A SIMPLE METHOD OF DESIGNING A SINGLE STAGE AXIAL FLOW FAN FOR PRESCRIBED SPANWISE CIRCULATION

By

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Notations

a, b, c	= constants
c_a	= axial velocity before the rotor
c_D	= drag coefficient
c_{D_A}	= annulus drag coefficient
c_{D_P}	= profile drag coefficient
c_{D_S}	= secondary drag coefficient
c_L	= lift coefficient
c_{ti}	= tangential velocity due to trailing vortices
c_u	= tangential velocity when the circulation is constant
$\bar{c}_u$	= tangential velocity when the circulation is not constant
K_{D_A}	= annulus drag loss coefficient
K_{D_P}	= profile drag loss coefficient
K_{D_S}	= secondary drag loss coefficient
K_R	= loss coefficient of rotor
K_{swirl}	= loss coefficient due to swirl
$\bar{K}_{act}$	= actual mean head rise coefficient
$\bar{p}_S$	= mean static pressure rise coefficient
K_{th}, $\bar{K}_{th}$	= theoretical and mean theoretical headrise coefficients, respectively
l	= chord length
l/s	= solidity
N	= number of blades
Q	= torque
Q_C	= torque coefficient
R, Θ, Z	= cylindrical polar co-ordinates
R_H	= hub radius
R_H/R_T	= $\frac{\text{hub}}{\text{tip}}$ ratio
R_T	= tip radius
S	= blade spacing
T	= thrust
T_C	= thrust coefficient
W_1	= relative inlet velocity
W_2	= relative outlet velocity
W_∞	= relative mean velocity
$\bar{W}_\infty$	= new relative mean velocity
η	= efficiency at a point
$\bar{\eta}$	= mean efficiency
$\Phi_{eff.}$	= effective angle of attack
$\Phi_{geo.}$	= geometric angle of attack
φ	= angle which the relative mean velocity, W_∞, makes with the tangential direction
$\bar{\varphi}$	= angle which the new relative mean velocity, $\bar{W}_\infty$, makes with the tangential direction

A	$= C_a/\omega R_T$	
ω	= angular velocity of the rotor	
Γ	= circulation	
Γ_H	= circulation at the hub.	

Suffixes: H denotes the value at the hub
T denotes the value at the tip.

1. Introduction

The solution of a three-dimensional flow through an axial turbo machine has been obtained by MARBLE [6—7], MIKHAIL [8], RUDEN [12], SMITH, TRAUGOTT and WISLICENUS [13] and others under the assumptions that the circulation is variable along the blade length and there are infinite number of blades in each row and $R_H/R_T > 0.45$. The fluid is considered as incompressible, frictionless and without heat transfer. HOWELL [4] obtains the solution of three-dimensional flow in an axial compressor by giving expressions for the slope of the velocity profile as a function of axial co-ordinate and considering the flow that occurs in the neighbourhood of the midradius of the flow annulus. BETZ [1] has obtained the three components of the induced velocity due to trailing vortices which are spirals, in the case of an airscrew, but has not integrated them. GLAUBERT [3] has obtained the solution of the direct problem in the case of a single wing of finite span by the isolated aerofoil method, by considering the variation of the circulation along the blade length. WALLIS [15] has obtained the solution of the three-dimensional flow in an axial fan with pre-rotator and straightener in the case of non-free-vortex flow without taking into account the effect of trailing vortices. NATH [9] has obtained the solution of the three-dimensional flow in an axial fan consisting of N (finite) blades, without pre-rotator and straightener and with R_H/R_T lying between 0.2 to 0.45 when the circulation is prescribed. He has taken the circulation as increasing along the span of the blade being minimum at the hub and maximum at the tip so that the derivative of the circulation vanishes at the hub and the tip. The trailing vortices are taken as straight lines parallel to the axis. Further NATH [10] has obtained the solution considering the trailing vortices as spirals and the number of blades as infinite.

The present author considers the same problem under the assumptions of ref. 9 by replacing the finite number of blades by an infinite number of blades so that the total circulation is $N\Gamma$. Therefore, the trailing vortices will form an infinite concentric cylinder. The assumption that there are infinite number of blades is justified because for a given circulation it gives almost the same efficiency and the chord distribution as in the case of finite number of blades. The assumption that the trailing vortices are straight lines instead of spirals is also valid, because in both cases, the efficiency is almost the same for the same circulation. The results obtained by the present method have been compared with a more exact method given in ref. 9.

2. Basic assumptions

We prescribe a circulation which increases along the radius and whose derivative vanishes at the hub and the tip. In addition, the following assumptions have been made:

a) The fluid is non-viscous and incompressible, but frictional forces are taken into account.

b) Each blade is treated as a lifting line for purposes of induced-velocity calculation.

c) The $\frac{\text{hub}}{\text{tip}}$ ratio lies between 0.2 to 0.45 and $c_a/\omega R_T$ lies between 0.1 to 0.3.

d) The axial velocity, c_a, is given and is taken as a constant before the rotor.

e) The tip clearance is considered to be zero.

f) For numerical calculation, $a = -0.01$, $c = 0.02$, $R_H/R_T = 0.2$. The aerofoil is R.A.F. 6E, Re. No. 0.312×10^6, $c_{D_p} = 0.177$, $c_L = 1$, $\Phi_{\text{eff.}} = 6°$. c_L, c_{D_p} and $\Phi_{\text{eff.}}$ have been considered as constant along the radius, as the effect of Reynolds number on them is very small.

3. Outline of the method

For a prescribed circulation, first the induced velocity due to trailing vortices is obtained. Then an aerofoil section is chosen and the effective angle of attack, corresponding to the design lift coefficient is obtained from isolated aerofoil data. Then the geometrical angle of attack is obtained. Knowing the lift coefficient and the new resultant velocity, the chord length can be obtained. Then the losses due to profile, secondary and annulus drag and swirl are obtained and hence the efficiency is determined. Further torque and thrust coefficients are also obtained.

4. Basic equation and solution

Consider the rotor of a single stage axial fan consisting of N blades placed at equal distance apart, whose circulation Γ varies from the inner radius, R_H, to the outer radius, R_T. We replace these N blades by infinite number of blades so that the total circulation is $N\Gamma$. We want to find out the induced velocity at any point due to trailing vortices which form an infinite concentric cylinder.

