− 9.9
	13	1.04	1.09	+ 4.8	1.26	1.20	− 4.8
	14	1.08	1.12	+ 3.7	1.22	1.19	− 2.5
	15	1.10	1.14	+ 3.6	1.25	1.23	− 1.6
	16	1.14	1.18	+ 3.5	1.27	1.25	− 1.6
	17	1.28	1.32	+ 3.1	1.44	1.34	− 7
50	10	1.23	1.07	−13	1.21	1.22	+ 0.8
	11	1.26	1.12	−11.1	1.23	1.15	− 6.5
	12	1.29	1.18	− 8.5	1.24	1.18	− 4.8
	13	1.22	1.20	− 1.6	1.24	1.26	+ 1.6
	14	1.16	1.18	+ 1.7	1.33	1.30	− 2.3
	15	1.14	1.19	+ 4.4	1.39	1.36	− 2.2
	16	1.16	1.26	+ 8.6	1.36	1.34	− 1.5
	17	1.32	1.42	+ 7.6	1.36	1.42	+ 4.4

Thus, nominal velocity v_x for the calculation of T_F is the following:

$$v_x^2 = v_0^2\left(\frac{a}{c_F} + b\right) = v^2\left(1 + \frac{A}{A_0}\,c_T\right)\cdot\left(\frac{a}{c_F} + b\right)$$

With this, the difference

$$T - R = \varrho/2 \cdot c_F \cdot v^2 \cdot A\left(\frac{v_x^2}{v^2} - 1\right)$$

and the thrust deduction fraction

$$t = \frac{c_F}{c_T}\cdot\left(\frac{v_x^2}{v^2} - 1\right).$$

Using the measured results of c_T and c_R from the model experiments of the Victory model family [1], [2], the following constants were obtained:

in loaded condition A

$$a = 1.205 \qquad b = 0.347$$

in light condition B

$$a = 1.295 \qquad b = 0.380$$

By comparing, the coefficient of the resistance-difference-means was calculated from the following equation:

$$(c_T - c_R)_{\text{calc}} = (a + bc_F)\left(1 + \frac{A}{A_0} c_T\right) - c_F$$

The percentages of errors of $(c_T - c_R)$ in the table and k in the diagrams did not show any regular differences in the cases of the investigated field of speeds. The errors in the table [the errors of the $(c_T - c_R)$] are also equal to the errors of the thrust deduction fraction:

below	$\pm$ 3	per cent in	57	per cent of the cases
between	$\pm$ 3 ~ 7	per cent in	22	per cent of the cases
between	$\pm$ 7 ~ 12	per cent in	18	per cent of the cases
above	$\pm$ 12	per cent in	3	per cent of the cases

The mean values of the errors in load condition 4.56 per cent
in light condition 3.92 per cent.

These are not very high, compared with the error of the measurement. In *Fig. 7* the measured points are given [1] and the curve [2] of the thrust coefficient (c_T), plotted against the ship speed (v_m), in the case of the model $a = 18$ in loaded condition.

We can say, that our assumption is practically true in the case of the Victory ship $v_s = 10 \sim 17$ *kt.*

As the errors are not higher than the errors of the measurement, and there is no effect of Froude's number in the investigated field of ship speed,

$$\left(Fr = \frac{v}{\sqrt{gL}} = 0.14 \sim 0.24, \text{ where } v \text{ m/s, } L\text{m}\right)$$

the difference of the thrust and resistance $(T - R)$ is equal to the difference of the viscous resistance in self-propelled and towed conditions.

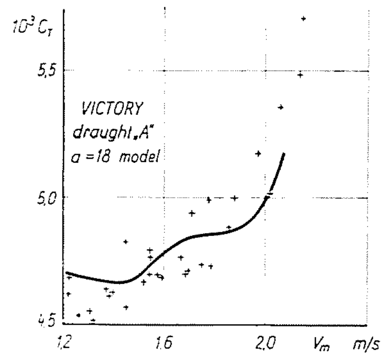

Fig. 7

The Victory ship was investigated in two conditions. The principal dimensions of the ship are as follows:

		Load condition	Light condition
Lp	m	133.045	133.045
L	m	135.562	133.177
B	m	18.898	18.898
T (mean value)	m	8.687	6.809
Δ	m³	15019	11370.3
A	m²	3687	3164
D (diam. of screw)	m	5.3	5.3
$c_B = \frac{\Delta}{LBT}$		0.6876	0.6575
$c_A = \frac{A}{L(B+2T)}$		0.750	0.731
$\frac{D}{T}$		0.610	0.669

Though it is premature to make a determination of the effect of the ship-form and the relative position of propeller from these two investigated model families, it is interesting to note that the constants a and b in the formula of k may be determined in the following way:

$$a = \frac{0.623}{c_B \cdot c_A} \qquad a_{load} = \frac{0.623}{0.6876 \cdot 0.750} = 1.208 \approx 1.205$$

$$a_{light} = \frac{0.623}{0.6565 \cdot 0.731} = 1.296 \approx 1.295$$

$$b = 0.568 \frac{D}{T} \qquad b_{load} = 0.568 \cdot 0.610 = 0.347$$

$$b_{light} = 0.568 \cdot 0.669 = 0.380$$

Conclusion

It may be seen from the investigation of the measured results of the Victory model family that the calculated values of k from the model experiment of a ship without high propeller load, plotted against c_F, gives a linear extrapolator. If we repeat the model experiment with the same model having different roughness, we can get the extrapolator more exactly. With this extrapolator we can obtain the value of k for the ship and determine the c_T thrust coefficient with the aid of c_R resistance coefficient

$$c_T = \frac{c_R - c_F + k}{1 - k \dfrac{A}{A_0}}$$

or the resistance coefficient

$$c_R = c_T + c_F - k\left(1 + \frac{A}{A_0} c_T\right)$$

with a good approximation.

Summary

The thrust deduction of a model family of a single screw ship is a linear function of the viscous resistance coefficient:

$$\frac{c_T - c_W}{1 + \dfrac{A}{A_0} c_T} = a\, c_F + b$$

This linear function gives us the possibility to determine the real value of the thrust deduction of a ship, from the results of measured model data without any scale effect.

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