THE WAKE FRACTION OF A GEOSIM

By

Z. BENEDEK

Department of Hydraulic Machines, Polytechnical University, Budapest

(Received September 1, 1966)

Presented by Prof. B. BALOGH

Symbols used

a = model scale
A = wetted surface of ship (m^2)
A_0 = propeller disc area (m^2)

$$A_0 = \frac{1}{4} D^2 \pi$$

c_F = viscous resistance coefficient

$$c_F = \frac{R_F}{\frac{1}{2} \varrho v^2 A}$$

d = mean value of the thickness of the boundary layer (m)
D = diameter of screw (m)
g = gravitational acceleration (m sec^{-2})
h = loss of kinetic energy of 1 kg water (m)
k = constant
L = length of waterline (m)
p = function of wake fraction
R_F = viscous resistance of ship (kp)
T = draught (mean value) (m)
v = shipspeed (m sec^{-1})
v_A = propeller advance speed (m sec^{-1})
w = effective wake fraction

$$w = \frac{v - v_A}{v}$$

ϱ = water density (kp m^{-4} sec^2)
γ = specific gravity of water (kp m^{-3})

The value of the effective wake fraction is the most important factor in the design of a propeller. We use a lot of simple approximative relations for precalculation, but the exact value is determinable only by the investigation of the model of the ship and the propeller.

However, the investigation of the Victory model family has indicated that models made in different sizes give us different values of the wake fraction for the same shipspeed [1]. E.g. the values of the wake fraction are the following in loaded condition of ship (at even keel) at 11 knots shipspeed:

model scale	6	18	23	30	40	50
$10^3 \cdot w$	269	329	352	364	358	403

The values are different in the same way also at other shipspeeds and in other ship condition (light condition, trimmed by the stern). If the model scale is greater the wake fraction is also greater. We could not say that there is some error of the measure but we must assume that it is a kind of scale effect. Therefore, we need a method for the extrapolation which gives us a possibility to calculate the wake fraction of the ship, from the measured wake fraction of her model.

The speed of advance of a propeller can be defined with its three components. The potential flow around the ship gives a relative velocity in the place of the propeller. The local speed of water in the stern wave system gives the second component. The third component is defined by the local velocities of the boundary layer of the ship hull.

The picture of a potential flow of a perfect fluid is determined by the ship form only. But there is a boundary layer around the ship in the viscous fluid and the thickness of the boundary layer is different at the ship and the model, therefore, the potential flow around the ship and model is also different. But this difference is negligible and we can say that the potential component is the same at the model and at the ship.

The second component is a function of the Froude number and the ship form. Because the Froude numbers of ship and model are equal, this component is also equal if we disregard the changes of the thickness of the boundary layer.

The velocities of the water in the boundary layer are defined by the Reynolds number and by the roughness of the ship hull. Therefore the third component, the so-called viscous component, is different at the ship and at the model.

Thus, there is a difference between wakes of the ship and model or of models made in different sizes owing to the difference of the viscous wake component [2].

The water going along the ship near the hull surface has a loss of its kinetic energy. The loss of the kinetic energy of one kg water is equal to the power of the frictional resistance divided by the mass of water going near the surface of the ship, during one second:

$$h = \frac{\frac{1}{2}\varrho c_F v^2 A v}{\gamma d \frac{A}{L} v} = \frac{1}{2 \cdot g} \frac{L \cdot c_F}{d} v^2 \tag{1}$$

where ϱ is the density of water, γ is the specific gravity of water, v is the shipspeed, A is the wetted surface of the ship, L is the length of the waterline of the ship, d is the mean value of the thickness of the boundary layer. But there is a defined mean value at each shipspeed and propeller advance speed which is constant at each shipspeed. Therefore, we can write the shipspeed:

$$v = X \frac{v + v_A}{2} \tag{2}$$

where X is a constant for a shipspeed. With this the first equation is:

$$h = \frac{1}{2 \cdot g} \frac{L c_F}{d} \frac{X^2}{4} (v + v_A)^2$$

If we introduce the k constant

$$k = \frac{4\,d}{L X^2}$$

then

$$h = \frac{1}{2 \cdot g} \frac{1}{k} c_F (v + v_A)^2 \tag{3}$$

Thus, the loss of kinetic energy of one kg water:

$$\frac{v^2 - v_A^2}{2 \cdot g} = \frac{1}{2 . g} \frac{1}{k} c_F (v + v_A)^2 \tag{4}$$