The circulation in non-dimensional form can be written as:

$$\frac{\Gamma}{\omega R_T^2} = \Gamma_1 = a\left(R/R_T - \frac{R_H}{R_T}\right)^3 + b\left(R/R_T - \frac{R_H}{R_T}\right)^2 + c \tag{1}$$

where

$$b = -\frac{3a}{2}\left(1 - \frac{R_H}{R_T}\right)$$

The induced tangential velocity for a constant circulation is given by:

$$\frac{c_u}{\omega R_T} = \frac{N\Gamma_{1H}}{2\pi\, R/R_T} \tag{2}$$

The induced tangential velocity when the circulation is variable is given by:

$$\frac{\bar{c}_u}{\omega R_T} = \frac{c_u}{\omega R_T} + 2\,\frac{c_{ti}}{\omega R_T} = \frac{N\Gamma_1}{2\pi\, R/R_T} \tag{3}$$

There is no induced velocity due to trailing vortices in the axial and radial directions.

The tangential induced velocity is given in Table I and Fig. 1. It decreases as N or circulation, Γ_1, decreases. It has the maximum value at the hub.

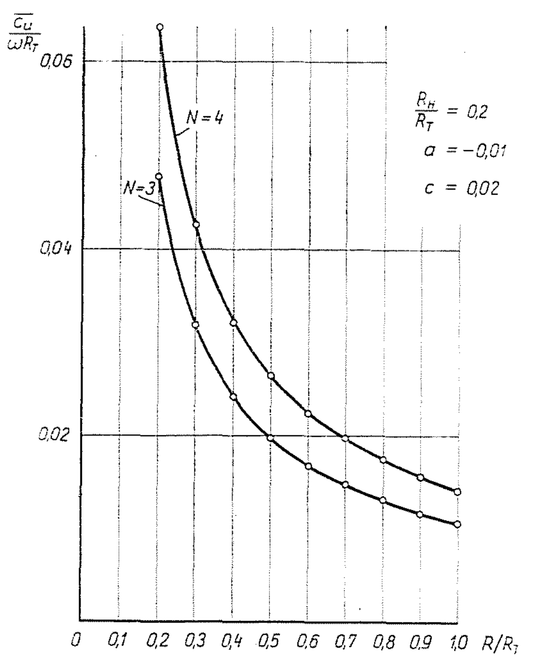

Fig. 1. Tangential velocity distribution

Table I

Tangential component of induced velocity

	Present method				Method of [9] (Finite Number of blades)	
	$N=4$		$N=3$		$N=4$	$N=3$
R/R_T	$\frac{\bar{c}_u}{\omega R_T}$	$\frac{\bar{c}_u}{\omega R_T}$	$\frac{\bar{c}_u}{\omega R_T}$	$\frac{\bar{c}_u}{\omega R_T}$	$\frac{\bar{c}_u}{\omega R_T}$	$\frac{\bar{c}_u}{\omega R_T}$
0.2	0.06366	0.06366	0.04774	0.04774	0.06318	0.04707
0.3	0.04244	0.04267	0.03183	0.03200	0.04175	0.03087
0.4	0.03183	0.03247	0.02387	0.02435	0.03160	0.02332
0.5	0.02546	0.02649	0.01910	0.01987	0.02581	0.01909
0.6	0.02122	0.02258	0.01591	0.01693	0.02227	0.01661
0.7	0.01819	0.01978	0.01364	0.01483	0.01994	0.01504
0.8	0.01591	0.01763	0.01194	0.01322	0.01826	0.01395
0.9	0.01415	0.01588	0.01061	0.01191	0.01685	0.01291
1.0	0.01273	0.01436	0.00955	0.01077	0.01512	0.01162

5. Geometrical angle of attack, chord length and headrise coefficient

The effective angle of attack is given by the experimental data of isolated aerofoil of infinite span by choosing design value of c_L [11]. The angles ψ and $\bar{\psi}$ can be obtained from the velocity diagram (Fig. 2).

Hence

$$\tan \psi = \frac{A}{R/R_T - \dfrac{N\Gamma_{1H}}{4\pi\, R/R_T}} \tag{4}$$

$$\tan \bar{\psi} = \frac{A}{R/R_T - \dfrac{N\Gamma_1}{4\pi\, R/R_T}} \tag{5}$$

It is known that

$$\Phi_{\text{eff.}} = \Phi_{\text{geo.}} - (\bar{\psi} - \psi) \tag{6}$$

The new relative mean velocity is given by:

$$\left(\frac{\overline{W}_\infty}{\omega R_T}\right)^2 = A^2 + \left(R/R_T - \frac{N\Gamma_1}{4\pi\, R/R_T}\right)^2 \tag{7}$$

The chord length is given by:

$$l/R_T = \frac{2\Gamma_1}{c_L \frac{\overline{W}_\infty}{\omega R_T}} \tag{8}$$

l/R_T is given in Table II and Fig 3.

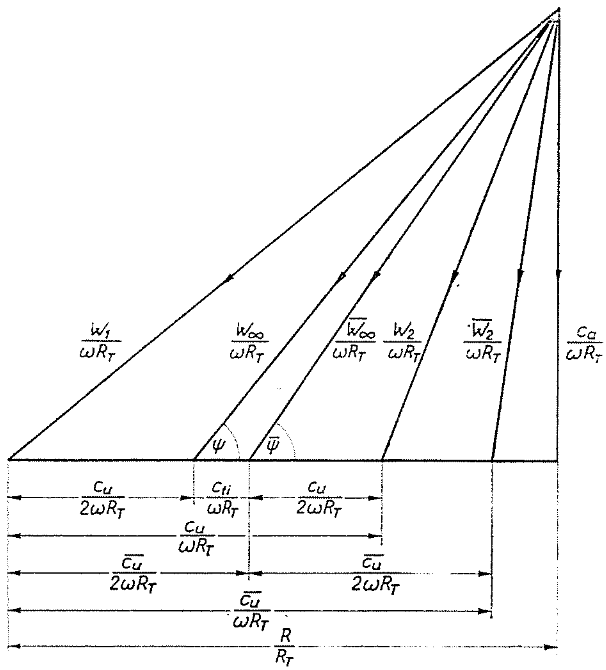

Fig. 2. Dimensionless velocity diagram of axial flow fan

Table II

Chord length, l/R_T

R/R_T	Present method				Method of [9] (Finite number of blades)			
	$N = 4$		$N = 3$		$N = 4$		$N = 3$	
	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$
0.2	0.11632	0.15308	0.11503	0.15010	0.11626	0.15298	0.11492	0.14997
0.3	0.09823	0.11726	0.09736	0.11579	0.09815	0.11713	0.09727	0.11563
0.4	0.08376	0.09617	0.08321	0.09350	0.08370	0.09420	0.08314	0.09340
0.5	0.07279	0.07909	0.07243	0.07863	0.07275	0.07904	0.07239	0.07858
0.6	0.06441	0.06845	0.06417	0.06816	0.06440	0.06843	0.06415	0.06814
0.7	0.05781	0.06054	0.05763	0.06034	0.05781	0.06055	0.05764	0.06035
0.8	0.05238	0.05431	0.05225	0.05417	0.05240	0.05433	0.05227	0.05419
0.9	0.04771	0.04911	0.04761	0.04901	0.04773	0.04014	0.04764	0.04904
1.0	0.04350	0.04455	0.04343	0.04447	0.04352	0.04457	0.04345	0.04449

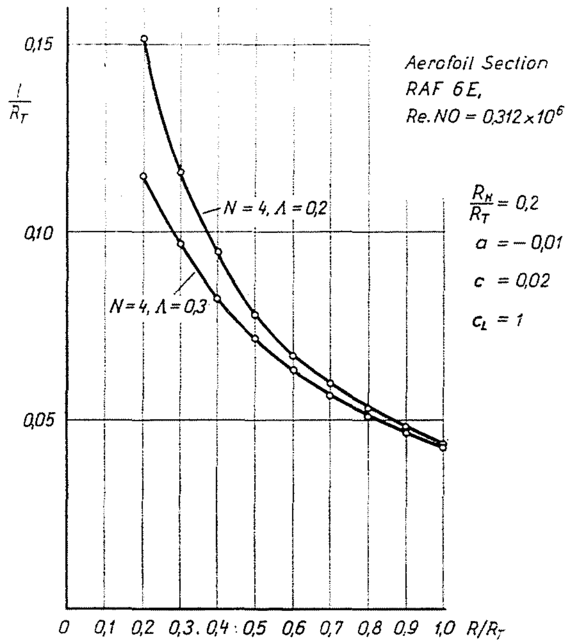

Fig. 3. Chord distribution

All pressure coefficients were constructed by using the dynamic head $\frac{1}{2}\ Pc_a^2$.

Hence

$$K_{th} = \frac{2\,\dfrac{\bar{c}_u}{\omega R_T}}{\Lambda^2}\,R/R_T \tag{9}$$

Table III

Theoretical head rise coefficient, K_{th}

R/R_T	Present Method				Method of [9] (Finite number of blades)			
	$N = 4$		$N = 3$		$N = 4$		$N = 3$	
	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$
0.2	0.2829	0.6366	0.2121	0.4774	0.2808	0.6318	0.2092	0.4707
0.3	0.2844	0.6400	0.2133	0.4800	0.2783	0.6263	0.2058	0.4630
0.4	0.2886	0.6494	0.2164	0.4870	0.2809	0.6321	0.2073	0.4664
0.5	0.2943	0.6622	0.2207	0.4967	0.2868	0.6452	0.2122	0.4774
0.6	0.3010	0.6774	0.2257	0.5079	0.2970	0.6682	0.2214	0.4982
0.7	0.3076	0.6923	0.2306	0.5190	0.3102	0.6980	0.2340	0.5265
0.8	0.3134	0.7052	0.2350	0.5288	0.3247	0.7305	0.2479	0.5578
0.9	0.3176	0.7146	0.2381	0.5359	0.3369	0.7581	0.2597	0.5844
1.0	0.3191	0.7180	0.2393	0.5385	0.3359	0.7559	0.2583	0.5811

K_{th} increases as R/R_T or $\frac{\overline{c}_u}{\omega R_T}$ increases, but decreases as Λ increases. It is given in Table III. Again $\overline{\Phi}_s$, $\overline{K}_{th}$ and $\overline{K}_{act}$ decrease as Λ increases and increase, as N increases. It is given in Table X.

N has very little effect on the chord distribution. It increases when the circulation increases but decreases when Λ increases.

There is no interference effect as $l/R_T < 0.55$ and hence the application of isolated aerofoil method is justified. The angle, $\Phi_{geo.}$ is given in Table IV and $\overline{\psi} + \Phi_{eff.}$ in Table V.

Table IV

$\Phi^\circ_{geo.}$

R/R_T	Present method				Method of [9] (Finite number of blades)			
	$N = 4$		$N = 3$		$N = 4$		$N = 3$	
	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$
0.2	6.0000	6.0000	6.0000	6.0000	5.9640	5.9600	5.9443	5.9460
0.3	6.0115	6.0110	6.0107	6.0084	5.9670	5.9672	5.9403	5.9543
0.4	6.0235	6.0196	6.0160	6.0098	5.9922	5.9930	5.9795	5.9834
0.5	6.0302	6.0207	6.0202	6.0161	6.0121	6.0069	6.0000	6.0000
0.6	6.0268	6.0206	6.0198	6.0146	6.0208	6.0160	6.0135	6.0099
0.7	6.0239	6.0176	6.0181	6.0131	6.0262	6.0193	6.0213	6.0154
0.8	6.0181	6.0154	6.0152	6.0113	6.0326	6.0210	6.0234	6.0177
0.9	6.0173	6.0116	6.0133	6.0085	6.0269	6.0180	6.0240	6.0156
1.0	6.0131	6.0092	6.0102	6.0069	6.0192	6.0135	6.0170	6.0117

Table V

$(\overline{\psi} + \Phi_{eff.})^\circ$

R/R_T	Present method				Method of [9] (Finite number of blades)			
	$N = 4$		$N = 3$		$N = 4$		$N = 3$	
	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$
0.2	66.7271	55.9317	65.5885	54.6385	66.6911	55.8918	65.5328	54.5845
0.3	53.1142	41.6671	52.5780	41.1551	53.0697	41.6233	52.5076	41.1009
0.4	44.0152	33.5250	43.7230	33.2771	43.9839	33.4984	43.6865	33.2507
0.5	37.6471	28.3374	37.4735	28.2004	37.6290	28.3236	37.4533	28.1843
0.6	33.0040	24.7644	32.8934	24.6802	32.9980	24.7598	32.8871	24.6755
0.7	29.4958	22.1636	29.4219	22.1091	29.4981	22.1653	29.4251	22.1114
0.8	26.8062	20.1881	26.7107	20.1490	26.8114	20.1937	26.7189	20.1554
0.9	24.5894	18.6368	24.5501	18.6106	24.5990	18.6432	24.5608	18.6177
1.0	22.8141	17.3914	22.7854	17.3712	22.8202	17.3957	22.7922	17.3760