From this

$$1 - \frac{v_A}{v} = \frac{c_F}{k} \left(1 + \frac{v_A}{v}\right) \tag{5}$$

The effective wake fraction defined by TAYLOR [3]:

$$w = 1 - \frac{v_A}{v}$$

from this

$$\frac{1 + \frac{v_A}{v}}{1 - \frac{v_A}{v}} = \frac{2}{w} - 1$$

therefore

$$\frac{c_F}{w} = \frac{k}{2} + \frac{c_F}{2} \tag{6}$$

Table 1

Model scale	Ship-speed kt	Loaded condition: 10^3 w calculated	Loaded condition: 10^3 w measured	Loaded condition: difference %	Light condition: 10^3 w calculated	Light condition: 10^3 w measured	Light condition: difference %
6	10	270	291	−6.9	292	291	+0.3
	11	268	269	−0.4	291	294	−1
	12	265	266	−0.4	288	293	−1.7
	13	262	262	0	286	290	−1.4
	14	260	259	+0.4	284	285	−0.3
	15	257	256	+0.4	282	281	−0.4
	16	255	251	+1.6	281	281	0
	17	253	247	+2.4	279	288	−3.1
18	10	337	333	+1.2	347	349	+0.6
	11	334	329	+1.5	344	343	+0.3
	12	329	333	+1.2	341	341	0
	13	327	328	−0.3	339	334	+1.2
	14	323	324	−0.3	335	328	+2.1
	15	321	235	−1.2	334	327	+2.1
	16	318	324	−1.9	333	326	+2.1
	17	315	315	0	331	325	+1.8
23	10	353	360	−1.9	355	359	−1.1
	11	348	352	−1.1	351	355	−1.1
	12	345	350	−1.4	349	350	−0.3
	13	342	349	−2.0	347	345	+0.6
	14	339	347	−2.3	345	340	+1.5
	15	336	346	−2.9	344	358	−3.9
	16	333	346	−3.8	341	339	+0.6
	17	330	339	−2.6	339	343	−1.2
30	10	374	370	+2.7	369	354	+4.2
	11	369	364	+1.4	366	356	+2.8
	12	364	366	−0.5	363	358	+1.4
	13	361	361	0	360	359	+0.3
	14	357	353	+1.1	358	359	−0.3
	15	353	347	+1.7	356	359	−0.8
	16	350	348	+0.6	354	359	−1.4
	17	348	349	+0.3	352	357	−1.4

Table 1 continued

Model scale	Shipspeed kt	Loaded condition			Light condition		
		10^3 w calculated	10^3 w measured	difference %	10^3 w calculated	10^3 w measured	difference %
40	10	400	376	+ 6.4	385	397	−3
	11	400	358	+11.9	382	391	−2.3
	12	390	349	+11.7	379	382	−0.8
	13	386	346	+11.6	377	372	+1.3
	14	382	351	+8.8	374	370	+1.1
	15	378	362	+4.4	372	370	+0.5
	16	374	367	+1.9	369	373	−1.5
	17	372	369	+0.8	367	373	−1.6
50	10	417	404	+3.2	397	429	−7.5
	11	412	403	+2.2	394	431	−8.6
	12	407	398	+2.2	391	430	−9.1
	13	404	400	+1	388	427	−9.1
	14	399	396	+0.8	385	427	−9.8
	15	396	400	−1	382	426	−10.3
	16	391	402	−2.7	380	429	−11.4
	17	389	403	−3.5	378	430	−12.3

The value of k contains the mean thickness of the boundary layer (d), the length of ship (L), and the quotient of shipspeed and mean speed (X) so it is the function of Reynolds number and the roughness of the surface at the same ship form (at geometrically similar models, so-called "geosims"). The c_F is also the function of these two, thus we may say the k is to be the function of c_F. In this way, the right side of equation (6) is the function of the viscous resistance coefficient:

$$p = \frac{k}{2} + \frac{c_F}{2} = f(c_F)$$

We have calculated the values of

$$p = \frac{c_F}{w} \tag{7}$$

with the measured data of the Victory model family [1] for all models at different shipspeeds. The calculated values of p are plotted on the c_F in Figs 1 and 2.

A linear function is defined by the plotted points, in both the loaded and the light condition of the ship:

$$p = a\ c_F + b \tag{8}$$

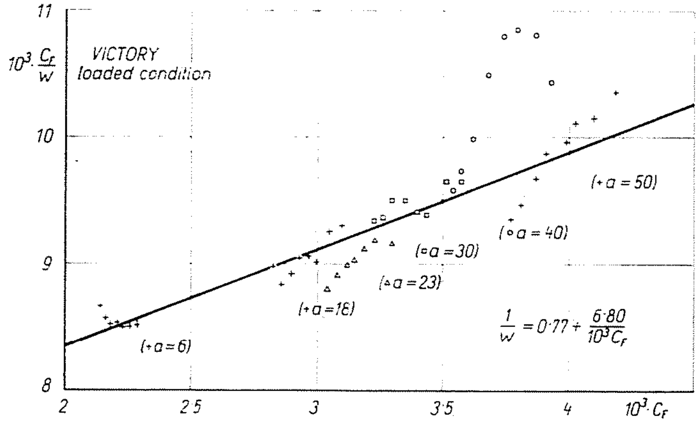

Fig. 1

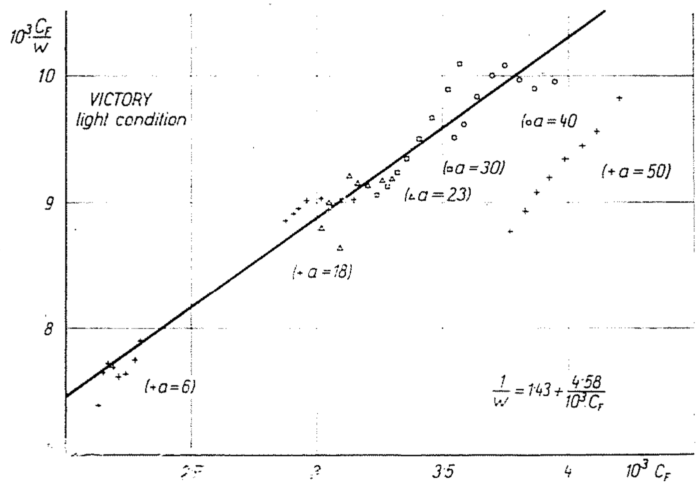

Fig. 2

The values of the constants a and b determined by the mean values of wake fractions and viscous resistance of the models

in loaded condition (at even keel)
$a = 0.77$ $\qquad b = 0.00680$

in light condition (trimmed by the stern)
$a = 1.43$ $\qquad b = 0.00458$

The most important geometrical data of the ship are the following:

			in loaded condition	in light condition
length of the waterline	L	(m)	135.562	133.177
draught (mean value)	T	(m)	8.687	6.809
wetted surface of ship	A	(m^2)	3687	3164
diameter of screw	D	(m)	5.3	5.3
disc area of screw	A_0	(m^2)	22.05	22.05
relations:	$\frac{A}{A_0}$		167.1	143.4
	$\frac{L}{T}$		15.61	19.58
	$\frac{T}{D}$		1.639	1.285

We can write the values of the constants a and b with a good approximation:

$$a = 13.3 \frac{\frac{L}{T}}{\frac{T}{D}\frac{A}{A_0}}$$

$$b = 2.48 \cdot 10^{-5} \frac{T}{D}\frac{A}{A_0}.$$

In the loaded condition

$$a = 13.3 \frac{15.61}{1.639 \cdot 167.1} = 0.758 \cong 0.77$$

$$b = 2.48 \cdot 10^{-5} \cdot 1.639 \cdot 167.1 = 0.00680$$

in the light condition

$$a = 13.3 \frac{19.58}{1.285 \cdot 143.4} = 1.441 \cong 1.43$$

$$b = 2.48 \cdot 10^{-5} \cdot 1.285 \cdot 143.4 = 0.00457 \cong 0.00458$$

The calculated values of the wake fraction according to equations (7) and (8), with the above-mentioned values of a and b, are given in the table

$$w = \frac{c_F}{a \cdot c_F + b}$$

The measured values of the wake fractions and the difference of the measured and calculated wake fractions in the percentage of the measured values are also given.