6. Fan unit efficiency

In a fan unit which consists of a rotor only, the loss in the efficiency is due to rotor and swirl losses, other losses are neglected. The value of losses along the blade span will not be the same. To overcome this difficulty, we calculate the mean value of the losses. The secondary and annulus losses in a non-free-vortex flow are not precisely known, hence they have been calculated

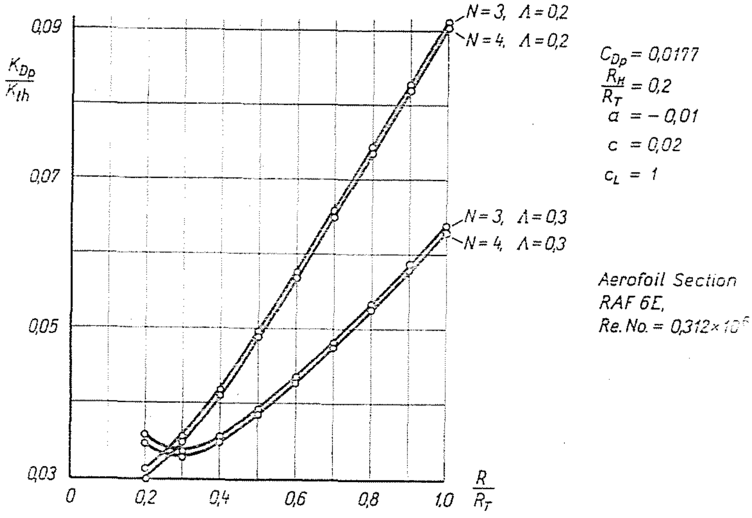

Fig. 4. Loss in efficiency due to profile drag

accordings to [15], where the efficiency at a point (R/R_T, 0.0) is given by

$$\eta = 1 - \frac{K_R}{K_{th}} - \frac{K_{swirl}}{K_{th}} \tag{10}$$

$$\frac{K_R}{K_{th}} = \frac{K_{D_P} + K_{D_S} + K_{D_A}}{K_{th}} = \left(\frac{c_{D_P} + c_{D_S}}{c_L}\right) \frac{A}{\sin^2 \overline{\psi}} \, \frac{1}{R/R_T} + \frac{K_{D_A}}{K_{th}} . \tag{11}$$

The profile drag coefficient, c_{D_p}, is calculated from the experimental data of isolated aerofoil of infinite span, by taking the design value of c_l (Say 1).

The secondary drag coefficient, c_{D_S}, can be obtained by the well-known empirical formula by carter i. e.

$$c_{D_S} = 0.018\, c_L^2 \quad [15] .$$

This equation has been used, as there is no better equation for the determination of c_{D_S}. The annulus loss, $\frac{K_{D_A}}{K_{th}} = 0.02$, for the fan unit [15].

The secondary loss should be evaluated at the radius where profile loss assumes its mean value.

The swirl loss at any point $(R/R_T, 0.0)$ is given by:

$$\frac{K_{swirl}}{K_{th}} = \frac{1}{2} \frac{\bar{c}_u}{\omega R_T} \frac{1}{R/R_T} \tag{12}$$

The mean efficiency is given by:

$$\bar{\eta} = 1 - \frac{2}{1 - \left(\frac{R_H}{R_T}\right)^2} \int_{\frac{R_H}{R_T}}^{1} \left[\frac{K_R}{K_{th}} + \frac{K_{swirl}}{K_{th}}\right] R/R_T \, d(R/R_T) \tag{13}$$

Equation (13) can be integrated numerically, graphically or otherwise. The profile loss is given in Table VI and Fig. 4.

Table VI

Rotor loss due to profile drag, $\frac{KD_P}{K_{th}}$

R/R_T	Present method				Method of [9] (Finite number of blades)			
	$N = 4$		$N = 3$		$N = 4$		$N = 3$	
	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$
0.2	0.03489	0.03021	0.03570	0.03142	0.3491	0.03025	0.03573	0.03148
0.3	0.03297	0.03471	0.03356	0.03559	0.03302	0.03478	0.03362	0.03569
0.4	0.03499	0.04143	0.03546	0.04213	0.03505	0.4151	0.03552	0.04221
0.5	0.03858	0.04901	0.03896	0.04959	0.03862	0.04907	0.03900	0.04965
0.6	0.04293	0.05702	0.04326	0.05751	0.04295	0.05705	0.04327	0.05754
0.7	0.04772	0.06527	0.04801	0.06570	0.04772	0.06525	0.04800	0.06568
0.8	0.05280	0.07367	0.05306	0.07406	0.05276	0.07362	0.05302	0.07399
0.9	0.05807	0.08218	0.05830	0.08253	0.05801	0.08210	0.05823	0.08244
1.0	0.06346	0.09077	0.06367	0.09109	0.06342	0.09071	0.06362	0.09101

For a prescribed circulation, Γ_1, the profile loss decreases when Λ or N or both increase. When $\Lambda = 0.2$, the loss rapidly increases towards the tip, but when $\Lambda = 0.3$, the increase is not so rapid.

The swirl loss is given in Table VII and Fig. 5.

Table VII

Swirl loss, $\frac{K_{swirl}}{K_{th}}$

Present method			Method of [9] (Finite number of blades)	
R/R_T	$N = 4$	$N = 3$	$N = 4$	$N = 3$
0.2	0.15915	0.11936	0.15796	0.11768
0.3	0.07112	0.05334	0.06959	0.05144
0.4	0.04058	0.03044	0.03950	0.02915
0.5	0.02649	0.01987	0.02581	0.01909
0.6	0.01881	0.01411	0.01856	0.01384
0.7	0.01413	0.01059	0.01424	0.01074
0.8	0.01102	0.00826	0.01141	0.00871
0.9	0.00882	0.00661	0.00936	0.00721
1.0	0.00718	0.00538	0.00756	0.00581

The swirl loss is independent of Λ and it decreases as R/R_T increases. It has a very high value at the hub. It decreases as R_H/R_T increases and increases as circulation or N increases.