The differences (the errors) are the following in the loaded condition:

in 33.3% of the cases the errors are below $\pm 1\%$
in 29.2% of the cases the errors are between $\pm 1-2\%$
in 16.7% of the cases the errors are between $\pm 2-3\%$
in 20.8% of the cases the errors are over $\pm 3\%$

The mean value of the errors is 2.1%. If we disregard the extremely high errors, the mean value is 1.3%. The errors are about 10% only at the model made in scale 40. We can assume that there is some error of measurement in the consequence of the low Reynolds number (lg Re = 6.2—6.5).

In the light condition of the ship the errors are the following:

in 33.3% of the cases the errors are below $\pm 1\%$
in 31.3% of the cases the errors are between $\pm 1-2\%$
in 12.5% of the cases the errors are between $\pm 2-3\%$
in 22.9% of the cases the errors are over $\pm 3\%$

The mean value of errors is 2.7%, but apart from the extremely high errors, the mean value is only 1.1%. The errors are about 10% only at the model made in scale 50.

The results of the investigation of the rough model made in scale 6 are not given in the mentioned paper [1]. Therefore it would not be possible to control this method in the field of the higher roughness of this motorboat ($c_F = 5-5.5 \cdot 10^{-3}$).

If we consider the extremely high errors of the mentioned models, we can obtain the following conclusion according to the investigations of the two Victory families at 10—17 knots shipspeed:

1. We can write the effective wake fraction of a ship as the function of the viscous resistance coefficient, with a good approximation (see equations (7) and (8)):

$$\frac{1}{w} = a + \frac{b}{c_F} \tag{9}$$

2. The gravitational component of the wake fraction is negligible, because the differences of the measured and calculated values of the wake fraction are about 2—3% at different shipspeeds.

3. The mentioned low values of errors prove that the potential component of wake fraction gives very low differences at different Reynolds numbers and different roughnesses.

4. But the presence of the potential and gravitational component is demonstrated beside the frictional component. In Figs 1 and 2 we can see that the line determined by the measured points of one of the models has a bigger slope than the line of the $p = f(c_F)$. We cannot obtain more exact information in this question. The results of further investigations are doubtful, if we take into consideration that the differences of the values of effective wake fractions are very low at different shipspeeds (we can assume that the errors of the test measurement have the same values), and the propeller has a different Reynolds number in open water and behind-condition at the tests.

5. If we make the model experiments of any ship with two models in different sizes or with one model but with two different roughnesses and we calculate the values of

$$p = \frac{c_F}{w}$$

in the two mentioned cases by the mean values of the measured wake fractions and viscous resistance coefficient, we can obtain the straight of $p = f(c_F)$. The effective wake fraction is determinable with this extrapolator, for the different roughenesses of the ship. If we investigate one model with two different roughnesses, we could obtain the tests of the rougher model instead of the usual overload tests.

Summary

The scale effect of the different measured data of ships were investigated by means of the results of geometrically similar models (geosim). The values of the wake fraction of models made at different scales are very different. But it is possible to write the wake fraction as a simple function of the viscous resistance coefficient with a good approximation. The wake fraction of the ship is determinable without scale effect by means of this function from the measured data of two models of ships. The method gives a possibility for the determination of the wake fraction of ship with different roughness, too.

References

1. LAP, A. J. W. and VAN MANEN, J. D.: Scale effect experiments on Victory ships and models (Parts III and IV). Transaction of the Royal Inst. of Naval Architects, 1962.
2. GROTHEUS-SPORK, H.: On geosim tests for the research vessel Meteor and a tanker. Transaction of the Inst. of Marine Engineers, 1965.
3. BALOGH, B.—VIKÁR, T.: Hajók elmélete. Akadémiai Kiadó, 1955.

Zoltán BENEDEK, Budapest XI., Sztoczek u. 2—4. Hungary