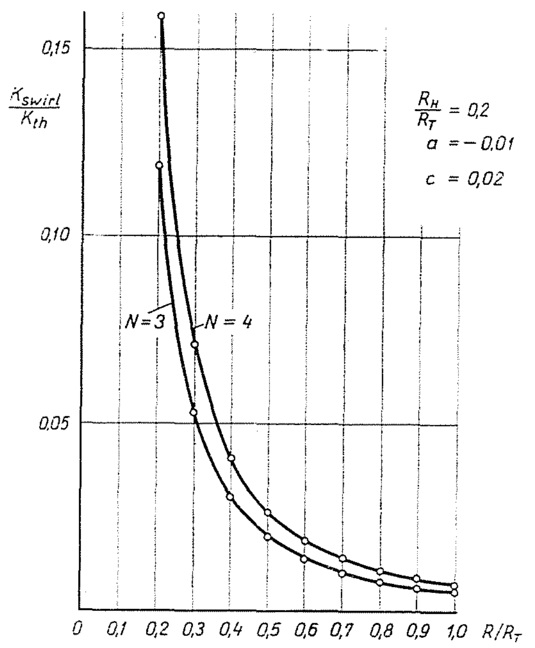

Fig. 5. Loss in efficiency due to swirl

The efficiency due to profile and swirl loss only and the mean efficiency of the rotor are given in Table VIII, Fig. 6 and Table IX, respectively.

Table VIII

Efficiency, η due to profile drag and swirl only

R/R_T	Present method				Method of [9] (Finite number of blades)			
	$N = 4$		$N = 3$		$N = 4$		$N = 3$	
	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.2$
0.2	0.80595	0.81063	0.84493	0.84921	0.80712	0.81179	0.84658	0.85083
0.3	0.89590	0.89417	0.91309	0.91106	0.89739	0.89563	0.91493	0.91286
0.4	0.92442	0.91798	0.93410	0.92743	0.92544	0.91898	0.93533	0.92863
0.5	0.93493	0.92449	0.94117	0.93054	0.93557	0.92511	0.94190	0.93125
0.6	0.93825	0.92416	0.94263	0.92837	0.93849	0.92439	0.94288	0.92862
0.7	0.93814	0.92060	0.94139	0.92370	0.93804	0.92050	0.94125	0.92357
0.8	0.93617	0.91530	0.93867	0.91767	0.93582	0.91497	0.93826	0.91729
0.9	0.93311	0.90899	0.93508	0.91085	0.93263	0.90854	0.93455	0.91035
1.0	0.92935	0.90204	0.93094	0.90352	0.92902	0.90173	0.93056	0.90317

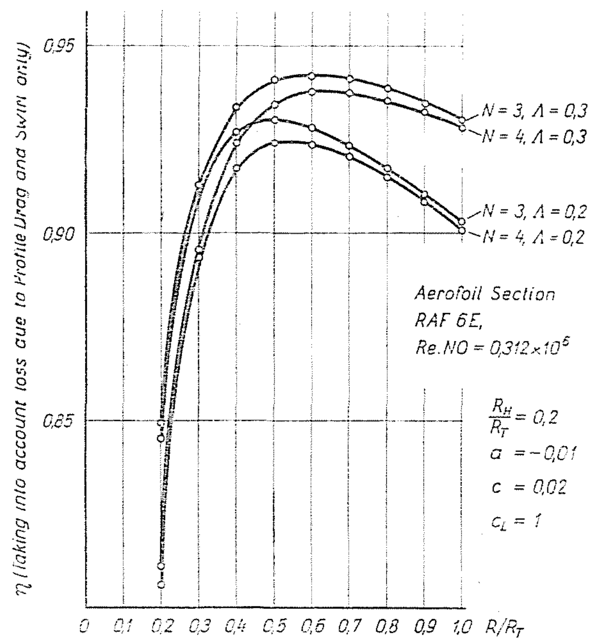

Fig. 6. Efficiency distribution

Table IX

Mean efficiency, $\bar{\eta}$

Present method				Method of Ref. 9 (Finite number of blades)				Method of [10] (When trailing vortices are spirals)		
$N=4$		$N=3$		$N=4$		$N=3$		$N=4$		$N=3$
$\Lambda=0.3$	$\Lambda=0.2$	$\Lambda=0.3$	$\Lambda=0.2$	$\Lambda=0.3$	$\Lambda=0.2$	$\Lambda=0.3$	$\Lambda=0.2$	$\Lambda=0.3$	$\Lambda=0.2$	$\Lambda=0.3$
0.8658	0.8348	0.8707	0.8394	0.8659	0.8349	0.8709	0.8395	0.8644	0.8315	0.8698

7. Torque and thrust

The torque coefficient and the thrust coefficient [15] are respectively given by:

$$Q_C = 4\int_{\frac{R_H}{R_T}}^{1} \frac{\frac{\bar{c}_u}{\omega R_T}}{\Lambda} (R/R_T)^2\, d(R/R_T) \qquad \text{where} \qquad Q_C = \frac{Q}{\frac{1}{2} P c_a^2 \pi R_T^3}$$

$$= a\left[\frac{7}{40}\left(\frac{R_H}{R_T}\right)^5 - \frac{5}{8}\left(\frac{R_H}{R_T}\right)^4 - \frac{1}{4}\left(\frac{R_H}{R_T}\right)^3 + \frac{3}{4}\left(\frac{R_H}{R_T}\right)^2 + \right. \tag{14}$$

$$\left. + \frac{7}{8}\,\frac{R_H}{R_T} - \frac{37}{40}\right] + \frac{c}{2}\left[1 - \left(\frac{R_H}{R_T}\right)^2\right]$$

Similarly,

$$T_C = 2\int_{R_H/R_T}^{1} K_{th}\left(1 - \frac{K_{th}}{K_R} - \frac{K_{swirl}}{K_{th}}\right) R/R_T\, d(R/R_T)\,. \tag{15}$$

Equation (15) can be integrated numerically or graphically.

The torque and thrust coefficients are given in Table X.

For prescribed circulation, Q_C and T_C increase as N increases, but decrease as Λ increases.

8. Approximate estimation of a and c

The approximate minimum and maximum values of a and c are obtained from Equation (3), which can be written as:

$$a\left(R/R_T - \frac{R_H}{R_T}\right)^3 - \frac{3a}{2}\left(1 - \frac{R_H}{R_T}\right)\left(R/R_T - \frac{R_H}{R_T}\right)^2 + c = \frac{2\pi\epsilon\Lambda}{N} R/R_T \tag{16}$$

where

$$\epsilon = \frac{\bar{c}_u}{c_a}$$

Table X

	Present method				Method of [9] (Finite number of blades)			
	$N = 4$		$N = 3$		$N = 4$		$N = 3$	
Q_c	0.0881	0.1321	0.0660	0.0991	0.0896	0.1344	0.0678	0.1017
T_c	0.2461	0.5305	0.1840	0.3992	0.2587	0.5613	0.1968	0.4269
$\bar{p}_s$	0.2564	0.5526	0.1917	0.4158	0.2695	0.5847	0.2050	0.4447
$\bar{K}_{th}$	0.2962	0.6620	0.2200	0.4954	0.3112	0.7003	0.2354	0.5297
$\bar{K}_{ad.}$	0.2630	0.5674	0.1952	0.4240	0.2764	0.6002	0.2089	0.4534
$\frac{\bar{K}_{DP}}{\bar{K}_{th}}$	0.04818	0.06490	0.04851	0.06538	0.04817	0.06488	0.0485	0.06537
$\frac{\bar{K}_{swirl}}{\bar{K}_{th}}$	0.02231	0.02231	0.01674	0.01674	0.02222	0.02222	0.01661	0.01661
$\frac{\bar{K}_{DS}}{\bar{K}_{th}}$	0.04366	0.05798	0.04399	0.05848	0.04367	0.05801	0.04401	0.05851

The minimum and the maximum values of ϵ are 0 and 1, respectively. If ϵ is zero, the inflow and the outflow directions coincide. If the maximum value of ϵ is taken as greater than 1, c_L becomes greater than 1.2, which is generally the maximum permissible value. If $c_L > 1.2$, c_D rises rapidly with the angle of incidence, hence $\frac{c_D}{c_L}$ increases, which reduces the efficiency. For a given A, R_H/R_T and N, the minimum values of a (a is negative) and c are obtained by prescribing maximum circulation at the tip and minimum at the hub by putting ϵ equal to 1 and 0, respectively. Similarly the maximum values of a and c are obtained by prescribing the circulation at the tip to be equal to the maximum circulation at the hub. The maximum and minimum values of a and c are given below:

$$a = 0 \quad \text{(maximum)}$$

$$a = -\frac{4\pi A}{N} \cdot \frac{1}{\left[1 - \frac{R_H}{R_T}\right]^3} \qquad \text{(minimum)}$$

$$c = \frac{2\pi A}{N} \frac{R_H}{R_T} \qquad \text{(maximum)}$$

$$c = 0 \quad \text{(minimum)} \tag{17}$$

Even within this prescribed range in the case of isolated aerofoil method, the values of a and c should be chosen in such a manner that the solidity, l/s, should be less them 0.66.

Boundary conditions at the hub and at the tip

Boundary conditions at the hub: Since the blade velocity at the hub is minimum, the relative velocity at the inlet, W_{1H} is also minimum. $\frac{\bar{c}_u}{\omega R_T}$ is maximum at the hub. This leads to large chord and to high values of c_L at the hub.

However, there exists a definite upper limit for both the chord and the lift coefficient, c_L, at the hub. The upper limit of the lift coefficient, c_L, is always taken to be less than the value at which stalling occurs. This results in minimum values for the blade speed at the hub and also for $\frac{\text{hub}}{\text{tip}}$ ratio. The chord also should not be increased arbitrarily, because high value of the solidity, l/s, introduces harmful interference effect between adjacent blades. Generally for the isolated aerofoil method it does not exceed 0.7 and for the cascade method it does not exceed 2. Similarly for the isolated aerofoil method, generally $\mu \cdot c_L$ also does not exceed 0.7 and for the cascade method it does not exceed 2.

From the equations (7), (8) and (9), it is possible to obtain the required value of $\frac{R_H}{R_T}$, for prescribed μ_H, A and K_{th_H}. The relation between them can be expressed as:

$$\frac{R_H}{R_T} = \frac{A}{2}\left[\left\{4\left(1 - \frac{1}{2}K_{th_H}\right)^2 + \left(\frac{16}{\mu_H^2} - 1\right)K_{th_H}^2\right\}^{\frac{1}{2}} - 2\left(1 - \frac{1}{2}K_{th_H}\right)\right]^{\frac{1}{2}}$$

where $\mu = c_L \cdot l/s$ (18)

This relation is very important, because now it is possible to know the value of $\frac{R_H}{R_T}$ to be taken in order to obtain the required value of K_{th} at the hub provided A and μ_H are also known. Values of $\frac{R_H}{R_T}$ calculated from equation (18) are given in Table XI and Fig. 7. They show that $\frac{R_H}{R_T}$ depends on A, K_{th_H} and μ_H. It increases as A or K_{th_H} increases or μ_H decreases. From Fig. 7 it can be concluded that low pressure rise axial fan requires small values of $\frac{R_H}{R_T}$.

Table XI

$\frac{R_H}{R_T}$ calculated from Equation (18)

K_{th_H}	$\mu_H = 1$					$\mu_H = 0.2$				
	$\Lambda = 0.1$	$\Lambda = 0.3$	$\Lambda = 0.5$	$\Lambda = 0.7$	$\Lambda = 0.9$	$\Lambda = 0.1$	$\Lambda = 0.3$	$\Lambda = 0.5$	$\Lambda = 0.7$	$\Lambda = 0.9$
0.2	0.0199	0.0597	0.0995	0.1394	0.1792	0.0803	0.2410	0.4017	0.5624	0.7231
0.4	0.0396	0.1188	0.1979	0.2771	0.3563	0.1279	0.3838	0.6397	0.8956	
0.8	0.0728	0.2185	0.3642	0.5099	0.6556	0.1925	0.5775	0.9625		
1.2	0.0989	0.2968	0.4947	0.6926	0.8905	0.2407	0.7222			
1.6	0.1205	0.3615	0.6025	0.8436		0.2809	0.8328			
2.0	0.1391	0.4174	0.6957	0.9740		0.3160	0.9481			

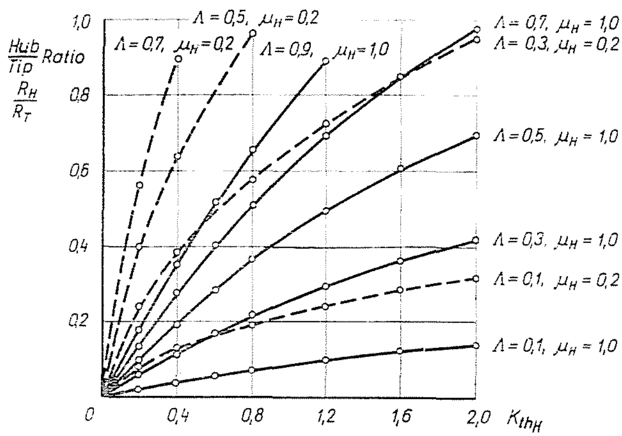

Fig. 7. Variation of R_H/R_T with K_{thH}, Λ and μ_H

In a non-free vortex flow, K_{th} generally increases with the radius. Normally a specified value of K_{th} at the tip is required. If a specified value of K_{th_T} is required for a prescribed Λ, N and μ_H, the corresponding values of K_{th} at the hub can always be chosen. The difference between K_{th_H} and K_{th_T} should not be large, due to design difficulties. In the section 10, the method is given by which it is possible to verify whether a prescribed K_{th} occurs when K_{th_H} has been chosen. Hence it is always possible to choose K_{th_H} corresponding to K_{th_T} and from the corresponding value of $\frac{R_H}{R_T}$ it can be chosen. Hence, the above equation can give an idea about the value of $\frac{R_H}{R_T}$ to be employed, because in the absence of such a relation, it is very difficult to foresee the value, and one has to follow the method of trial and error.

Boundary conditions at the tips of blades : The upper limit of the rotational velocity at the blade tip can be fixed at 550 ft/sec, because at higher values, the air can no longer be regarded as incompressible and will invalidate the assumption of constant air density. Moreover there will be a considerable decrease in efficiency and large increase in noise.

10. Circulation corresponding to prescribed headrise

Great difficulty is normally encountered in obtaining the circulation which can give prescribed K_{th_T}, when Λ, N and $\frac{R_H}{R_T}$ are known. Until now the method of trial and error has been employed to obtain the required K_{th_T} in a rotor of the fan. Here attempt has been made to calculate it approximately, when the circulation increases with the radius. From equations (2) and (9) the relation can be expressed as:

$$\left.\begin{aligned} \Gamma_{1H} &= \frac{\Lambda^2 \pi K_{th_H}}{N} \\ \Gamma_{1T} &= \frac{\Lambda^2 \pi K_{th_T}}{N} \end{aligned}\right\} \qquad (19)$$

Values of Γ_{1T} calculated from equation (19) are given in Table XII and Fig. 8. Γ_{1H} or Γ_{1T} varies directly as K_{th_H} or K_{th_T} and Λ and inversely as N.

Table XII

Γ_{1H} or Γ_{1T} calculated from Equation (19)

K_{th_H} or K_{th_T}	$N = 3$					$N = 6$				
	$\Lambda = 0.1$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.5$	$\Lambda = 0.7$	$\Lambda = 0.1$	$\Lambda = 0.2$	$\Lambda = 0.3$	$\Lambda = 0.5$	$\Lambda = 0.7$
0.25	0.0026	0.0105	0.0235	0.0654	0.1283	0.0013	0.0052	0.0118	0.0327	0.0641
0.50	0.0052	0.0209	0.0471	0.1309	0.2566	0.0026	0.0105	0.0236	0.0654	0.1283
1.0	0.0105	0.0419	0.0942	0.2618	0.5132	0.0052	0.0209	0.0471	0.1309	0.2566
1.5	0.0157	0.0628	0.1414	0.3927	0.7698	0.0078	0.0314	0.0707	0.1964	0.3849
2.0	0.0209	0.0838	0.1885	0.5236	1.0264	0.0105	0.0419	0.0942	0.2618	0.5132
2.5	0.0262	0.1047	0.2356	0.6545	1.2829	0.0131	0.0524	0.1178	0.3273	0.6415
3.0	0.0314	0.1256	0.2828	0.7855	1.5395	0.0157	0.0628	0.1414	0.3927	0.7698
3.5	0.0366	0.1466	0.3299	0.9164	1.7961	0.0183	0.0733	0.1649	0.4582	0.8981
4.0	0.0419	0.1675	0.3770	1.047	2.0527	0.0209	0.0838	0.1885	0.5236	1.0264

In a non-free-vortex flow when the circulation increases with the radius, K_{th} also increases with the radius. The difference between K_{th_H} and K_{th_T} should not be much due to the design difficulties. For a prescribed K_{th_T}, it is always possible to chose K_{th_H}. For a given Λ and N, the value of the circulation at the hub corresponding to K_{th_H} can be obtained from Fig. 8.

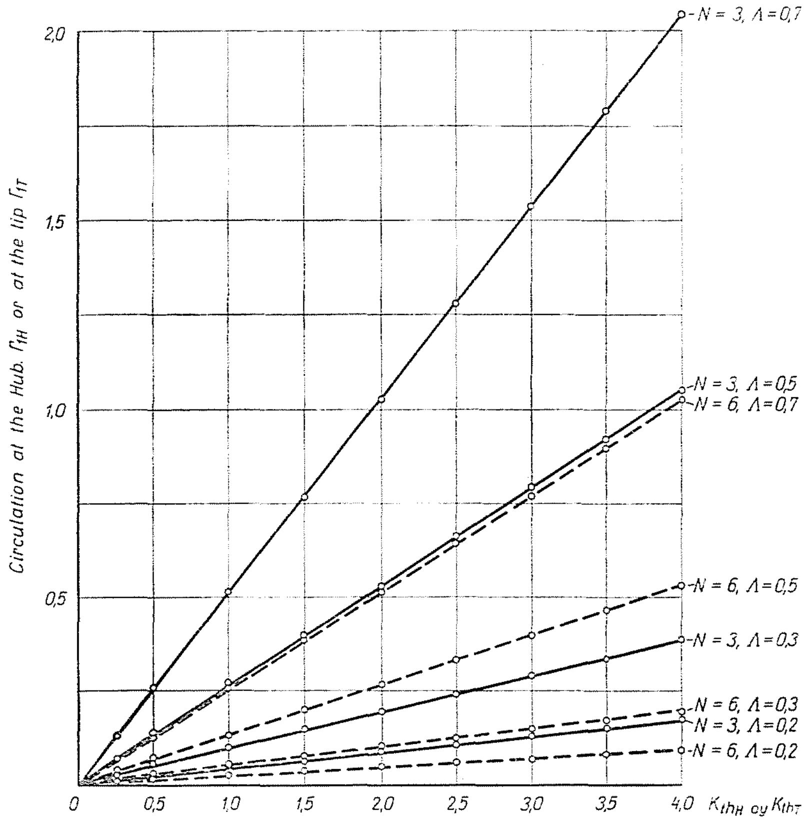

Fig. 8. Variation of Γ_{1T} with K_{thH}, Λ and N

Similarly Γ_{1T} corresponding to K_{th_H} can also be obtained from Fig. 8. In the present case, circulation at the hub will give c and the circulation at the tip will give a. Hence knowing a and c, the circulation at any point can be determined. Knowing the circulation at every point along the radius, the distribution of K_{th} and hence $\overline{K}_{th}$ can be obtained. Hence, with the aid of the above equation, it is now possible to obtain the required circulation which can give a prescribed K_{th_T}. This equation is important from the point of view of design because it gives the designers of a non-free-vortex flow fan a method by which they can immediately obtain the required circulation which gives a prescribed K_{th_T}, when Λ, N and $\frac{R_H}{R_T}$ are known. It can be concluded from Fig. 8 that for low pressure rise fan using isolated aerofoil method, small values of a and c are employed as Λ is also small.

11. Design limitation of some parameters

The properties of the fan are influenced by four major parameters: Λ, N, Γ_1, and $\frac{R_H}{R_T}$. It is not easy to give a precise lower or upper limit to them as these limits vary according to the nature of the fan and the method of designing it i. e. whether a low pressure rise or a high pressure rise fan is desired or whether the isolated aerofoil method or the cascade method is employed. Hence, only the tentative limits can be prescribed.

Normally, for the isolated aerofoil method, lower values of Λ, N, Γ_1, R_H/R_T are employed as compared to those employed in the cascade method. Generally the isolated aerofoil method is employed in low pressure rise and the cascade method in high pressure rise fan. In prescribing the limit in case of the isolated aerofoil method, it must be borne in mind that the solidity, l/s, should not exceed 0.7. Of course, other considerations like efficiency, static pressure rise etc. must also be taken into account.

The parameters a and c which determine the circulation in the present case depend on Λ, N and $\frac{R_H}{R_T}$. Moreover it is difficult to give a finite upper limit to N. Hence in order to remove the difficulty of prescribing lower or upper limit to Λ, N, $\frac{R_H}{R_T}$ and Γ_1 individually, efforts will be made to prescribe the upper and the lower limits to $\frac{R_H}{R_T}$, Λ and $\frac{\bar{c}_u}{\omega R_T}$ only. $\frac{\bar{c}_u}{\omega R_T}$ depends on N, Γ_1 and $\frac{R_H}{R_T}$.

For the isolated aerofoil method, generally, $\frac{R_H}{R_T}$ lies between 0.2 and 0.6. For cascade method it is greater or equal to 0.6. Similarly for the isolated aerofoil method, $\frac{c_u}{\omega R_T}$ should be less than 0.19 and Λ should lie between 0.04 and 0.48. If $\frac{\bar{c}_u}{\omega R_T}$ exceeds 0.19 and Λ exceeds 0.48, $\bar{\psi}$ becomes greater than 40°, μ_H exceeds 0.7 and swirl loss at the hub becomes very high. Similarly if Λ is taken as less than 0.04, there is great reduction in the efficiency. At the same time care should be taken so that $\frac{\bar{c}_u}{c_a}$ should not exceed 0.4. For the cascade method, $\frac{\bar{c}_u}{\omega R_T}$ should lie between 0.19 and 0.9 and Λ should lie between 0.48 and 0.9. If $\frac{\bar{c}_u}{\omega R_T} > 0.9$ and $\Lambda > 0.9$, the stalling and other design difficulties may occur. At the same time attention should be paid to see that $\frac{\bar{c}_u}{c_a}$ may not exceed 1. In case of the isolated aerofoil method optimum c_L should be taken slightly less than 0.8 for low pressure rise type of fan. Normally optimum c_L should not exceed 1.2 as stalling may occur. Even maximum value of angle of incidence along with optimum c_L can be prescribed, but they vary according to the nature of the aerofoil and also according to Reynolds number. In general the blade Reynolds number lies between 2.10^5 and 10^6.

12. Effect of parameters on the characteristics of the fan

There are four major parameters which affect the characteristics of the fan. They are: (I) axial velocity, Λ. (II) number of blades, N. (III) circulation, Γ_1, (IV) $\frac{\text{hub}}{\text{tip}}$ ratio, $\frac{R_H}{R_T}$. It is essential to know their effects on the characteristics of the fan when they are increased or decreased.

(I) Effect of axial velocity: l/R_T, $\bar{K}_{th}$, $\bar{\Phi}_{eff}$, $\frac{K_{D_P}}{K_{th}}$, $\frac{K_{D_S}}{K_{th}}$ Q_C and T_C decrease as Λ increases, but η increases as Λ increases. $\frac{\bar{c}_u}{\omega R_T}$ and $\frac{K_{swirl}}{K_{th}}$ are independent of Λ.

(II) Effect of number of blades, N: $\frac{\bar{c}_u}{\omega R_T}$, $\bar{K}_{th}$, $\bar{\Phi}_{eff}$, Q_C, T_C and $\frac{K_{swirl}}{K_{th}}$ increase as N increases. The change in $\frac{K_{D_P}}{K_{th}}$, $\frac{K_{D_S}}{K_{th}}$, l/R_T and $\bar{\eta}$ is small, if the change in N is also small.

(III) Effect of circulation, Γ_1: $\frac{\bar{c}_u}{\omega R_T}$, l/R_T, $\bar{K}_{th}$, $\bar{\Phi}_{eff}$, Q_C, T_C and $\frac{K_{swirl}}{K_{th}}$ increase as N increases, but $\frac{K_{D_P}}{K_{th}}$ and $\frac{K_{D_S}}{K_{th}}$ decrease as Γ increases.

(IV) Effect of $\frac{\text{hub}}{\text{tip}}$ ratio, R_H/R_T : $\frac{\bar{c}_u}{\omega R_T}$, l/R_T, $\bar{K}_{th}$, $\bar{\Phi}_{eff}$, Q_C, T_C and $\frac{K_{swirl}}{K_{th}}$ decrease as R_H/R_T increases, but $\frac{K_{D_P}}{K_{th}}$ and $\frac{K_{D_S}}{K_{th}}$ increase as $\frac{R_H}{R_T}$ increases.

13. Conclusions

In the present paper, a method has been developed to design an axial fan without prerotator and straightener with three or more blades having R_H/R_T lying between 0.2 to 0.45. The circulation is prescribed and taken as variable along the span and the effects of straight trailing vortices are taken into account. The difference in the chord distribution and the mean efficiency etc. obtained by different methods is very small. The present method is simple and different from the methods given by other authors.

Summary

The solution of three-dimensional flow in the rotor of a single stage axial flow fan, with prescribed variable circulation was obtained by considering that there are infinite number of blades whose total circulation is $N\Gamma$. R_H/R_T lies between 0.2 to 0.45 and A lies between 0.1 to 0.3. The efficiency, chord length and other design parameters have been obtained. For a prescribed circulation, Γ_1, and N, the difference in the mean efficiency and the chord length obtained by the present methods and by the methods of Refs. 9 and 10 is very small. The fluid is taken as incompressible and frictionless.

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